

A Machine Learning-based Model to Predict the Cap Geometry of Anatolian Seljuk Kümbets

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Abstract

The funerary structures known as kümbets emerged as a unique typology during the Anatolian Seljuk period (1077–1307). The term "kümbet" refers to a monumental tomb that has a tetrahedral, polyhedral, or conical cap. Although the majority of Anatolian Seljuk kümbets underwent renovation work in the 20th century, a lack of guidance and insufficient documentation has resulted in very few of them retaining their original characteristics. To support the decision-making processes of experts in future renovation work, this study introduces a machine learning (ML)-based model that predicts the cap geometry of kümbets through the use of section drawings. The model development process begins with the determination of the methods to be employed (Pix2Pix and SSIM). This is followed by data collection, data preparation and refinement, and the training of the machine learning model. Finally, there is testing and validation of the model. The results of both a two-step validation process and objective evaluations show that the ML-based model presented in this study has the potential to use section data to provide predictions of the cap geometries of kümbets.

Keywords

Anatolian Seljuk architecture, funerary structures, kümbet, machine learning, Pix2Pix

1 Introduction

The construction activities of the Anatolian Seljuks resulted in the first examples of Islamic architecture in Anatolia, an area that now comprises the bulk of modern-day Turkey. In addition to their other building typologies, including mosques, madrassas, and caravanserais, the intensive construction of monumental tombs became a defining feature of the Seljuk dynasty (1077–1307). Initially, these funerary structures followed the tradition for their use in Iran. However, as the Seljuks spread further across Anatolia, their tombs underwent a process of architectural development while retaining their basic functions and iconic features (Bates, 1971; 1978; Türkiye Kültür Portalı; Önkal, 2006).

The funerary structures of the Anatolian Seljuk period can be divided into two types according to their formal organization. In regard to the outer shell, the term "türbe" is used for those structures that are covered with a dome, while "kümbet" refers to those with a tetrahedral, polyhedral, or conical cap. Kümbets are generally classified as square, polygonal, or circular according to the plan geometry of the prayer room (Arık, 1967; Kuran, 2018;

Önkal, 2015) (Fig. 1). A typical kümbet has a compact, vertical form and its boundaries are defined by a combination of the mass of the prayer room and the cap raised above it. As a result of these architectural features, kümbets acquired a distinct identity and became a common style of monumental tomb (Bates, 1978; Kuran, 2018; Önkal, 2006; 2015). In terms of location, kümbets were built in a variety of locations throughout Anatolia, including cemeteries, villages, and hills, and therefore differ from the monumental buildings and structures found in city centers and on main transportation or trade routes (Arık, 1967). Today, kümbets remain easily recognizable within both rural and urban settings due to their formal organization and their architectural components such as their caps.

The majority of kümbets have undergone renovation at some level and few of them have retained their original characteristics as a result. Most of the renovations have included (but are not limited to) overall reconstruction, partial renovation, and the changing of an architectural element or material. However, there were no attempts at systematic documentation of these structures before the 20th

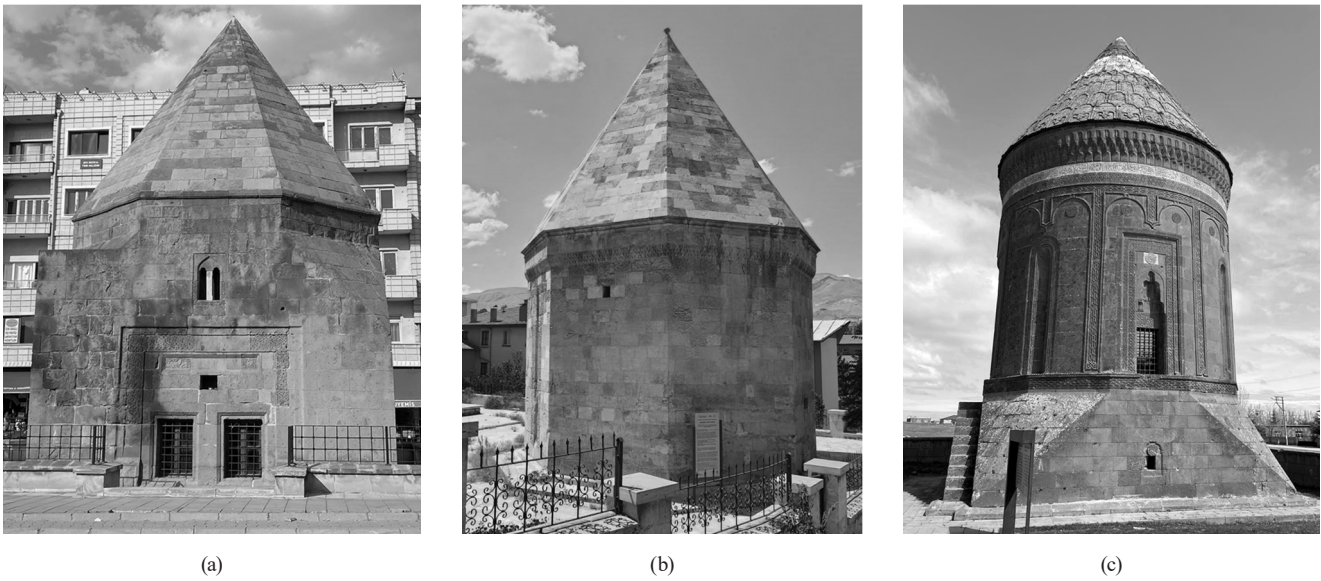


Fig. 1 Kumbets with different base geometries; (a) Cubic, (b) Polygonal, (c) Cylindrical

century, resulting in a lack of reliable data regarding their original state. Partial renovation may refer to the reconstruction of the cap or inner dome or the repair of the main outer wall. Changing an architectural element may consist of replacing the masonry cap with a metal or timber structure, covering the outer structure with metal cladding, or refunctioning the window openings as doors. In addition, materials might have been changed from stone to brick, or involve the replacement of deformed stonework.

