

Structural Performance of Topologically Interlocked Flat Vaults According to Joint Details

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Abstract

Although the notion of topological interlocking (TI) is not new, today's environment of parametric design tools encourages architects and engineers to reconsider the potential of TI systems and alternative geometries for TI blocks. This paper aims to generate alternatives for TI blocks by increasing the contact surface area of interlocking using *X*-joints to demonstrate the relationship between contact surface incrementation and structural performance of slab-like TI assemblies. A case study is presented in which two flat vaults with dimensions of 50 × 50 cm were designed using truncated octahedra and a new geometry based on *X*-joints added to the truncated octahedra. Each flat vault was subjected to a finite element analysis using SimScale to compare structural performances using displacement and von Mises stress maps. According to the findings, there was a 74.7% improvement in displacement and 76.8% reduction in von Mises stresses. Another analysis was conducted over a seven-element system by applying force to the centre element to measure the results on a single element. Results indicated a 41.7% improvement in displacement and a 39% reduction in von Mises stresses. The improved structural performance was demonstrated through these two-stage evaluations.

Keywords

topological interlocking assemblies, platonic solids, parametrical and computational design, contact surface area

1 Introduction

Traditional masonry construction is a structured system that small-sized elements put in a particular order to create surfaces with structural qualities. As Jacques Heyman (1995) clarifies, the fundamental knowledge of masonry is found in the correct comprehension of geometry. Such structural systems, in which the form directly determines the structural behaviour, have traditionally been a focus of research at the intersection of architecture and engineering. Stereotomy is the art and the technique of cutting three dimensional solids, especially the stone and it is the base of the geometric relationships that are inherent in masonry (Vella and Kotnik, 2017). The masters of stereotomy have created modular blocks that provide uniform load transmission in interlocking systems, which raises the concept of topological interlocking. The advent of advanced design and fabrication tools that can easily solve complex geometries has resulted in a renewed interest in stereotomy and topological interlocking in recent years (Fallacara, 2012; 2016; Fallacara et al., 2019; Moreno Gata et al., 2019; Weir et al., 2016).

Previous studies defined topological interlocking (TI) as a design concept in which special-shaped elements (blocks) are organized in such a way that the entire structure can be held together by a global peripheral constraint, while the elements are held in place locally by kinematic constraints imposed by the element's form and mutual arrangement. No glue or mortar is required for TI assemblies, since the sheer geometry and relative locations of the blocks support them within the structure (Lecci et al., 2021; Pfeiffer et al., 2020). Kanel-Belov et al. (2009) proposed the following theoretical definition of "topological interlocking": "Interlocking is achieved if in every row of elements one can identify two sections normal to the assembly plane such that while one section ensures kinematic constraint in one direction (normal to the assembly plane), the other section provides the same elements with constraint in the opposite direction" (cited from Kanel-Belov et al. (2008) in the paper of Weizmann et al. (2015:p.110)).

The application of a primitive and relatively simplistic concept of interlocking can be traced back to the

construction of the arch in the 2nd century BC. As one of the oldest and simplest techniques of utilization, topological interlocking applications were discovered in Inca structures for building stable, self-aligning structures, without using mortar or additional joints.

The theory of reciprocal structures was introduced during the early Renaissance, and was studied by many scientists, including Leonardo da Vinci. A reciprocal structure uses tension-compression to lock its parts and transmit loads. Topological interlocking systems can be described as three-dimensional reciprocal arrangements in which the blocks receive moments when one pushes against another (Moreno Gata et al., 2019). In 1699, Joseph Abeille invented "flat vault" which can be described as slab-like assemblies formed by using tetrahedrons in a way that each element supports two neighbouring elements and limit them kinematically. The organizational design of the Abeille vault resembles the structural arrangement of reciprocal timber frame structures of Villard de Honnecout or Sebastiano Serlio in a way that both designs solve the difficulty of finding an innovative way to cover a space with a flat floor composed of small discrete elements (Vella and Kotnik, 2016). However, it is necessary to clarify here that the Abeille's vault acts structurally differently from a reciprocal frame structure since flat vaults with topological interlocking systems differ from compression-active vernacular arches, vaults, shells, and brickwork buildings because the interconnecting interfaces provide structural coherence to TI systems (Lecci et al., 2021). Therefore, his invention is accepted in the literature as the origin of the topological interlocking systems (Lecci et al., 2021; Pfeiffer et al., 2020).

As alternatives to Abeille's module, theorists started to debate on new possible geometries of TI blocks. In order to fill the voids on Abeille's vault's upper surface, Jean Truchet

created a new geometric shape for the repeating module (Lecci et al., 2021). Frézier also explored a series of alternatives to Abeille's module and amended Truchet's block by increasing the concavity of the blocks. While the block alternatives were initially theoretical searches on form, they later turned into varying surface experiments on the blocks of similar shapes. The topological interlocking modules of Abeille, Truchet and Frézier are shown in Fig. 1.

In recent years, there has been a period in which digital design has come to the fore thanks to digital design. The computational studies on masonry construction and stereotomy rely on the methods that compute discrete elements rather than continuous surfaces. Since topological interlocking systems are one of these systems consisting of discrete elements, they have found a place in current research on computational design. The main objective of this study is to discuss the possible transformation of existing interlocking geometries benefiting from digital design technologies. To that end, this paper focuses on the transformation of topological interlocking blocks to generate new TI geometries while increasing contact surface area in order to compare structural performances of similar geometries which have different joint details. The incrementation of interconnecting interfaces was provided by adding X joints. This study has been designed to investigate the following research question: *How does the incrementation of contact surface area with X -joints in a topological interlocking system affects the structural performance of the system?*

Following an introduction to the background of topological interlocking systems, this paper focuses on current research on the topic and includes a literature review. The structural performances of topological interlocking blocks with similar geometry but different contact surface areas of interlocking were compared within a case study.

