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Eszter Fehér^{1,2*}, Tamás Baranyai¹

RESEARCH ARTICLE

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Abstract

A simple method is presented to carry out a retrospective analysis to examine the development of load-bearing structures. The idea is to eliminate the differences coming from technological changes (such as joints, profiles, loads) by using relative numbers to express the relation of the structures to the possible theoretical solutions under the same circumstances. The method is demonstrated by investigating the impact of historical changes focusing on metal Pratt trusses spanning about 100 ft, located in Indiana, U.S., erected between 1870 and 1937. Data of 87 structures was collected and compared to the results of a multi-objective optimisation computed using a genetic algorithm. Using the relative numbers acquired by evaluating the objective functions for the historical structures, a large time-scale optimisation process through history can be visualised. Plotting them on the Pareto-front diagram determined by the genetic algorithm and examining the historical background of the state revealed that the economic and industrial changes, in fact, had a considerable impact on the design trends, which manifests in changes of the weights of the objective functions.

Keywords

truss evolution, Pratt truss, Pareto-front, truss bridge

1 Introduction

There is a huge interest for the topology optimisation of load-bearing structures from mathematical, computational and engineering aspects (Lógó, 2005; Ezzat, 2016; Rozvany, 2014). In the case of load-bearing structures, the commonly used objective functions to be minimised are the weight of the structure, maximum deformation, and maximum internal forces or stresses. Although these are indeed significant factors in engineering practice, they are not of equal importance, meaning that these functions are weighted depending on the design conditions in real design situations.

One of the widely used optimisation techniques is the Genetic Algorithm (GA). It is an evolutionary algorithm and considered to be analogous to natural selection, the optimisation process of our nature. While this connection is straightforward for biological changes, an interesting question arises considering the development of engineering structures: are load-bearing structures developed in a similar evolutionary manner through history due to the advancement of technology? What makes the problem complex is that although technological advances might have produced better-optimised structures, there were several factors having an impact on the weights of the objective functions.

We aim to compare data of historic structures to the Pareto-front: a curve in parameter-space representing equally fit solutions computed by a multi-objective optimisation algorithm. To make the bridges from different years, comparable, objective functions were defined to take relative values comparing the structures only to the theoretically possible arrangements under the same circumstances. Considering the historical data as a result of a large time-scale optimisation process carried out by engineers, both the development and the changes in the importance of the objective functions can be visualised. The historical background of the structures is also considered to account for the changes in their design philosophy. A reverse analysis can be performed where the weights of the objective functions are unknown.

To carry out such an investigation, a load-bearing structure is needed that was built in large numbers for several years in various configurations but with a small number of changing

¹ Department of Mechanics, Materials and Structures, Faculty of Architecture, Budapest University of Technology and Economics, H-1111, Budapest, Műegyetem rkp. 3., Hungary

² Morphodynamics of Solids Research Group, Hungarian Academy of Sciences – Budapest University of Technology and Economics, H-1051, Budapest, Nádor u. 7., Hungary

* Corresponding author, e-mail: fehereszter@szt.bme.hu

parameters. One such structure is a truss with fixed topology and span length, where the changing parameters are the number of nodes and the height of the structure (Fig. 1). To be able to demonstrate the effect of technological advances and historical changes, the location of the structures should also be restricted to a well-developed area, where state of the art technology was accessible to the engineers. To provide the same historical, economic and climate conditions, we examined simply supported Pratt metal bridge trusses spanning about 100 ft, erected between 1870 and 1937 in Indiana State, U.S. Here bridge fabrication was based on scientific background and aimed to produce cost-effective structures (Cooper, 1987) in contrast to some of the other parts of the U.S. (Parsons Brinckerhoff and Engineering and Industrial Heritage, 2005, Hufstetter, 2014).

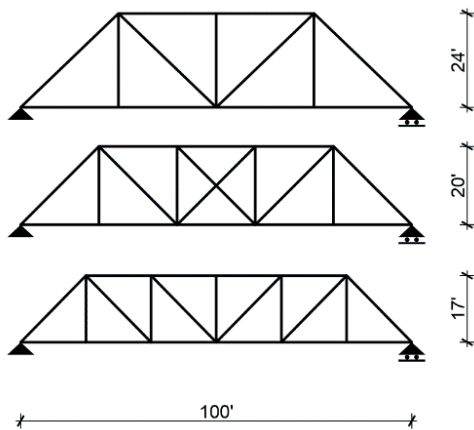


Fig. 1 100 ft Pratt trusses having different geometry.