Due to a lack of documentation (photographs or drawings), the renovation work done on Anatolian Seljuk kumbets was guided by the empirical knowledge and prior experience of experts rather than by scientific data. However, when the drawings of these experts are examined and compared, there are clear mismatches between them, even when the subject is the same. Most of these drawings were produced between the 1910s and the 1990s by experts from Europe and Turkey. In addition to the few studies that produced detailed surveys (Akok, 1967; 1969), others have included scaled/proportional drawings made during excursions or field trips (Bachmann, 1913; Gabriel, 1931). In recent years, a number of kumbets were re-documented with advanced digital measurement methods at the request of the relevant authorities.

As masonry structures, kumbets were constructed with cut stones, bricks, or a combination of both. However, despite the limitations of both the available materials and construction techniques, kumbets still display a great deal of variety. Each structure is unique in terms of its width, height, width-height ratio, inner shell (dome) angle, and outer shell (cap). This differentiation can be clearly

observed in the sections of the masonry caps. However except for a few morphological analyses, the section geometry has been neglected in a vast number of studies that provide classifications for kumbets (Ashkan and Ahmad, 2010; 2012). The previously mentioned mismatching problems regarding the drawings of kumbets are also more visible in the sections (Fig. 2).

Since there are no guides for designing kumbets or a rule-base for the formation of their sections, there is some motivation to employ machine learning algorithms that have the ability to detect both implicit patterns and the relationships between distinct and measurable features. This study therefore introduces a machine learning (ML)-based model that was trained using existing drawings to predict and complete the cap geometries for renovation purposes. Specifically, this study examines 60 Anatolian Seljuk kumbets that were built as independent structures and focuses on their section drawings. In addition, it is intended to offer answers to the following research questions:

- Can machine learning-based models make predictions from section drawings to complete the cap, which is a unique architectural element of Anatolian Seljuk kumbets?
- Is there any similarity between the predictions made by the machine learning model and the solutions suggested by the experts in the context of renovation of kumbets?
- Can machine learning-based models contribute to the work of renovators, architects, and other experts as a decision-support tool?

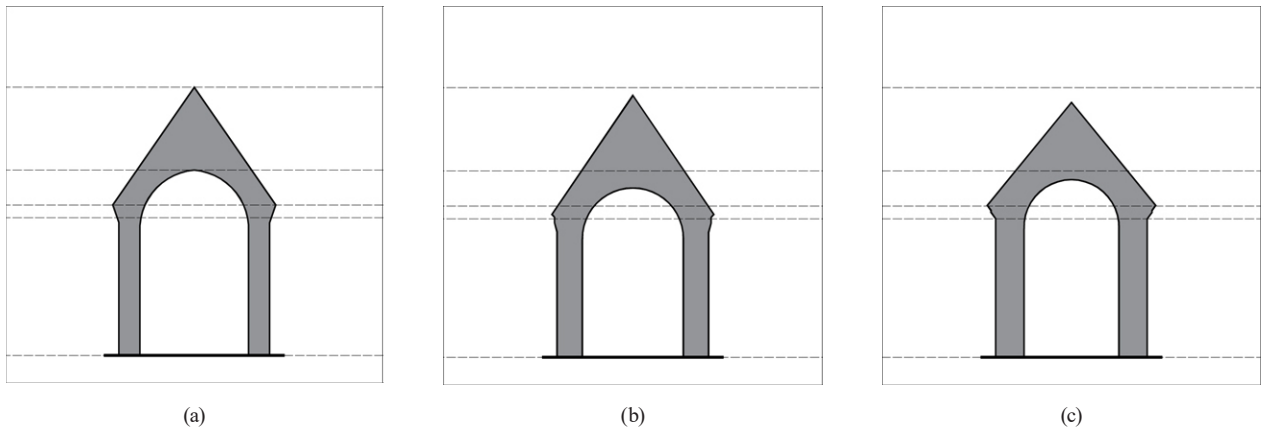


Fig. 2 Mismatching section drawings of Döner Kümbet in Kayseri from different sources; (a) Tuncer (1986), (b) Akok (1969), (c) Gabriel (1931) (reproduced by the author)

2 Background

The literature on kümbets consists of studies that examine a single kümbet (Bekmez, 2020; Blessing, 2015; Çetintaş, 2020; Danık, 2009; Özgüç and Akok, 1954; Parla, 2010; Toruk, 2014), a group of kümbets (Tuncer, 1976), and kümbets located within a particular city (Erdal, 2018; Önkal, 1980; Ünal, 1973) or town (Kındıgılı, 2019). There have also been studies comparing kümbets built within Anatolia to those located elsewhere (McClary, 2015; Nazer et al., 2020). Other studies have investigated the funerary structures by focusing on various periods (e.g. Anatolian Seljuk, Period of Principalities, Ottoman) rather than location (Gündüz, 2010; Turkan, 2009). Kümbets have also been the subject of studies in disciplines other than architecture in which the primary objective was to document architectural heritage through photogrammetry and 3D modeling (Doğru et al., 2017; Ulvi et al., 2019; Yakar et al., 2016).

The aforementioned studies examine Anatolian Seljuk kümbets in detail and provide a variety of information that includes their location, dimensions, formal organization, components, epitaphs/inscriptions, construction and renovation dates, current condition, building materials, construction techniques, and ornaments. They also provide a wealth of photographs and scaled orthographic drawings. In particular, the studies of Orhan Cezmi Tuncer (1986) and Hakkı Önkal (1996) represent a large body of information and were both published as books. In light of more recent discoveries, Hakkı Önkal updated the content of his book and published a second edition in 2015.