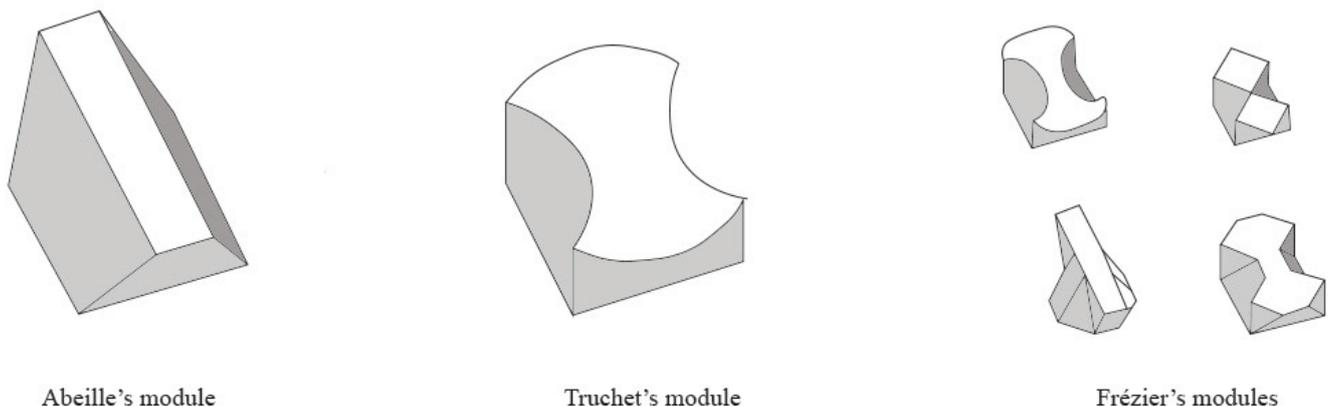


Fig. 1 Alternative geometric modules for Abeille's vault (Adapted from Lecci et al. (2021))

The Starfish plug-in for Grasshopper was used to create truncated octahedra from a hexagonal tessellation, which was then turned into a new geometry in Rhino by adding *X*-joints. Adding *X*-joints is the simplest way to increase the contact surface area of the blocks' to improve the structural performance of the flat vaults. Two flat vaults were designed from two different joint design and a finite element analysis was carried out using SimScale to compare the structural performances of blocks with different contact surface areas. While it is known in the literature that interconnected interfaces in topological interlocking systems increase the structural performance, the structural effect of increasing the contact surface area by increasing these interfaces using *X*-joints has not been tested. The effect of increasing the contact surface area, will be revealed by the FEM method and a numerical contribution will be made to the related literature.

2 Literature review

Recent studies on TI systems have focused on the topological interlocking assembly of various geometries, thanks to advancing digital technologies and fabrication tools. As a result, the design, investigation, and manufacture of identical and customized objects and their topological interlocking assemblies, have been the subject of much current research. Although the platonic solids and the osteomorphic blocks are the most common examples of topological interlocking assemblies in the literature, there are studies aimed at discovering alternative geometries of interlocking blocks.

One objective of this literature review is to list the different geometries of topological interlocking blocks that have been discovered so far and to describe the methods created for the discovery of new geometries. In Table 1, the reviewed studies are summarized by focusing on the different geometries of the topological interlocking blocks and their method of generation.

The most commonly used of these methods, developed by Glickman (1984), is *truncating the blocks* with the section planes (Fig. 2 (a) and (b)). Michael Glickman's innovation of the G-Block system was the first polyhedral assembly in contemporary studies (Fig. 2 (c)). Truncated tetrahedra are used for the first time in a modern structure for a paving system. Then, Lecci et al. (2021) used the truncation method to generate interlocking blocks from the cubes and to design a flat vault. With a section through the mid-plane of the cubes, they created a honeycomb pattern for the flat vault.

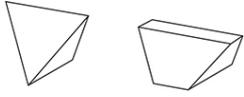
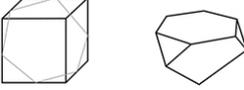
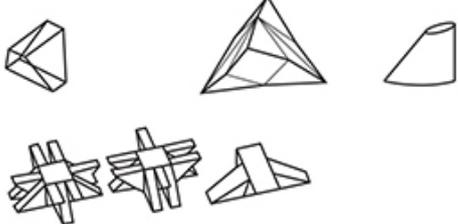
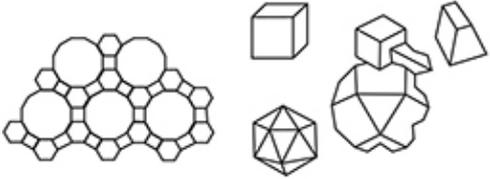
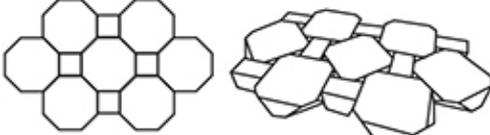
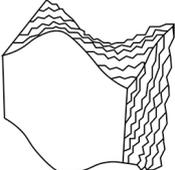
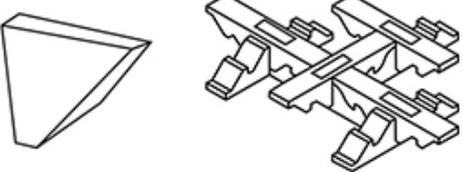
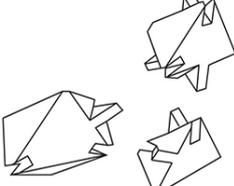
Kanel-Belov et al. (2009) developed a method called "*the moving cross-section procedure*" for generating TI blocks and TI systems. This method aims to create interlocking structures using regular lattices using the following steps (Fig. 3):