The development of historical trusses was strongly influenced by the advancement of technology and the continuous economic changes. A comprehensive qualitative analysis of the scientific impact on the design concept and truss topology in Western Europe is carried out in (Rinke, 2010) and there are also works examining the development of the structures designed by a specific engineer (Shotton, 2015). Paik computed theoretical Pareto-fronts with a multi-objective optimisation carried out on unstructured domains for various truss topologies used in the engineering practice and without further historical investigation a hierarchy of topologies was determined (Paik, 2005). However, limiting the examination to a fixed topology eliminates the differences coming from conceptual choices and sheds light on the impacts of external factors. That is different solutions can be constructed by varying the geometry of the structure (Fig. 1). These alternatives have different properties regarding their load-bearing capacity, economic and aesthetic aspects. Therefore, taking not only the topologies but also the exact geometry into consideration can provide a deeper insight into the effects of historical changes. Using relative numbers expressing the relation between the realised arrangements and the theoretically possible solutions, our method enables us to compare structures built with different technologies and in different ages.

In the following sections, firstly, the historical background of trusses in the U.S. is summarised, then a short introduction is given about structural optimisation techniques used in case of trusses focusing in detail on the genetic algorithm. Finally, after the description of our implementation of the method, the results and consequences are presented.

2 Development of trusses

2.1 U.S.

Trusses are intuitive, ancient load-bearing structures originating from wooden roofs. In the United States, the first wooden truss bridge was designed by Timothy Palmer in 1792. The development of the truss bridges was influenced by several factors such as the Civil War, the industrial revolution, the foundation of engineering academies, technological advancements and the appearances of bridge companies (Parsons Brinckerhoff and Engineering and Industrial Heritage, 2005).

There were huge differences between the states. The extent of governmental involvement, the academic background of the bridge designers and the involvement of locals (e.g. carpenters) show great variety. Although from the late 18th century, self-taught engineers were gradually replaced by educated engineers, the poor academic background of the employed engineers was a significant problem in the country until the mid-20th century. In 1905, the president of the American Society of Civil Engineers stated, that “bridges are frequently designed by incompetent or unscrupulous men, and the contracts are awarded by ignorant county officials, without the advice of competent engineers” (Parsons Brinckerhoff and Engineering and Industrial Heritage, 2005; Hufstetter, 2014).

2.2 Materials

Although metal could be processed far long before the first trusses appeared, in the beginning, due to financial issues and the time-consuming production of metal elements, wooden trusses dominated the industry. However, after the industrial revolution, metal structures became cheap enough to make mass production possible. Metal can be characterised as cast iron, steel and wrought iron. The list is ordered according to decreasing carbon content. Carbon makes the metal brittle and rigid while bridge elements require tensile strength and resistance against vibrations. On the other hand, carbon gives hardness to the metal, which is needed to resist abrasion and greater loads. Due to this combination of requirements, steel became the ideal material for bridge building. Also, wrought iron and steel can be welded, which can greatly simplify the construction process.

Since cast iron was the first metal available in industrial quantities, the shape of the first non-wooden structures was determined by the properties of cast iron. Although it has negligible tensile strength due to its conductivity of cracks, it can bear compressive stress well, which made it perfect for truss elements subjected to compressive forces. Being a prolific production technique,

trusses made completely of cast iron appeared. Even though the first iron bridge called “The iron bridge” in Coalbrookdale is still standing, use of the material led to many accidents, due to the proneness to brittle failure referred to earlier.

An efficient method to produce wrought iron was introduced in the US in the mid-nineteenth century, making it a metal of choice for bridge building, until steel became available.

Steel could be manufactured in great quantities relatively cheaply after the perfection of the Siemens-Martin process. This happened in the US in the 1870s and 1880s, with the country becoming the largest steel producing country in 1889. Consequently, steel was readily available and was preferably used for bridge construction over other metals.