Systematic studies on Anatolian Seljuk kümbets date back to the 1960s. The most common approach in the literature is to categorize kümbets as square, polygonal, or circular due to the plan geometry of their prayer

room (Arık, 1967; Bates, 1971; Önkal, 2006). Although it is uncommon, there are also rectangular plan types. The higher variety that exists between kümbets, compared to other building typologies, is related to the influence of local traditions, the availability of materials, the necessities of the building typology, and the demands of the persons who erected them (Arık, 1967; Bates, 1971).

Most Anatolian Seljuk kümbets have two stories and consist of four main parts: the base (crypt), the main body (prayer room), the transition element (drum), and the cover surmounting the main body (cap) (Fig. 3). Although rare, there are also examples without a base. Functionally, the base serves as a burial chamber (crypt to protect the mummies), but it is also a structural requirement since it provides a foundation for the main body. Symbolic wooden coffins are generally placed in the main body which has a

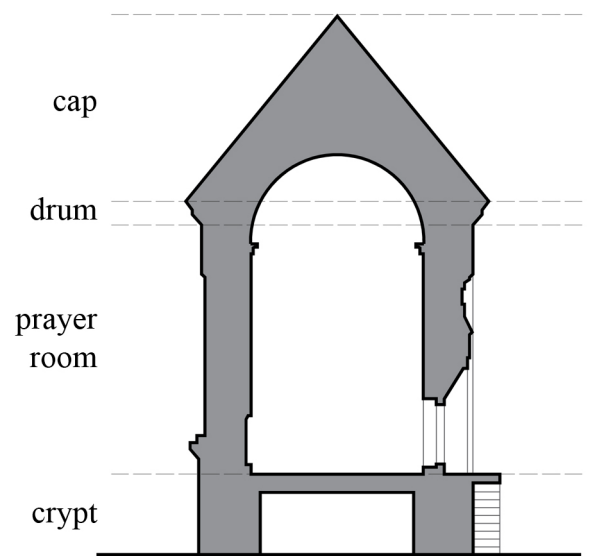


Fig. 3 Representation of common components on the section of Döner Kümbet in Kayseri (produced by the author based on Gabriel, 1931)

single door/portal and functions as a prayer room for visitors. In some kümbets, the transition between the geometries of the base to the main body, and the main body to the cap, is carried out by the chamfering of any right angles. In contrast to the outer shell (cap), the inner shell (as perceived from the interior of the main body) is generally dome-shaped (Arık, 1967; Bates, 1971; Kuran, 2018; Önköl, 2006; Yetkin, 1952).

Unlike the existing body of literature, this study focuses on the overall sections of the kümbets. Therefore, details regarding the materials, ornamentation, and openings of the kümbets are not included. Even without a strict description of the typological characteristics of a given kümbet, it remains possible to extract parameters (Fig. 4) such as: main wall thickness; main wall height; interior space volume; drum angle; inner shell type; and outer shell geometry. This study argues that the overall section geometries of a given kümbet can be learned and generated through machine learning methods by considering each parameter as it stands in relation to the others from a holistic perspective.

3 Machine learning and architectural heritage

There are a vast number of studies that have adapted machine learning (ML) techniques to the field of architectural heritage (Alani and Al-Kaseem, 2021; Grilli and

Remondino, 2020; Llamas et al., 2017; Varinlioglu and Balaban, 2021). Especially in recent years, it has become a major topic of interest. Grilli and Remondino (2020) focused on the automatic segmentation of point cloud data derived mostly from photogrammetry techniques and belonging to different architectural scales and contexts. Automatic segmentation and classification include the analysis of various architectural elements such as walls, windows, doors, roofs, floors, facades, arches, vaults, etc. (Grilli and Remondino, 2020). The important contributions of the study by Grilli and Remondino (2020) can be listed as the workflow integration of point cloud data and a 3D BIM model, the application of a pre-trained ML model to more complex tasks and novel contexts, and the implementation of the random forest model to 3D architectural heritage problems. Another segmentation study with a specific focus on the extraction and correction of facades in photographs by using deep learning and computer vision algorithms was presented by Ali et al. (2021). Moreover, the convolutional neural network (CNN) method was used by Llamas et al. (2017) to achieve results for 10 architectural elements including (but not limited to) the altar, apsis, bell tower, column, and dome. The ML model of Llamas et al. (2017) was trained using 8188 images of 128×128 pixels with a further 1404 images used for validation data. Varinlioglu and Balaban (2021) investigated