1. They use a two-coloured regular lattice as a two-dimensional grid. They placed arrows, of the same size, at the middle of each edge and considered a plane based on an edge and sloping towards to the direction of a corresponding arrow.
2. They placed a plane for each edge that had the same tilt angle. When tilted at the right angle, these planes form a tetrahedron.

Digital processes and parametric design allowed the discovery of new methods for generating topological interlocking blocks. Weizmann (2016) developed a parametric software allowing the parametric generation of various patterns and topological interlocking blocks (Starfish). According to him, this new method is divided into two stages: the first is responsible for creating 2D patterns, and the second is creating 3D blocks from the pattern's polygons. Although creating 2D patterns is a typical task for many computer-aided design-based technologies, they rely on established pattern configurations either based on a single polygon or a small number of polygon types. On the other hand, the method he suggested provides more flexibility in terms of generating alternative results. Using Starfish, Weizmann et al. (2015) also looked at topological interlocking systems as organizational methods for facades, utilizing different tetrahedra for planar and curved tessellation systems (Weizmann et al., 2015). In another research, they investigated how TI systems were employed in constructing building floors and ran numerical simulations using the finite element method (FEM) (Weizmann et al., 2017). Tessmann (2012) also benefitted from parametric design to create differentiating interlocking blocks for planar and curvilinear structures. He recently published an article on using of non-continuous logic in architecture, which included an important case study on topological interlocking (Tessmann and Rossi, 2019).

As topologically interlocking geometries were discovered, the focus of research shifted to how to transform them into new geometries. One of these methods is to change the geometry by forming a new interlocking pattern on the surface of a topologically interlocked geometry. Djumas et al. (2017) put forth the idea of hierarchical

Table 1 Topological interlocking summary table literature review

Method	Researchers	TI geometry
Truncation	Glickman (1984)	
	Lecci et al. (2021)	
Moving cross-section procedure	Kanel Belov et al. (2009)	
Parametric design	Tessmann (2012)	
	Weizmann et al. (2015)	
	Weizmann et al. (2017)	
Surface modifications	Djumas et al. (2017)	
Volumetric modifications	Weir et al. (2016)	
	Tessmann and Rossi (2019)	
	Pfeiffer et al. (2020)	

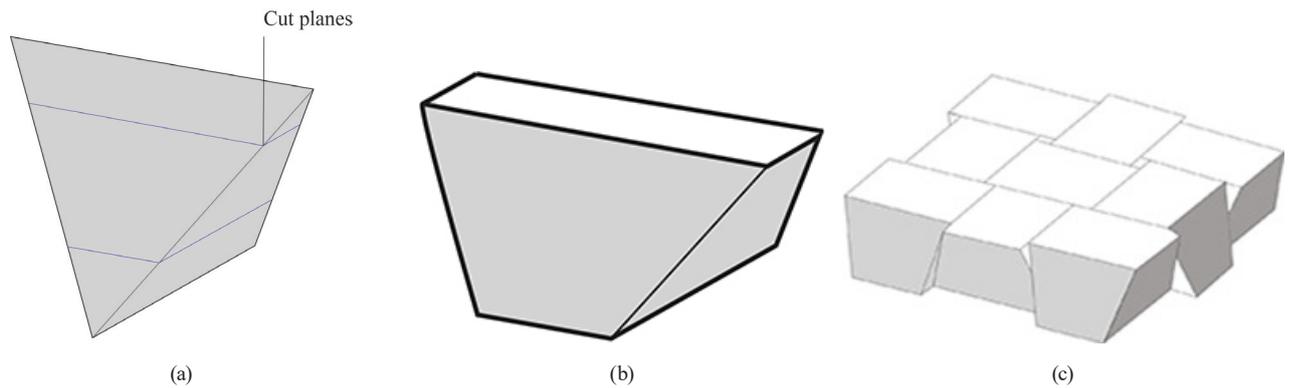


Fig. 2 (a) Regular tetrahedron including truncation planes outlines, (b) The G-Block solid within the tetrahedron, (c) Interlocking principle of the G-Block system (Adapted from Lecci et al. (2021))

