MARKING POINTS FOR CRANK-ROCKER LINKAGE ON THE CENTERPOINT CURVE

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The theoretically possible four bar linkages achieving the prescribed four positions of the coupler plane can be determined by means of the centerpoint and circlepoint curves. Using these Burmester curves by synthesis, it appears that some of the solutions fulfil the prescribed geometrical conditions theoretically but not constructionally. Namely, these mechanisms — although mountable in the four prescribed positions — either cannot move continuously between the four prescribed positions or the movement is continuous but the order of the positions is wrong. After the first unsuccessful trial, new frames have to be chosen until the constructionally correct solution [1]. But in some cases there is no solution to fulfil the conditions and therefore all attempts could get frustrated.

A survey of possible solutions could help to select two points on the centerpoint curve which directly result in a working mechanism. The proposed new diagram has not yet been published in the literature. This diagram belongs to the four prescribed positions of the plane and together with the centerpoint and circlepoint curves, it characterizes all the solutions without marking a definite rotation center.

This method may be applied before computation and by limiting the number of possibilities, it may considerably reduce the running time. It is useful either as self-contained graphic procedure or can be built into the computer program.

Fig. 1a shows the four prescribed positions of the coupler plane C and the resulting Burmester curves. In this present case, the curves have single branches, which tend to infinity, but the method lends itself for two disconnected branches, one closed and the other tending to infinity. To graphically determine or calculate the centerpoint curve is a tedious procedure [2—6]. For given opposite pole quadrilateral it is possible and helpful to prepare a sketch of the centerpoint curve to relieve the uncertainty of the order of connecting the available points and to indicate how to change the basic data as necessary. [7—9]. To every revolute centerpoint on the centerpoint curve belongs a link R and the angles $\Phi_{12}\Phi_{13}\Phi_{14}$ (Fig. 1b). Wether a link belonging to an arbitrarily

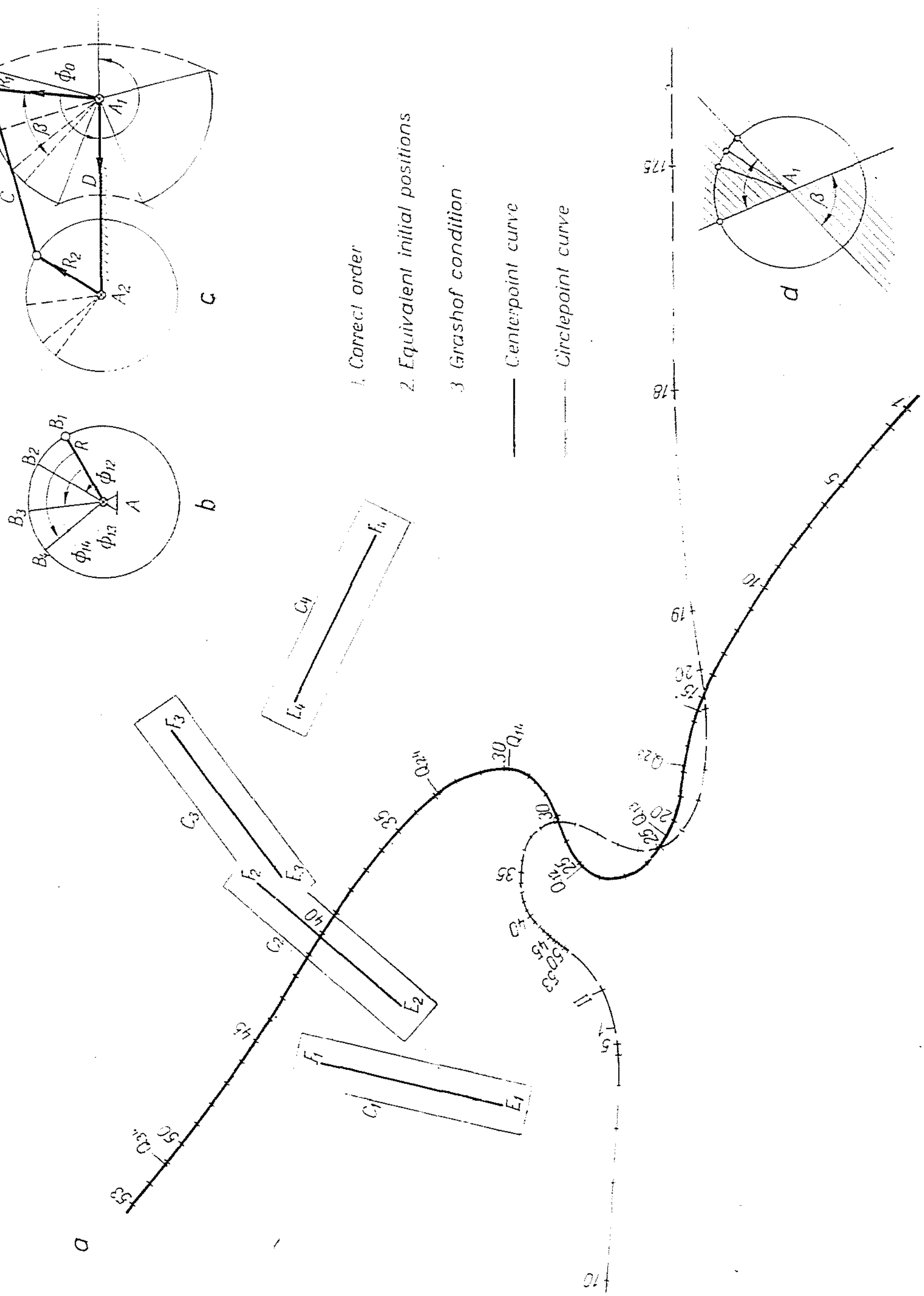
1. Correct order
2. Equivalent initial positions
3. Grashof condition
Centerpoint curve
Circlepoint curve
a
b
c
d