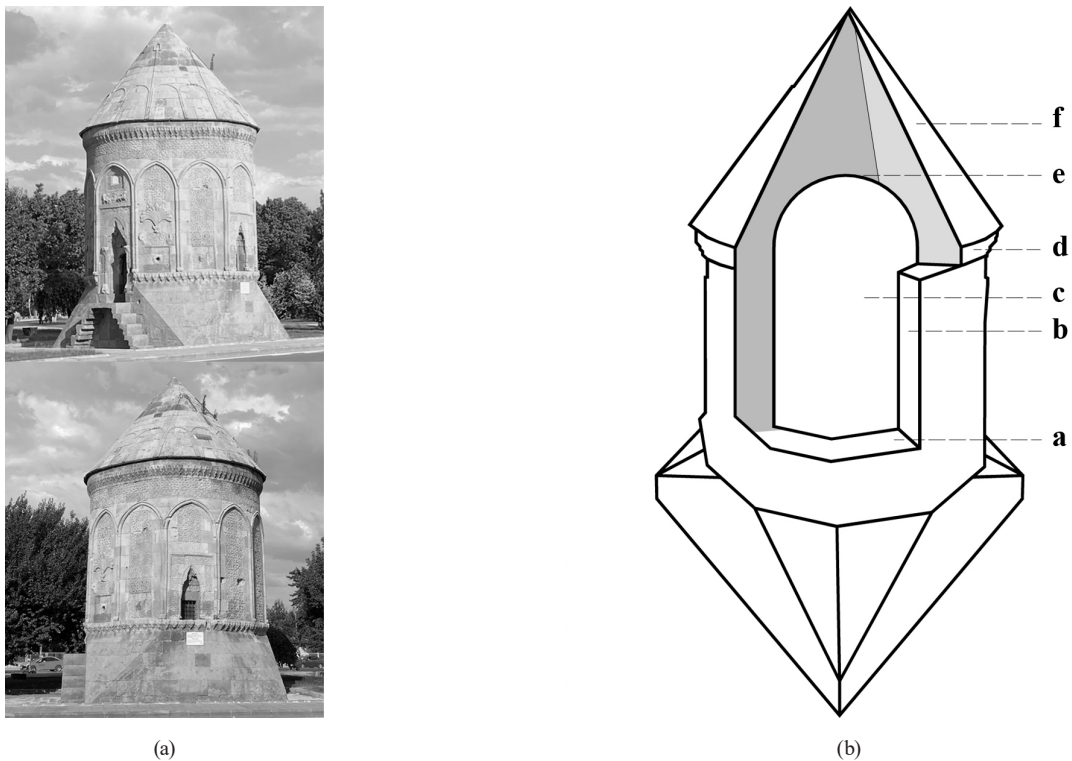


Fig. 4 (a) Photos of the Döner Kümbet in Kayseri and (b) an axonometric view showing its physical features: a – main wall thickness, b – main wall height, c – interior space volume, d – drum angle, e – inner shell type, f – outer shell geometry

the usability of supervised learning techniques within the context of heritage site prediction through a case study. Their experimental case study employed the Unity Game Engine and ArcGIS as tools, GIS data, location labels, and a terrain model as data sources, and 256×256 pixels Google Earth images as training material (Varinlioglu and Balaban, 2021). Alani and Al-Kaseem (2021) implemented a Deep Convolutional Generative Adversarial Network (DCGAN) to the Islamic pattern generation problem in relation to a given noise input. Nogales et al. (2021) focused on developing a generative adversarial network (GAN) model to predict missing architectural elements by using image data from archaeological sites. The research of Nogales et al. (2021) has common points with this study, including context (working on in-situ and historical heritage), problem definition (prediction of the missing piece of an architectural entity), and approaching architectural representations merely as visual data instead of as a typological classification. However, this study differs from that of Nogales et al. (2021) in terms of its data type (2D section versus 3D axonometric projection), its data preparation process, the size of the dataset, and its use of restitution drawings for comparison. Mesanza-Moraza et al. (2021) proposed a machine learning model to automate reading sections from given images in the context of archaeological sites with a specific focus on stone building types, namely ashlar and rough stone. Adhikary et al. (2021) implemented machine learning techniques to restore the missing parts of a piece of artwork. Finally, Pix2Pix as a generative adversarial network (GAN) was utilized in a study by Adhikary et al. (2021).

In brief, there have been studies which use Pix2Pix that take 2 dimensional images for use as training materials and which apply cultural heritage as context to predict the missing parts of a given subject. However unlike previous studies, this research presents a unique approach to the representation of architectural sections, to the conceptualization of the problem, and to the examination of an original context.

4 Machine learning-based model

This section presents Pix2Pix and structural similarity (SSIM) as the employed methods. It also describes the data collection from the available plan and section drawings in the literature, the data preparation and refinement to train the ML model, the training of the ML model with section drawings, and the testing and validation of the ML model through its ability to predict cap geometry.

4.1 Pix2Pix and structural similarity method

The presented ML-based model employs Pix2Pix, which was developed by Isola et al. (2017) in their paper "Image-to-Image Translation with Conditional Adversarial Networks", which was presented at CVPR in 2017. The Pix2Pix (Isola et al., 2017) model is a type of conditional GAN in which the output image is produced according to a source image. The architecture of the GAN model (Goodfellow et al., 2014) is designed to create competition between a "generator" that creates new logical synthetic images and a "discriminator" that classifies these images as real (from the dataset) or fake (generated). The generator is updated via the discriminator, while the discriminator is updated directly. As a result, the model is trained in an adversarial phase in which the generator tries to deceive the discriminator while the discriminator tries to identify the fake images. Adversarial loss is used to train the generator to produce reasonable images within the target domain. Losses between the generated image and the planned output image are also used to update the generator. The generator is encouraged to construct reasonable translations of the source image as a result of these additional losses. In Pix2Pix, a source image and a target image are given to the discriminator in a manner similar to that employed by the GAN architecture. Following this, the discriminator decides whether the target is a reasonable translation of the source image or not. Pix2Pix has been tested on a variety of image-to-image transformation tasks, including translating maps to satellite images, black-and-white photographs to color, and product drawings to product photographs.

The structural similarity method (SSIM) is an objective evaluation metric to quantify the overlap percentage between the target and the generated image. SSIM is linked to the quality and perception of the human visual system (HVS color model). In SSIM evaluation processes, a value of "0" indicates that there is no overlap between the compared images, whereas a value of "1.00" indicates perfect structural similarity with complete overlapping (Wang et al., 2004).

4.2 Data collection

In addition to their plan type, Anatolian Seljuk kümbets are also commonly classified according to two groups, namely:

1. free-standing, and
2. structurally connected to another structure/building complex (Blessing, 2015; Önköl, 2015).

In this study, only free-standing kümbets have been selected due to their fully perceivable overall forms. Over time, some of the structures/building complexes that hosted kümbets have been completely ruined, but the kümbets themselves have survived (Arık, 1967). This type of kümbet is also included in the research. The scope of this study covers the 83 independent funerary structures given by Hakkı Önkal (2015) in his book *Anadolu Selçuklu Türbeleri* (Anatolian Seljuk Tombs). Since these 83 funerary structures include both kümbets and türbes, a second refinement was made, and 67 structures in 18 Turkish cities were identified as kümbets. For this refinement, photographs taken from written sources and the Culture Portal of Türkiye (a website hosted by the Republic of Türkiye Ministry of Culture and Tourism) were used (Türkiye Kültür Portalı). The location and numerical distribution of the 67 Anatolian Seljuk kümbets given by Önkal (2015) are shown on the map (Fig. 5).