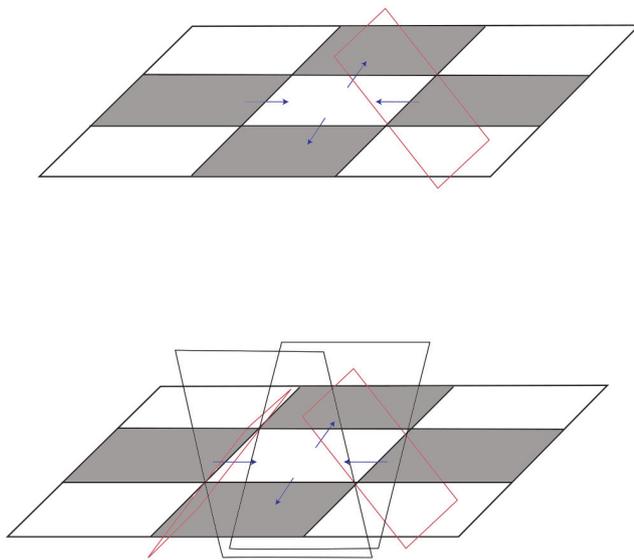


Fig. 3 Stages of "the moving cross-section procedure" (Adapted from Kanel-Belov et al. (2009))

structures for the topological interlocking assemblies. In the study, a secondary interlocking pattern was introduced at the surface of an osteomorphic block considered as the reference element. Introducing the secondary surface that they called as "interacting surfaces" increases the contact surface area of the interlocking blocks. The incrementation of contact surface area with surface patterning prevents the interlocking block from sliding under a concentrated load (Djumas et al., 2017).

The studies generated new geometries with significant volumetric adjustments besides surface-level modifications in topological interconnecting blocks. By integrating wave joints into osteomorphic blocks, a new geometry was designed and produced with robotic fabrication in research conducted by Weir et al. (2016). Again, as a *volumetric change* on the interlocking blocks, *X*-joints have been used in a few studies to achieve new geometries of

TI blocks. In previous studies, Philipp Mecke employed *X*-joints to increase the interlocking directions for edge modules. Blocks can constrain each other in more directions with the incrementation of interlocking directions (Tessman, 2012). In experimental research, topological interlocking blocks with *X* joints were used as edge modules to absorb lateral thrusts. Besides, *X*-joint concept edge modules were used to address the concept of removable structures (Pfeiffer et al., 2020).

To sum up, it can be said that some of the earlier research has concentrated on digital design in order to achieve differentiation in terms of geometries. Some have prioritized simulations with finite element analysis for structural performance assessment of the TI systems, while others have focused on the digital fabrication for prototyping the TI assemblies.

3 Research methodology and case study

The geometric solids known as Platonic Solids are identical because their faces are made up of regular polygons. According to Dyskin and his colleagues, some convex polyhedra, particularly the five platonic bodies (tetrahedron, cube, octahedron, dodecahedron, and icosahedron), as well as their variants, allow for topological interlocking (Dyskin et al., 2003; Estrin et al., 2011). Among these platonic solid; cubes, octahedra, and dodecahedra can be assembled with a hexagonal tiling with the truncated blocks generated from the middle sections of them (Fig. 4 (a)). As it can be seen in Table 1, Lecci et al. (2021) studied the design of flat vault using cube because of its simplicity in design and manufacture. Therefore, in this paper, topological interlocking of octahedra was studied since it is the geometry with least complexity after the cube and also with the aim of adding a new discussion to the literature with the use of different geometry.

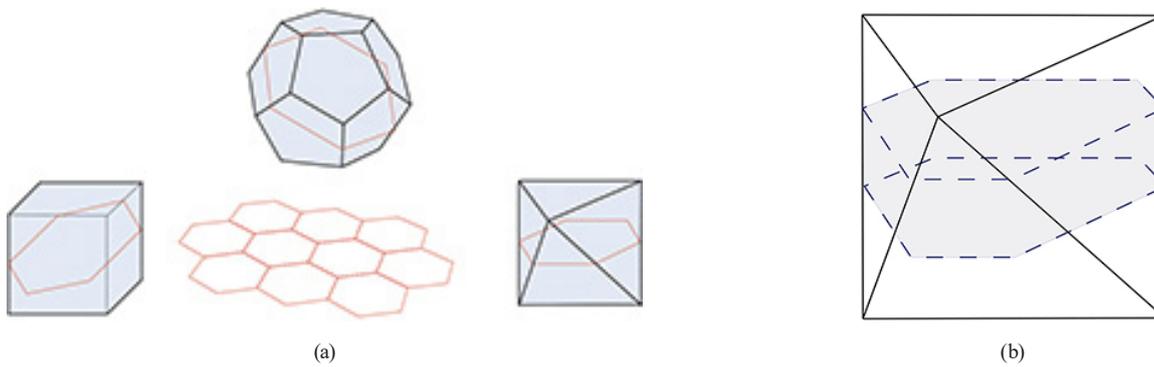


Fig. 4 (a) Hexagonal tiling at middle sections of cubes, octahedra and dodecahedra (Adapted from Dyskin et al. (2003)),
 (b) Diagram showing the sections planes to truncated octahedra

Since the purpose of this study is to design flat and walkable vaults, the transformation method of Glickman (1984) called "truncation" was used to transform octahedra into a flat geometry by truncating with section planes (Fig. 4 (b)).

From a hexagonal tessellation, truncated octahedra solids were generated using the Starfish plug-in in Grasshopper. Starfish is a Grasshopper plug-in that enables parametric pattern generation. It concentrates on two-dimensional tessellations that can be utilized to build structural systems using the topological interlocking principle.

From the interlocking edges of truncated octahedra, (Fig. 5 (a)) cubical *X*-joints were added in Rhino Ceros using volumetric modification, and a new topological interlocking module, which increased contact surface area, was generated (Fig. 5 (b)). Adding and removing cubical parts for *X* joints allowed the female and male joints to have the same geometry and thus make a more controlled transformation on the second geometry.

With the addition of each male or female *X* joints to the truncated octahedra, an increase of 10 cm^2 in contact surface area was achieved. This provided a total increase of 60 cm^2 of the contact surface in one block, in this way the contact surface was increased approximately 2 times compared to the block without *X*-joints.