Fig. 1

chosen revolute center, will be a crank or a rocker can only be determined after having choosen the second revolute center of the four bar linkage. But it is immediately evident — without marking any revolute center — what are the parts of the centerpoint curve with points unsuitable for a crank because the order of the positions would be wrong.

Two arbitrary revolute centers will determine all dimensions of the desired linkage (fig. 1*c*). The Grashof condition can be checked with dimensions R1, R2, C and D.

The movement of plane C, starting from one of the possible initial positions, is a geometrically periodical movement. All the possible equivalent initial positions which produce the same movement of the plane C are all the positions of one possible periodical movement of the plane C. In the Burmester synthesis problem it is primordial that all the arbitrarily chosen four prescribed position should be equivalent initial positions. All the initial positions are equivalent if the Grashof conditions, relating to the existence of a crank in the four bar linkage, is not satisfied. In these cases the linkage is a double rocker. If this type of solution is suitable the problem is much simplified. Else there are two different periodical movements and the linkage can be either a double rocker, a double crank or crank and rocker type. In many cases a crank and rocker type solution is desired and as not all possible initial positions are equivalent, further checking is needed.

The problem is: how to find a crank and rocker which will work well in practice?

Three conditions need to be fulfilled:

— The crank must go over the four prescibed positions in the right order;

— All the four prescribed positions must be equivalent initial positions. This means that all the positions of the rocker must be either above the frame or below it (Fig. 1*c*);

— The Grashof condition is satisfied and the crank must be the shortest link.

Can the first revolute center designated so that the pertaining link is a crank?

Fig. 2 is a graph of Φ_{12}, Φ_{13}, Φ_{14} and R as a function of the arch length of the centerpoint curve. It appears that that part of the centerpoint curve will be suitable as a revolute center for operational cranks where either $\Phi_{12} < < \Phi_{13} < \Phi_{14}$ (anti-clockwise rotation) or $\Phi_{12} > < \Phi_{13} > < \Phi_{14}$ (clockwise rotation). Functions appear to cross each other only at intersections Q of the opposite sides of the opposite pole quadrilaterals. Points Q divide the centerpoint curve into six parts. It is clear without determining the functions Φ, that if there is one point between two points Q where the order Φ is good, all points of this section will be good, and vice versa. Two bad sections can be next to each other, but two good ones never.