Although the work by Önkal (2015) describes the Anatolian Seljuk kümbets in detail, it does not provide plans and sections for all 67. Therefore, the literature review was extended, and drawings were gathered from books (Bachmann, 1913; Gabriel, 1931; Tuncer, 1986), articles (Akok, 1967; 1969; McClary, 2015; Özgüç and Akok, 1954; Parla, 2010), dissertations (Altınsapan, 1997), public (T.R. Directorate General of Foundations) and private (Salt Research) archives, and the databases (Aygör, 2010 (Konya Metropolitan Municipality); Kayseri Metropolitan Municipality, 2013) of local governments.

As a result, except in the cases of three ruined kümbets, at least one plan drawing was obtained for the remaining structures. Of the 64 kümbets with plan drawings,

sections were obtained for 60 of them. However, during this research process it was observed that the section drawings of some kümbets differ in terms of their dimensions and forms between the aforementioned sources. As a result, due to having two or three different alternative drawings for some kümbets, a total of 91 different section drawings were collected for 60 structures.

4.3 Data preparation and refinement

As stated, the dataset for this study consists of 91 section drawings belonging to 60 kümbets. All the plan and section drawings obtained by scanning and retrieval from digital archives were formatted as raster images. Following this, the drawings were reinterpreted at the same level of detail and redrawn using AutoCAD software to allow their use as input data. As the Pix2Pix model only works with pixels, any dataset to be utilized for its training has to be converted from vector drawings to pixel-based graphics.

The overall forms of the kümbets were then analysed according to the section drawings alone. Architectural elements such as doors, windows, muqarnas, and niches, and features such as building materials and construction techniques were not included in these reinterpreted section drawings. Moreover, not all kümbets have crypts, and for those that do some are underground. Therefore, the crypts were also not included in the dataset to provide a degree of consistency in the analysis of different kümbets.

To prepare the dataset, first, the kümbet sections were represented by contour lines (Fig. 6 (a)). The dataset was then divided into two subparts. The first of these gives the interior space in the sections (Fig. 6 (b)) and the second shows the outer shell covering the interior (Fig. 6 (c)).

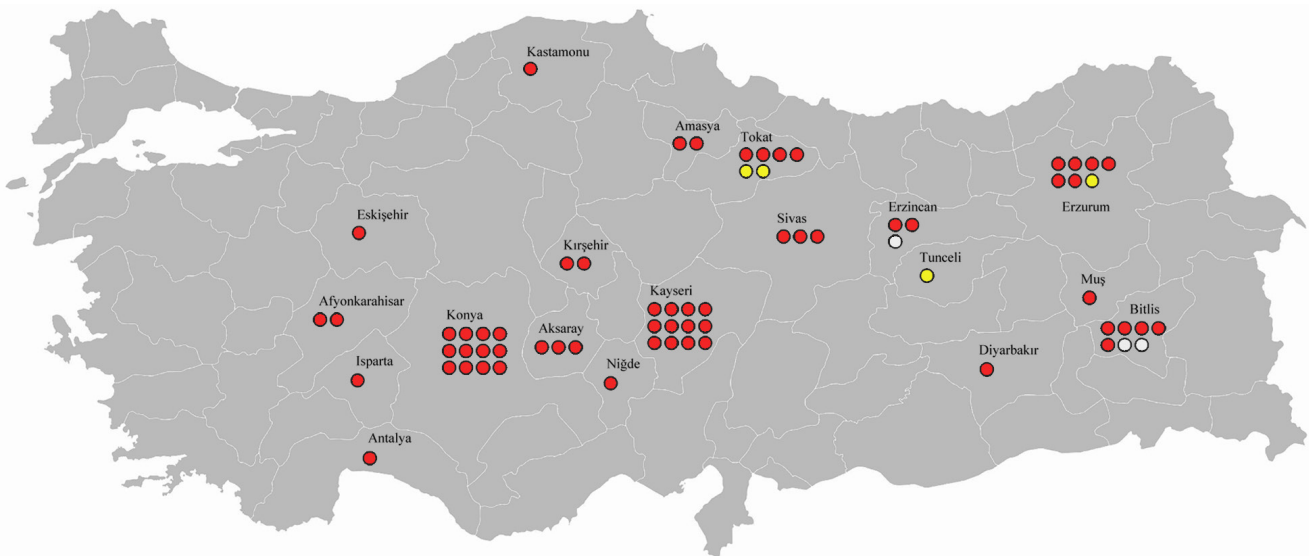


Fig. 5 Map of Anatolian Seljuk kümbets; existing (red dots), ruined (white dots), documented without section (yellow dots)

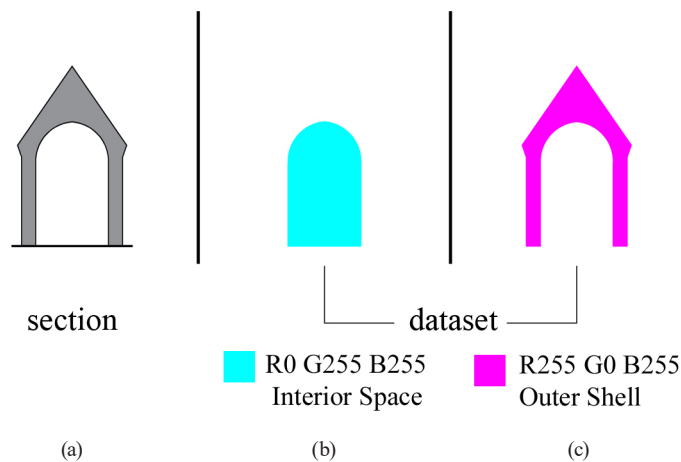


Fig. 6 Representation types; (a) kümbet section, (b) the dataset of the interior space, (c) the dataset of the outer shell

Because the kümbets in the dataset are standing (not ruins), there are definite contour lines for all of them. These contour lines enabled the data set to be reorganized into 91 interior spaces and 91 outer shells through the use of the "boundary detection" operation in AutoCAD. To avoid confusion during the training of the ML model and to increase its efficiency, the detected boundaries of the kümbets are represented by colors that clearly differ from each other in terms of their RGB values (Fig. 6).