In brief this case study describes a simple *truncation* method for flat vault design and *volumetric modification* method for increasing the contact surface area of interlocking blocks by employing *X*-joints to create a new block geometry. Rather than using *X*-jointed blocks exclusively as edge blocks, flat vault designs were created using fully *X* jointed blocks. As a result, the focus is on how *X* joints could affect the overall system's structural performance by increasing the contact surface area between interlocking blocks. In order to measure the effect of the contact surface area on the structural performance, two flat vaults were designed from two truncated octahedra blocks with different joint details.

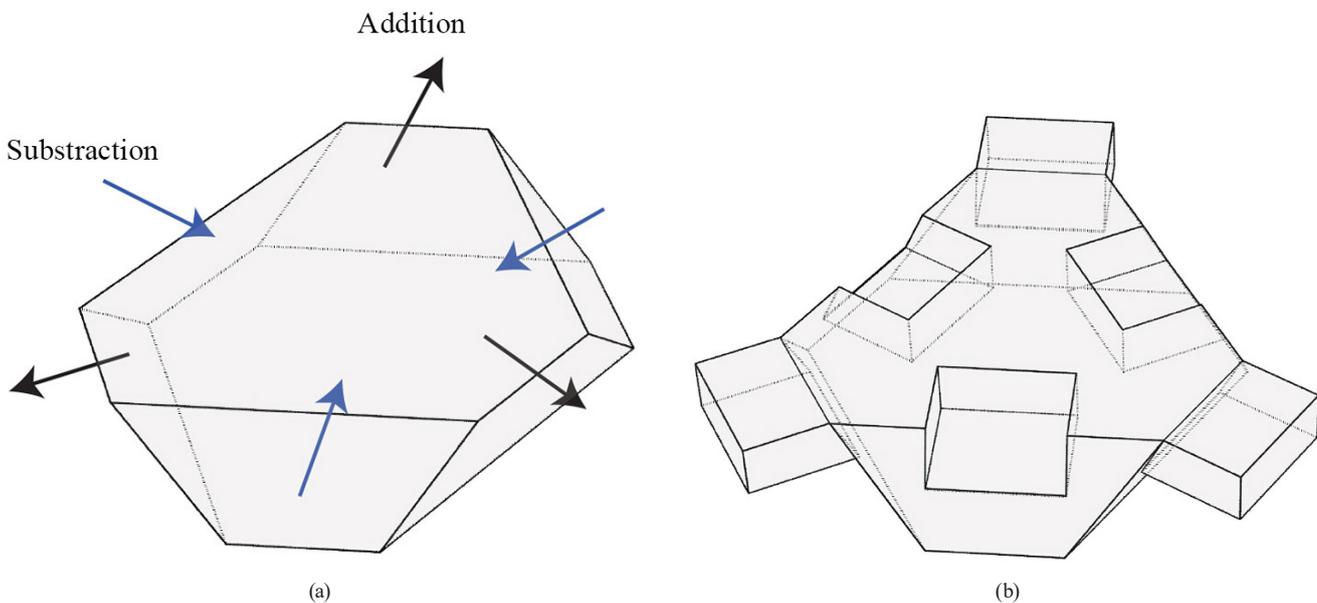


Fig. 5 (a) Diagram of the transformation of truncated octahedra, (b) New interlocking module with *X*-joints

To unfold load-bearing behaviour, topological interlocking systems require rigid peripheral constraints like an external frame (Djumas et al., 2016; Tessmann, 2012). Therefore, a 50×50 cm frame was used as a border in these two flat vault systems. For the border conditions, customized interlocking blocks are required as shown in Fig. 6 (a) and (b).

Finite element analysis was carried out to compare the structural performance of two different geometries using SimScale. It is a cloud-based computer-aided engineering (CAE) software tool that includes Computational Fluid Dynamics, Finite Element Analysis, and Thermal Simulation (Heiny et al., 2012). The most prevalent type of structural analysis utilizing the FE approach is static analysis.

The model of flat vaults was designed in Rhino and imported to SimScale. As the analysis was done under Earth conditions, the gravity was chosen as 9.81 m/s^2 in the negative Z direction. Regular concrete with (E) Young's modulus $3e+10 \text{ Pa}$, (ν) Poisson's ratio 0.2, and (ρ) Density 2240 kg/m^3 was chosen as a material without reinforcement for both blocks and for the frame. Following the material selection, the external constraint, which is the 50×50 cm frame with the thickness of 2 cm, was defined as fixed support. In addition to the self-weight of the sample material, a concentrated single -50 N force

(approximately 50% of the total weight) in the Z direction was applied at the centre of the flat vaults to analyse stress concentration.

After investigating the behaviour of the system as a vault, analysis was also carried out within the scope of the discrete element method to examine the interactions between each block. For that purpose, the interlocking blocks were arranged in a 7-element composition whose exterior surfaces were fixed in all degrees of freedom to perform a static analysis. -50 N force in Z direction applied on the central element.