2.3 Topologies

Investigating the topology of historical trusses, there is a wide variety of design concepts developed by engineers and companies each having very special structural behaviour (Haupt, 1867; Parsons Brinckerhoff and Engineering and Industrial Heritage, 2005; Calvert, 2000). The US patent system provided a framework for engineers and entrepreneurs to share their ideas with the public in a regulated and protected way. Many of the early bridge types are named after people patenting certain geometries and topologies while providing tools to compute the forces.

Three main structures that can be considered as a basis for other statically determinate concepts are demonstrated in (Fig. 2). William Howe patented the Howe truss in 1840. It was the first statically determinate concept having no superfluous elements. The vertical rods bearing compressive forces were originally made of iron, while the other elements were wooden. The Pratt truss developed by Thomas and Caleb Pratt in 1844 has a similar structure, but the direction of the diagonals causes tension in the vertical rods. In the United States the most common, standard bridge form was the Pratt truss for 35 years until the Warren truss took over its role. James Warren and Theobald Monzani patented the Warren truss, which is composed of equilateral triangles, therefore compressive and tensile elements alternate. The diagonals of equal length make this concept economic and ideal for mass production or prefabrication. At first, these trusses were also made of wood since the search for the appropriate form for iron bridges was in its very early stages.

The area of development changed as the technology enabled the economic production of wrought iron and later steel. The emphasis shifted from creating new truss types to the effective production of parts and using the existing bridge types efficiently.

2.4 Indiana

The state was part of the then industrial heartland of the US, with numerous ironworks and later steel mills, as well as automobile factories. This resulted in the availability of quality metal products, state-of-the-art machinery and a need to construct good infrastructure, on which cars and lorries could run.

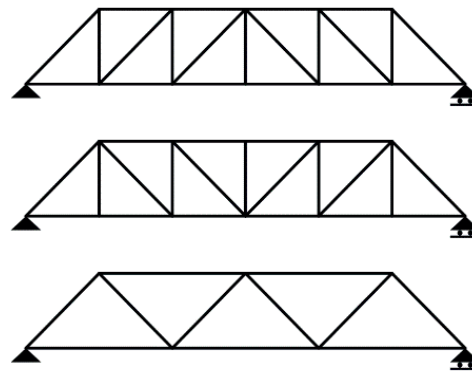


Fig. 2 Howe truss (top), Pratt truss (middle), Warren truss (bottom)

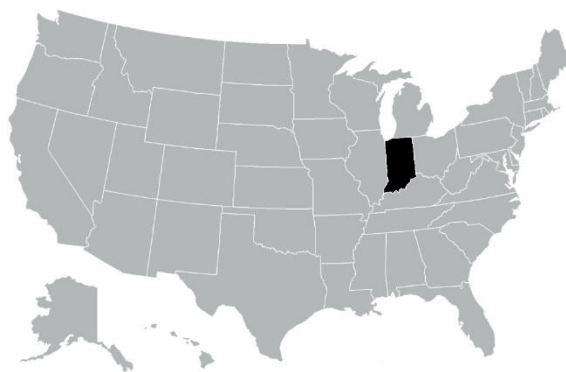


Fig. 3 Indiana State (source: Wikimedia Commons)

According to (Mead and Hunt, 2007), the history of truss engineering in Indiana can be differentiated into two distinct periods. During the first period, the bridge companies were protected by the US patent system. However, Indiana companies, in general, submitted fewer patents and concentrated on the fabrication of efficient bridges and the improvement of manufacturing techniques (e.g. Lafayette Bridge Company). The second period can be characterised as the time of consolidation and standardisation with increasing governmental control. In Indiana, the governmental influence on the bridge designs was significant: more than 70 percent of the bridges were built under the jurisdiction of counties.

The technological development of the state, the aims and design behaviours of the companies and the significant governmental control make Indiana an appropriate candidate to examine the evolution of trusses.

3 Structural optimisation, GA

The topology optimisation of load-bearing structures gained interest in the last century, and it has many branches (Rozvany, 2009). The purpose of this technique is to minimise an objective function assigned to the structure subjected to some constraints. There are several analytical approaches (Michell, 1904; Hemp, 1973), but after the 1980s, numerical methods and algorithms were actively researched and applied (Bendsoe and Kikuchi, 1988; Rozvany, 2014;

Maraveas et al., 2014). Most general problems involve material distribution in a design space with given loads and boundary conditions. The design space can be analysed using the finite element method (Rozvany, 2001) or structure-specific methods with the optimisation process carried out by several techniques such as topological-derivatives, level-sets or genetic algorithm.