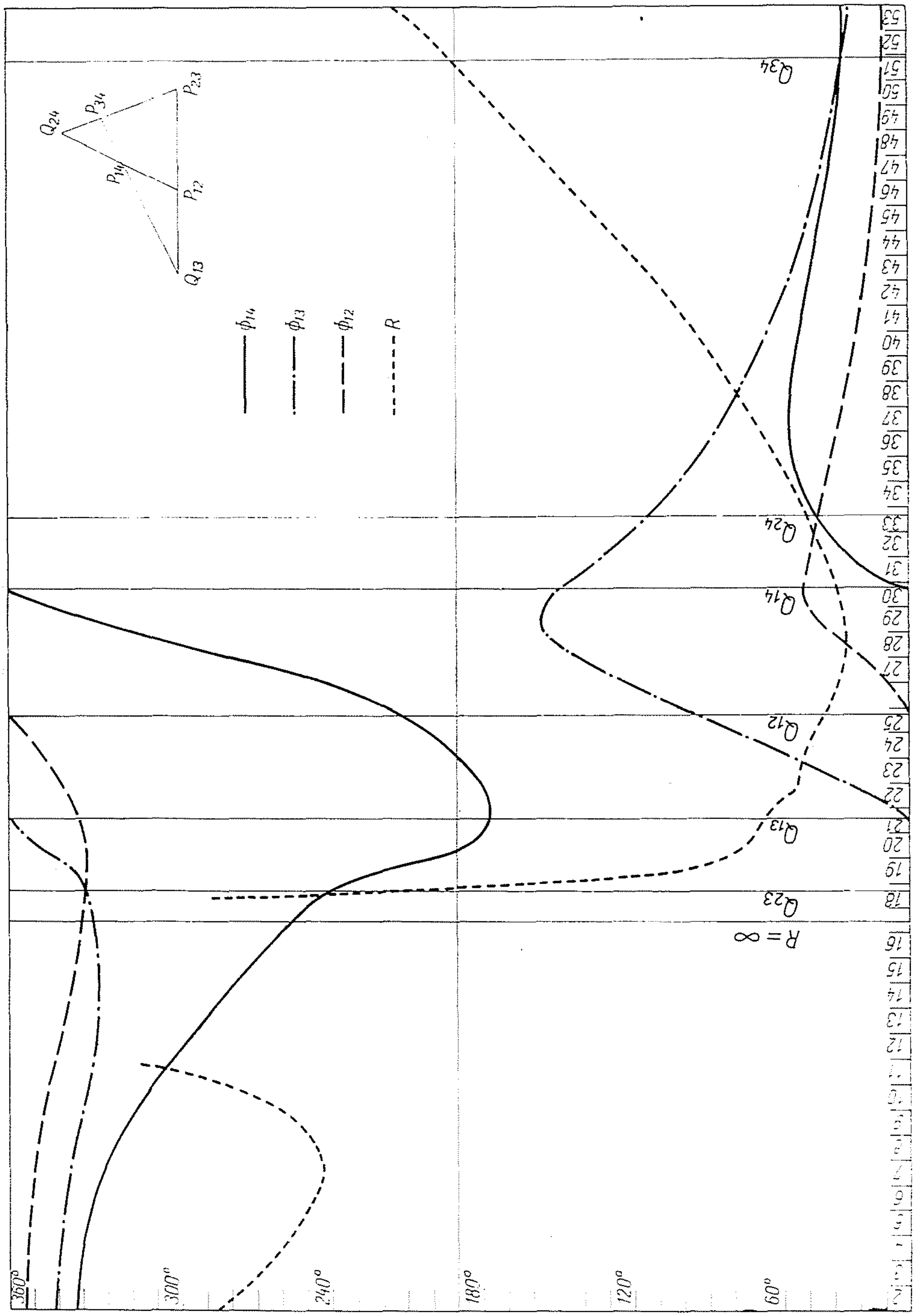
Q_{24}
P_{34}
P_{23}
P_{14}
P_{12}
Q_{13}
ϕ_{14}
ϕ_{13}
ϕ_{12}
R
Q_{34}
Q_{24}
Q_{14}
Q_{12}
Q_{13}
Q_{23}
$R = \infty$
360°
300°
240°
180°
120°
60°

Fig. 2

There will be no rocker of crank and rocker linkage along that section of the centerpoint curve, where the graph Φ is partly below and partly above the line of 180°. If all the graphs are above the line of 180° (and $\Phi_{12} < \Phi_{13} < \Phi_{14}$) the rocker will turn anti-clockwise and if all the graphs Φ are below the line of 180° (and $\Phi_{12} >< \Phi_{13} >< \Phi_{14}$) the rocker will turn clockwise, between positions 2, 3 and 4.

Any point on the marked section will be the intersection point of the frame directions for an infinite number of solutions. Taking this point as a base, all the remaining points of the centerpoint curve define an angle Φ_0. Determining the graphs Φ_0 and $\Phi_0 + 180°$ the territory between them must not be crossed by graphs Φ_{12}, Φ_{13} and Φ_{14} if the four prescribed positions of the plane C are to be equivalent initial positions.

It is possible that from the included zone, it appears that there is no rocker to operate satisfactorily with the marked crank.

A new rocker could only be marked after choosing a new revolute center and plotting the new graphs Φ_0 and $\Phi_0 + 180°$, hence by trial and error method. It is easier to check a crank belonging to a given rocker. Hence it is advisable to choose the rocker first. The shaded area in Fig. 1d belonging to any arbitrary revolute center of rocker is inconvenient as a revolute center of a crank. The shaded area can be determined from the two extreme rocker positions, belonging to the four prescribed positions. Using an angle meter the whole centerpoint curve can be surveyed for marking points of the centerpoint curve suitable as revolute centers of a rocker.

As a final test, only these parts need be examined to see if they satisfy the Grashof condition. In the example, the Grashof condition is not satisfied, there is not a single crank and rocker solution, which would be feasible.

In other cases, after the final test, there might be a finite section of the centerpoint curve the points of which contain good solutions. One among them can be chosen arbitrarily or by the best possible ratio of Φ_{12}, Φ_{13} and Φ_{14} of the crank, or in accordance with a suitable optimum criterion.

The Grashof condition cannot be checked first to every arbitrary revolute center because different graphs of Grashof condition are inhibitively laboursome and inaccurate to be assumed again and again for every point of the centerpoint curve.

The whole process can be computerized. Computer plotting of R and angles Φ_{12}, Φ_{13} and Φ_{14} simplifies and increases the accuracy of the additional control.

Two revolute centers for other than crank and rocker type four bar linkages can be marked similarly.

Summary

Formerly the only solution proposed for identifying frame points, for crank-rocker linkages, was to choose arbitrarily and to proceed by the trial and error. This paper suggests the use of a new diagram demonstrating all the possibilities simultaneously, offering a solution immediately or after a far fewer trials. On the other hand, the diagram shows if there is no suitable linkage, making all trials useless.

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