4 Result and discussions

The widespread availability of parametric modelling technologies has created excellent opportunities for more complex designs, which are often architecturally and geometrically complicated. Although the point of form production in the digital environment is considered to be almost unlimited, there is still a gap that cannot be ignored regarding the constructability of structures designed in complex forms. This study aimed to search for form and integrated structural analysis carried out in the digital design environment, seeking answers to the manufacturability of complex geometries, to investigate the potentials of X -joints that can be applied as a flexible and easy detail at the beginning, and to receive feedback on the structural

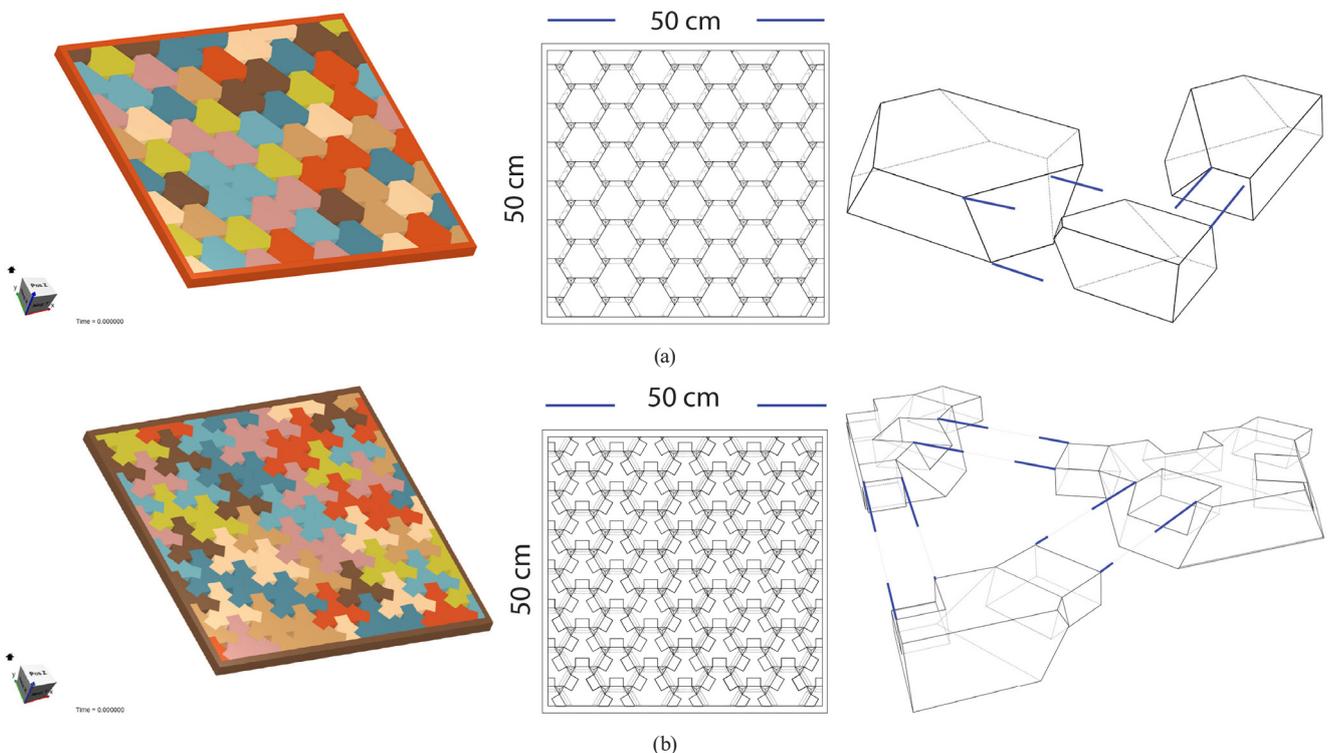


Fig. 6 Design of TI flat vaults; (a) with truncated octahedra, (b) with X jointed truncated octahedra

behaviour. The displacement values presented the deflection, and von Mises stresses values showed the stress concentration of the flat vaults.

When the flat vault composed of truncated octahedra blocks was examined, the average displacement was calculated as -6.00×10^{-7} m (Fig. 7 (a)). The average displacement of the flat vault generated with *X* jointed truncated octahedra was -1.51×10^{-7} m (Fig. 7 (b)).

The average value of von Mises stress of the flat vault with truncated octahedra was found as 24.18×10^3 Pa (Fig. 7 (c)). For the flat vault with *X* jointed truncated octahedra, the average von Mises stress was 5.59×10^3 Pa

(Fig. 7 (d)). As it can be deduced from the results, thanks to the incrementation of contact surface area with *X*-joints, the second flat vault designed with *X* jointed truncated octahedra has a better structural performance than the flat vault designed with truncated octahedra.

According to the numerical values obtained from FEM analysis, increasing the contact surface area, with the addition of *X* joints, between interlocking blocks, improves the structural performance. In the flat vault with *X* jointed truncated octahedra design, a 74.7% improvement in displacement and 76.8% reduction in von Mises stresses contributed to the structural performance as shown in Table 2.