Genetic algorithm is an evolutionary algorithm inspired by the processes in biological evolution. The population is composed of a set of candidate solutions having varying values of the design parameters. A new generation is created from the previous population in each iteration step. The initial population consists of randomly generated individuals. The quality of the solutions is represented by their fitness values computed from the objective functions and evaluated for each member of the population in every iteration step. The next generation is generated through crossover and mutation processes from the previous generation. Similar to natural selection, the more fit individuals have a higher probability to be reproduced.

In the case of a multi-objective algorithm, the iteration does not converge to a single solution but to a set of solutions called the Pareto optimal solutions. It consists of individuals having fitness values from which none can be improved without degrading another fitness value. From the mathematical point of view, solutions on the so-called Pareto-front are considered to be equally fit. However, in engineering practice, additional external objective or subjective factors influence the final geometry.

4 Implementation

Data of historical structures was collected, and a simplified mechanical model of the structure and the loads was established. We determined the parameterisation of the structures relying on the available data and created relative objective functions depending on the parameters. These functions were used as fitness functions in the genetic algorithm and were also evaluated for the historical data.

4.1 Historical data

The analysed examples were taken from an online database 'Bridgehunter.com', which is an extensive collection of historical bridge trusses of the U. S. (<https://bridgehunter.com>). According to the website statistics, data of 1,753 truss including 389 through Pratt truss bridges from Indiana are presented in the database. For some examples see figures Fig 4., Fig 5. and Fig. 6. The following properties of the structures were collected: topology, the number of nodes, span length, height and the year of erection. This information clearly characterises the truss concept and geometry. The data show a wide dispersion: while Fig. 4 and Fig. 5 have the same number of internal nodes, their span length differ; the bridges in Fig. 5 and Fig. 6 have close span lengths, but they have a different number of internal nodes.



Fig. 4 Lost Bridge (142 ft), Dearborn County Indiana, 1916
(Photograph by Anthony Dillon)



Fig. 5 Mill Creek Bridge (100,7 ft), Crawford County, Indiana, 1885
(Historic American Engineering Record)



Fig. 6 Hibbs Ford Bridge (89,9 ft), Putnam County, Indiana, 1906.
(Photograph by Evan Dillon)

To make the examples comparable, an examination set is formed by 87 structures having almost the same span. To eliminate the small differences in the span length (allowing $\pm 15\%$) the exact spans and heights are rescaled to an average value (100 ft).

4.2 Mechanical model

The theoretical model consists of pin-jointed rods with uniform cross section (A) and Young-modulus (E). The truss is simply supported on its two ends. Supports were assumed to have a finite but much greater stiffness than the rods. Both deflection and the internal forces were computed with the displacement method.

Choice of the loads is a crucial point of the analysis since it determines the deformation and the internal forces of the structure. Since all our historical examples are road bridges, the common role implies the common distribution of live loads. Their exact magnitude is irrelevant for our purposes; it would only act as a scaling factor. Another question is the role of the self-weight. Taking into consideration that the differences between the weights of the different configurations are negligible compared to the number of external loads, the self-weight has been neglected, and the same load has been assumed for each structure. This is a fixed amount of static, vertical loading (T) applied evenly on the lower nodes (Fig. 7).

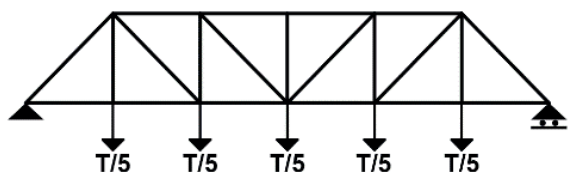


Fig. 7 The basic concept of loads, T denoting the total load of the truss.

4.3 Objective functions

The choice of a suitable objective function describing the (economic) cost of the structure is a complex question. In general, the cost is connected to the amount of material used in the structure. The accessible information from the photographs is limited, in particular, the practical cross sections of the rods are hard if not impossible to reconstruct. On the other hand, since Indiana is a well-developed state, it can be assumed that the builders had access to economical building materials. Therefore, the design concept determined the final weight. This explains our choice to assume uniform cross-sections in the theoretical model. With this assumption, we defined as economic objective function the relative weight of the truss, that is

$$w_{rel} = \sum_i l_i / L, \quad (1)$$

where l_i is the length of the i -th rod and L is the span of the truss. (The summation is on the rods of the structure.)