By using the fill command in AutoCAD, parts of the kümbets were colored and placed on two matrices, and each grid was exported as images with a size of 1024×1024 pixels and a resolution of 72 dpi. Following the export operation, the interior spaces shown in Fig. 7 were stored in the "Input" folder, and the outer shells in Fig. 8 were placed in the "Output" folder. The images in both folders were labelled identically according to the identification number of the kümbet in question (for example, label 3_1 refers to kümbet number 3 and its first alternative drawing). Finally, the necessary training folders with matched images for Pix2Pix were created.

4.4 Training and validation of the ML-based model

In the training phase of the ML Model, 10 sections were excluded for later use in the validation process of the model (marked with red frames in Fig. 7 and Fig. 8). This validation material consisted of 7 restitution drawings and 3 drawings showing the condition of damaged kümbets. By removing these from the complete dataset of 91 drawings, a new dataset comprising 81 drawings was then used to train the ML model. Keras is a deep learning library that provides a convenient way to define and train almost any type of deep learning model, and in this study it was used to provide an interface to TensorFlow,

an open-source library for creating artificial neural networks. The ML model was then trained with the dataset until it reached 20250 (81×250) iterations of 250 epochs. The trained model was expected to predict or complete the outer shell of a given kümbet after receiving the colored boundary of the interior space as an input. Fig. 9 shows the training process of the model based on 2 randomly selected kümbets, namely 38_2 (the kümbet of Melik Gazi in Kayseri) and 42_3 (the kümbet of Tacül Vezir in Konya) given in Figs. 7 and 8. It is important to mention that both source and target columns include the existing drawings obtained from the literature (Fig. 9).

To test the success of an ML model, the validation phase has to be performed with inputs that are not included in the datasets used in its training. In this study, and as already mentioned, the validation material included 7 restitution drawings. The interior spaces were extracted from these restitution drawings and given to the ML model as inputs. Fig. 10 illustrates the damaged situation of the kümbets (survey), the interior spaces extracted from the surveys of damaged kümbets (source column), the sections produced by the trained model (generated), and the restitution drawings (target) prepared by experts. The generated sections and restitution sections were compared using SSIM. The calculated SSIM values were 0.95, 0.94, 0.94, 0.96, 0.40, 0.96, and 0.90, respectively for the kümbets coded 3_1, 5, 26, 33_2, 41, 52_3, and 57_2.

In addition, the generative abilities of the trained model were further tested with drawings of 3 damaged kümbets that do not have sections prepared from any restitution or restoration phases. For the given inputs, the ML model generated kümbet sections by relying on its previous training (Fig. 11).