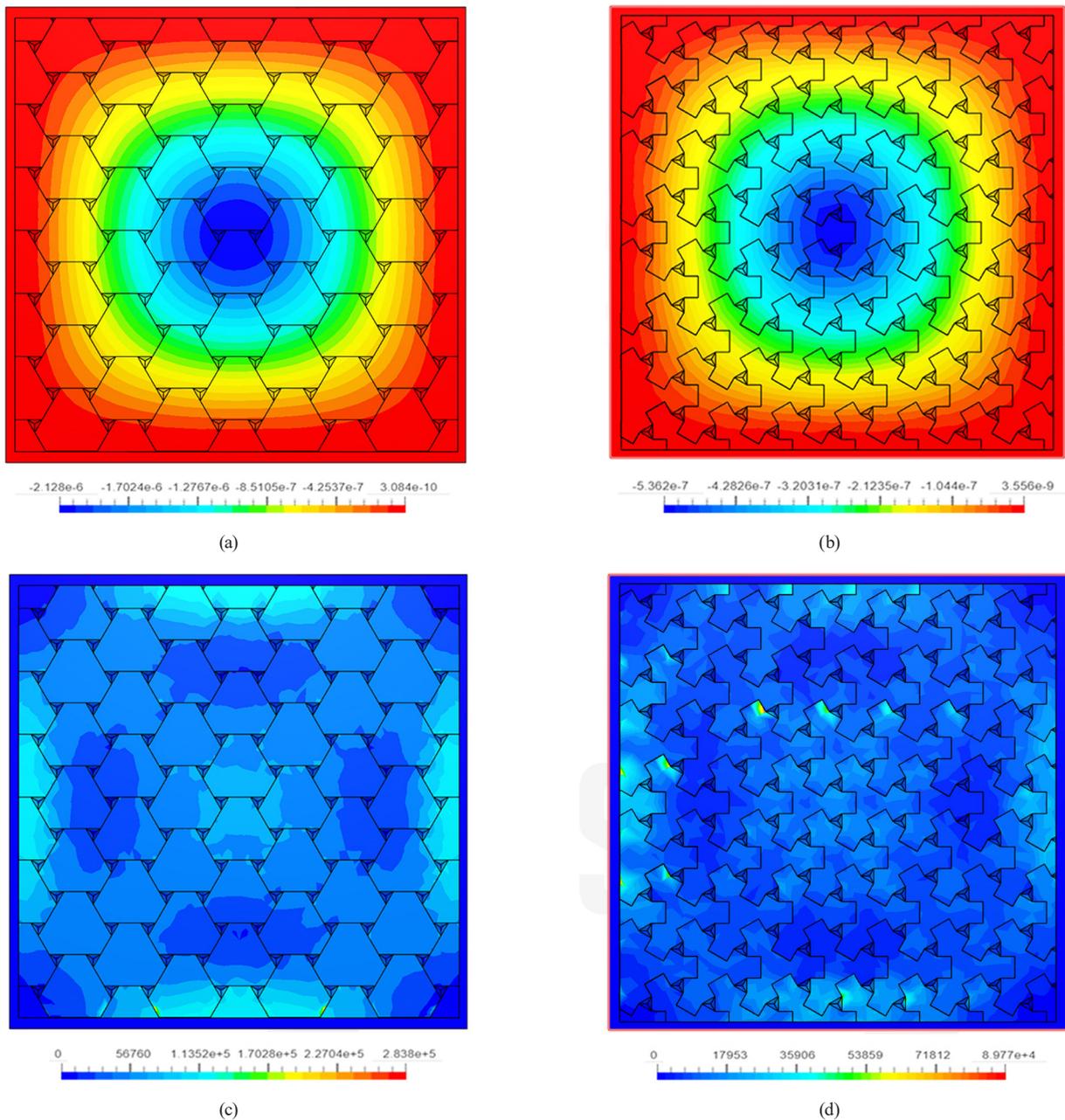


Fig. 7 Displacements of Flat vault (a) with truncated octahedra, (b) with *X* jointed truncated octahedra; von Mises stress distribution of Flat vault (c) with truncated octahedra, (d) with *X* jointed truncated octahedra

Table 2 Average vertical displacement and von Mises Stresses of 50–50 flat vault systems with changing block geometry

	Truncated octahedra	<i>X</i> jointed truncated octahedra	
Average displacement (m)	-6.00×10^{-7}	-1.51×10^{-7}	74.7%
Average von Mises Stresses (Pa)	24.18×10^3	5.59×10^3	76.8%

The impacts of contact surface area on a single block were evaluated in the second step of the finite element analysis. This second stage analysis aims to show the impact of *X* joint details on the structural performance of

a TI block. Under a -50 N force on the central elements, Fig. 8 illustrates the displacement and von Mises Stress maps for the 7-element composition of the truncated octahedra and *X*-jointed truncated octahedra. When the displacement magnitude was analysed, it was seen that the truncated octahedra had an average displacement magnitude of -2.91×10^{-7} m (Fig. 8 (a)). In contrast, the average displacement magnitude of the *X*-jointed truncated octahedra block was -1.69×10^{-7} m (Fig. 8 (b)). For each interlocking block composition, stress concentration was observed with the von Mises stress maps. Von Mises stress of the central element in the truncated octahedra composition