The most common non-economic objective functions are displacements and forces. Since they are related through the stiffness of the truss and the Pareto front requires independent functions, using one of them is sufficient. We chose the displacements and constructed a maximal relative deflection function. Denoting the vertical displacement of node k with u_k it can be given as:

$$u_{rel} = \sup \{u_k\} EA/TL \quad (2)$$

where the supremum is taken on the nodes of the truss.

4.4 Pareto-front

The population of the GA consists of 100 ft long Pratt trusses. The design variables of the candidate solutions are the number of internal nodes and the height of the truss. These parameters can be accurately determined from the photographs and show enough dispersion to detect changes.

The topology type and the position of the supports are fixed. The two design variables define a truss that can be explicitly constructed by the algorithm. The number of internal nodes is an integer between 3 and 10, and the height is between 10 ft and 60 ft. The initial population consists of 500 individuals. Fig. 8 represents the Pareto-front computed by the algorithm. The jumps on the diagram are caused by the fact that the number of internal nodes can have only discrete values. It can be seen, that the maximal relative deflection and the maximal relative internal force would give similar Pareto-fronts with respect to relative weight.

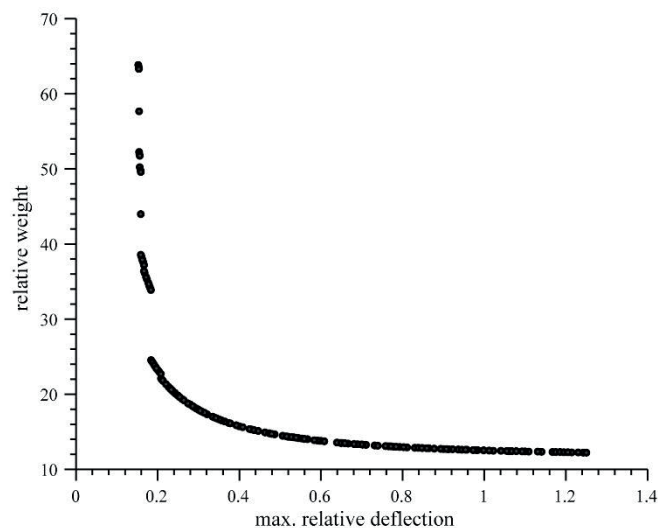


Fig. 8 Pareto-front using the max. relative deflection as the fitness function

5 Results and discussion

Having the Pareto-front in hand, we were interested in the fitness values of the historical trusses. Fitness values were computed by the evaluation of the objective functions for the collected data, then the values were plotted on the Pareto-front diagram to examine the optimisation process of human

engineering, and whether the examples approximated the Pareto-front. Finally, the historical data was investigated in time to detect changes in the weights of the objective functions.

5.1 Comparison to the Pareto-front

The early structures have dispersed fitness values, while late structures are much closer to each other and the Pareto-front (Fig. 9). The diagram shows that in the beginning, structures had higher reserve: heavier structures with smaller deflection were constructed. Later, structures were lighter and had larger deflections. These changes in the design manner can be attributed to the appearance of steel, better material quality and higher standards of construction.

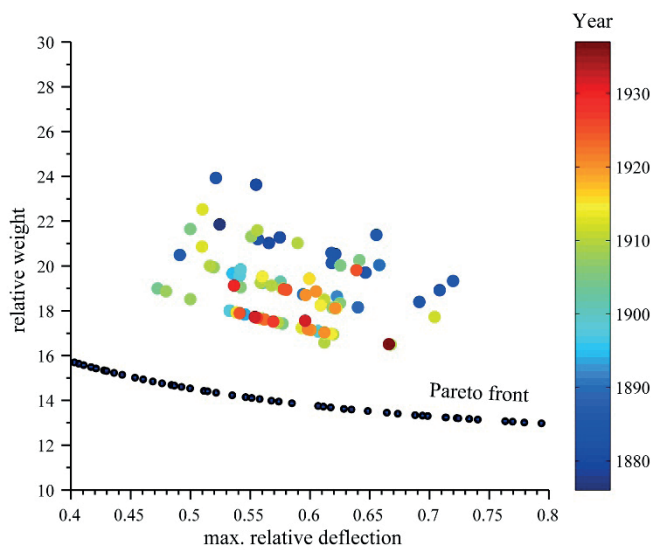


Fig. 9 Fitness values of historical bridges compared to the Pareto-front

5.2 Chronological analysis

To examine the changes chronologically, the fitness values were also plotted against time (Fig. 10). The data show three distinct behaviours in three distinct time periods. They are analysed below.