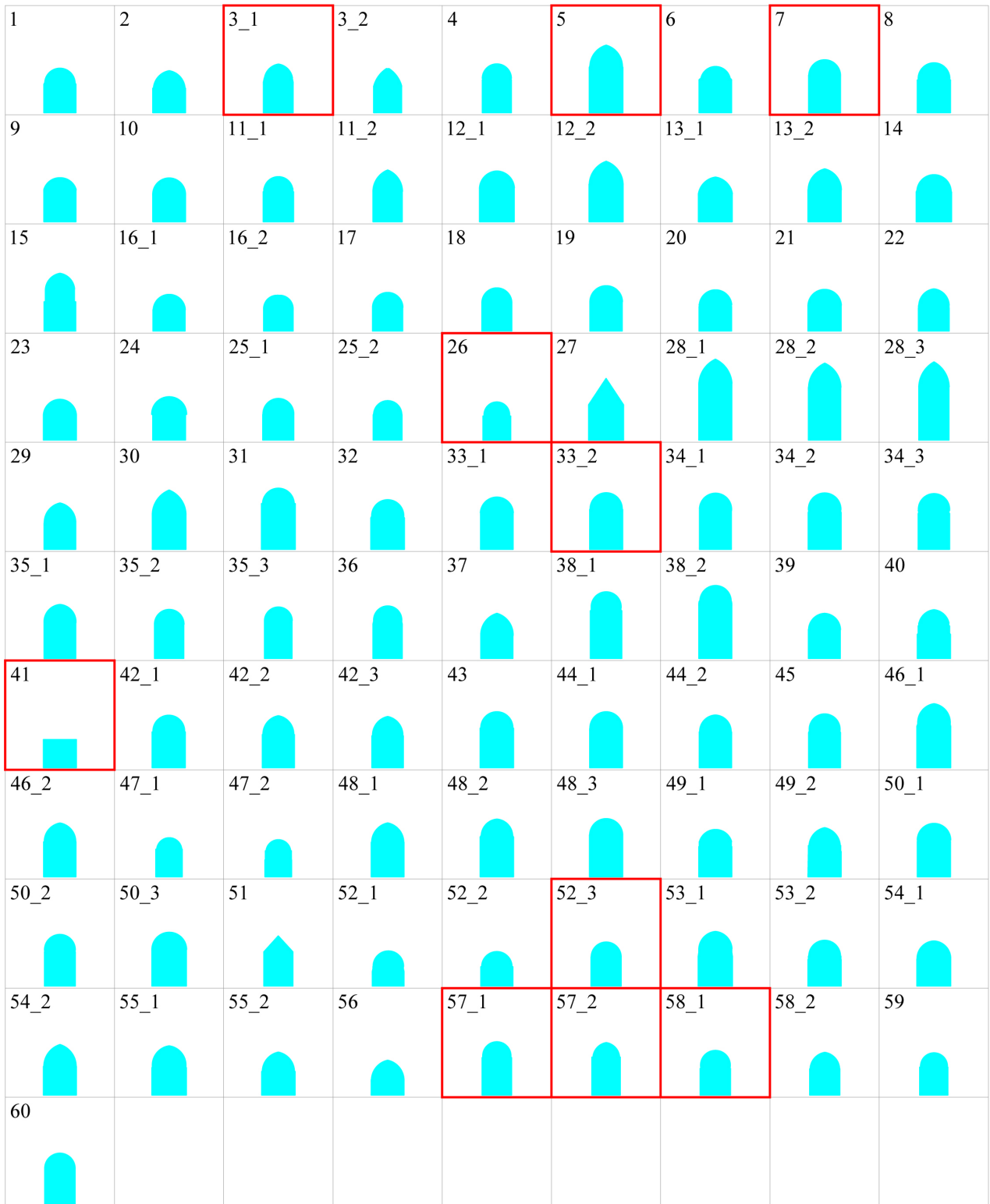


Fig. 7 Dataset part 1: the interior spaces of the kümbets

5 Conclusion

This study contributes to several fields of research. Primarily, by documenting funerary structures that belong to a unique typology (kümbet) and certain time period (Anatolian Seljuk) from a variety of sources and collecting

their section drawings, it contributes to the field of architectural heritage and preservation. In addition, by following a documentation and refinement process, a new dataset that consists of section drawings of kümbets is here presented publicly in order for it to be of use in future studies.

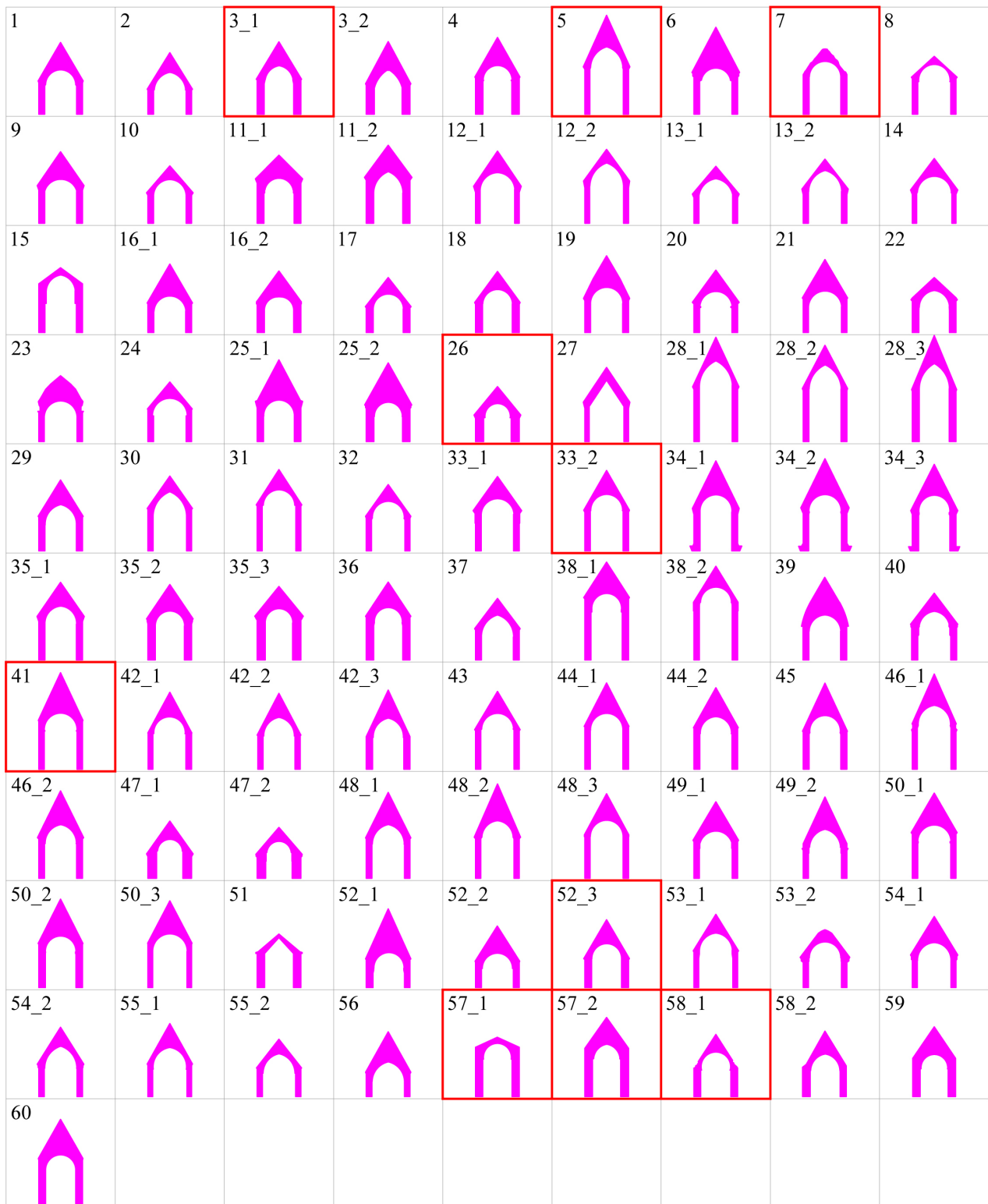


Fig. 8 Dataset part 2: the outer shells of the kümbets

The results obtained from the validation phase and the objective evaluation of the generated kümbet sections show that the presented machine learning-based model has the potential to provide satisfactory results. A common criticism of ML-based models is their incapability

to generate solutions when they encounter an unfamiliar input instead of materials they have been trained with. With regard to this issue, following the training and test (Fig. 9) processes of this study, a two-step validation was performed. The results of the first and second validation