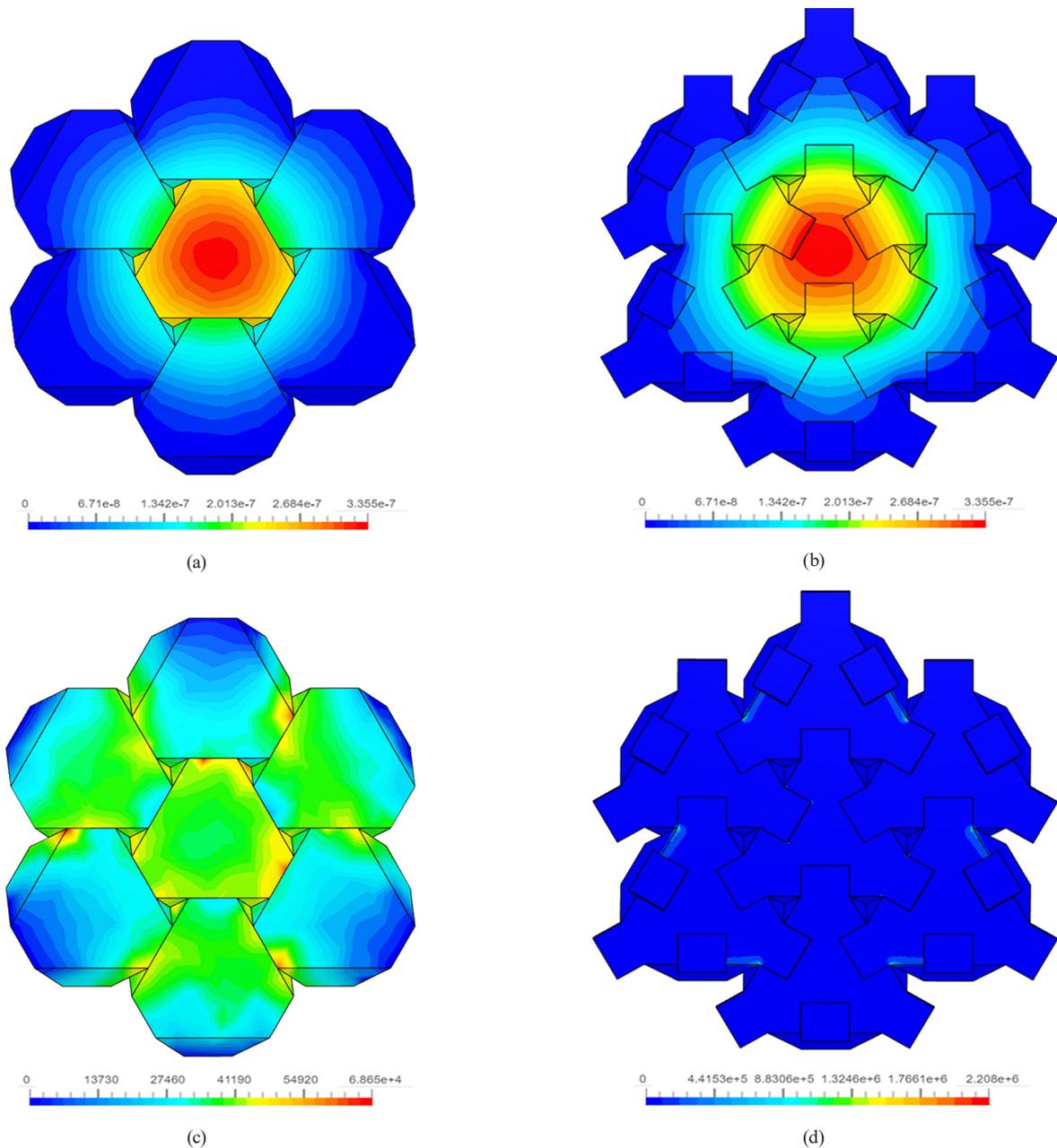


Fig. 8 Displacements of blocks (a) with truncated octahedra, (b) with *X* jointed truncated octahedra; von Mises stress distribution of blocks (c) with truncated octahedra, (d) with *X* jointed truncated octahedra

is, on average, 3.79×10^4 Pa (Fig. 8 (c)). The average von Mises stress of the central element in the X jointed truncated composition was 2.31×10^4 Pa (Fig. 8 (d)). From these analyses, it can be concluded that there is a direct relationship between the contact surface area of topological interlocking blocks and their structural stability.

As it can be observed from the analysis (Table 3), according to displacement values, a 41.7% improvement and according to von Mises Stresses, a 39% reduction in stress concentration in the central element improved the structural performance. It can be stated for further research that TI systems can become tolerant to loss of blocks by increasing the contact surface area of interlocking blocks.

5 Conclusions

Topological Interlocking systems are self-supporting systems and since they are composed of discrete elements, structural performance of TI systems is directly related to the form of the structure, geometries of blocks and joint details. This study provides significant insight into the change of structural performance of topological interlocking assemblies designed with different joint detailed blocks.

According to the finite element analysis, the major findings of this study can be listed as follows:

- Both von Mises Stresses and vertical displacement responses of the flat faults are affected by the geometry of the interlocking blocks.
- In the case of topological interlocking systems, there is a direct connection between the contact surface area of geometry and the structural performance of the system. Doubling the contact surface area with the addition of X -joints to the interlocking blocks

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Table 3 Average vertical displacement and von Mises Stresses of 7-element composition

	Truncated octahedra	X jointed truncated octahedra	
Average displacement (m)	-2.91×10^{-7}	-1.69×10^{-7}	41.7%
Average von Mises Stresses (Pa)	3.79×10^4	2.31×10^4	39%

ensures the assembly's structural integrity and overall stability.

- Even in the simplest geometries, X -joints improves the structural performance of the TI systems and TI blocks. This fact will result in the creation of new geometries of topological interlocking blocks by adding X -joints to increase structural performance. Adding X -joints can transform any topological interlocking block into a new geometry.
- The design of X -joints may take many variations. The size and positions of the X -joints can be adjusted, to accommodate a range of structural capacities.
- While using the same amount of material with addition and subtraction, the contact surface area can be increased between interlocking blocks.

This study on TI flat vault configurations is conducted to emphasize on the reanimation of traditional construction techniques in digital design environment and to demonstrate how the joint details of TI blocks affect structural performance. The designers can benefit from the results obtained from the digital modelling and finite element analysis of the study which enable design of topological interlocking structures with enhanced structural performance.

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