Period I: Until the early 1900s, both the cost of the structures and the maximal deflection decreased. This period coincides with wrought iron being the dominant bridge building material, from which smaller local companies built increasingly efficient bridges.

Period II: From around 1900 to 1919 the solutions show high dispersion: cheap structures with high deflection and expensive structures with low deflection were also built. According to Parsons Brinckerhoff and Engineering and Industrial Heritage “During the first 20 years of the century, bridge engineering was in an experimental stage, resulting at times in bridges that were over-engineered”, which coincides with our findings. Among the reasons behind this experimental approach are the change of material and the change of company sizes. The years 1890-1910 was the time of transition from wrought iron to steel bridges, and very few companies continued to function both

before and after it. As the market for iron bridges vanished, the majority of older companies either closed or were bought (and converted to build steel bridges).

New companies were also created to fill the gap in the widening market of steel bridges. This resulted in larger companies operating in a liberal market environment. Since mass production was dominant in the production of the bars of trusses; apart from structural efficiency, financial efficiency also played a role. Note, that, at the beginning of the period, the dispersion grows, while at the end it decreases.

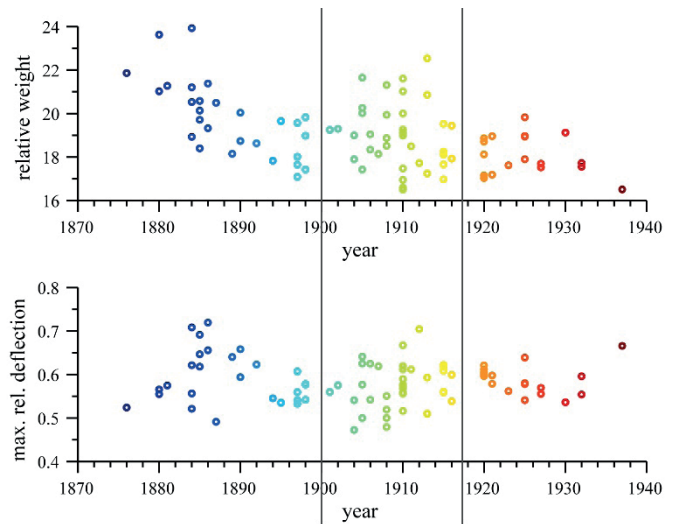


Fig. 10 Fitness values plotted against the completion year of the given bridge

Period III: From 1919 onwards. Due to the appearance of cars around 1914, the federal government of the US began the creation of a nationwide, standardised road system supporting motorisation. Consequently, in 1917, the Indiana State Highway Commission was created to fund bridge projects fulfilling the federal norms. A continuation of the law passed in 1919, further increased government participation and regulation. As these laws set a clear aim, bridges began to adapt to it: they get cheaper while allowing larger deflections.

6 Summary

This paper has demonstrated a method, which determines relative numbers for built structures expressing their relation to the theoretically possible solutions under the same circumstances, making it possible to carry out a comparative analysis of structures through history. The framework of structural optimisation provides a great tool to carry out such a comprehensive examination. In our work, we were able to visualise how engineers optimised Pratt trusses in Indiana from 1870 to 1937. It is important to emphasise that the development was observed at the very conceptual stage regardless of the available materials, technologies or construction techniques. Subsequently, the paper shows the scientific progress behind the design of these structures. It also illustrates how non-engineering considerations such as economic and legal changes, along with

governmental involvement affect how objective functions are weighted in real situations.

Our investigation confirms the words of James L. Cooper about Indiana bridge trusses (Cooper, 1987): “Even as the price of iron and steel dropped, metal remained too expensive to waste on unnecessary or unnecessarily heavy members. (...) Over time, scientific bridge fabrication produced cheaper and sounder structures, a happy coincidence of private and public interest.”

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