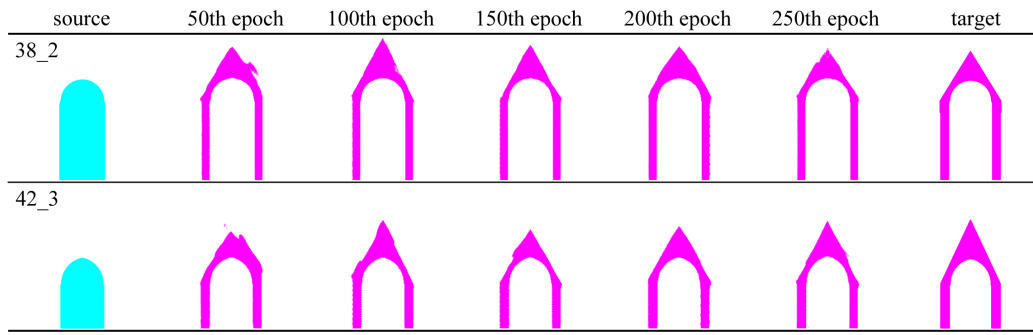


Fig. 9 Training process of the model

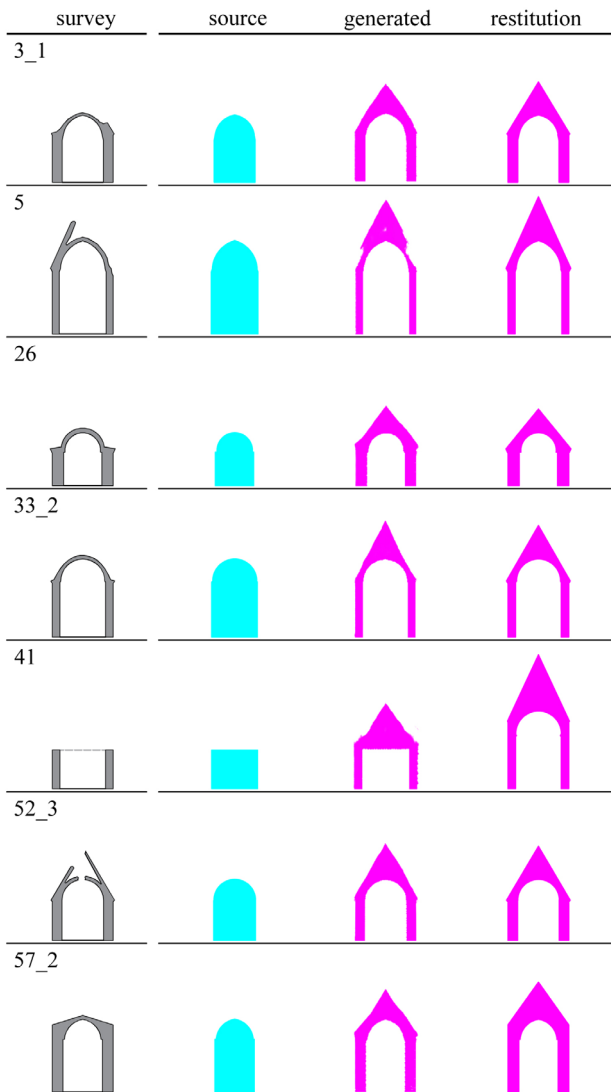


Fig. 10 Test results of the trained model with validation material

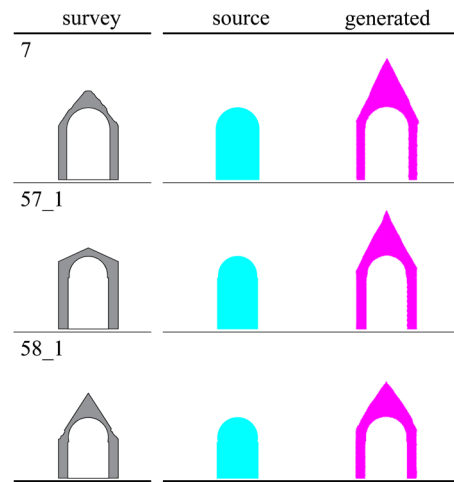


Fig. 11 Predicting the cap geometries of damaged kümbets with the trained model

show that the ML-based model can produce solutions for foreign validation data (Fig. 10 and Fig. 11)

As illustrated in Fig. 10 and calculated using SSIM, the outer shells generated by the trained ML model show similarity with the restitution drawings. However, they differ in detail. For example, the generated drum of the kümbet coded as 57_2 is thinner than that drawn in the restitution

drawings. In the case of the half kümbet coded as 41, the generated section was not successful due to insufficient input data to generate a complete section.

The small size of the dataset is a prominent limitation of the study. The scope of the dataset is limited to those kümbets built within a certain period and geography, and for this reason augmenting the dataset of this study was not possible. However, the ML-based model trained with this small dataset is able to make predictions for the cap geometry of kümbets and to generate satisfactory outputs.

Given that the restitution drawings are dependent on the empirical knowledge and experience of experts rather than objective documentation, it would not be appropriate to assert that these drawings reflect the original state of the structures. Similarly, it can be claimed that the proposed model does not generate certain outputs, but makes predictions relying on the sections of the kümbets (dataset) used for its training. With this ability, the proposed ML-based model may support the decision-making processes of experts in the fields of architecture, conservation, restoration, art history, and cultural heritage.

As the proposed model works with single-space structures, in future studies a similar framework can be applied to other historical typologies such as the single-domed mosques of the Early Ottoman period.

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