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The Design of Slender Columns Made of Solid or Glued Laminated Timber

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Abstract
In this study, a method for the design of timber slender elements, subjected to simple compression is presented. The presented method is analytical, and is in accordance with Eurocode 5 and the new Italian technical standards (NTC 2018). The proposed method can be applied in the design of compressed elements with a generic doubly symmetric section.

Keywords
timber, buckling, strut, columns

1 Introduction

The current standards (Eurocode 5 (CEN, 2005a); NTC 2018 (Ministero delle Infrastrutture e dei Trasporti, 2018)) clearly indicate which calculation procedure is to be followed in order to check the stability of slender timber elements subjected to simple compression. However, they do not say anything about the procedure that must be followed to carry out the design of the elements themselves. This forces engineers to make numerous attempts to optimise sections. To overcome this difficulty, and to provide engineers with a simple calculation method, an analytical method is developed in this study, which allows the immediate cross sectional design of slender timber elements under simple compression. The proposed procedure represents the first direct design method in the field of buckling. This is significant, if we consider that the discovery of the phenomenon of buckling can be traced back to the field of timber structures. In fact, it is believed that the first experimental observation carried out on wooden elements dates back to 75 B.C. by Heron of Alexandria. In the first phase, studies on this subject consisted essentially of observing the phenomenon and documenting it. This approach continued until the 16th century, the period in which we find the observations made by Leonardo da Vinci (1493). From the 17th century onwards, experimental observations begin to be accompanied by a number of propositions that summarise the results obtained from experimentation in simple rules. In this period we find the studies of Mersenni (1644) and van Musschenbroek (1729). The main conclusion of these studies is that the strength of compressed elements decreases with the square of the length of the elements. In accordance with these observations, Euler (1744; 1759), formulated the first analytical method for the study of the buckling phenomenon. Euler's model describes the phenomenon by considering geometric non-linearity, but does not take into account either mechanical non-linearity or imperfections. The importance of imperfections was later highlighted by Thomas Young (1807). Later, in 1833, Navier (1833) studied slender elements, with linear behaviour, subject to eccentric loading. In this study, Navier also defines the verification formula which, in the presence of eccentric loading, must be applied, in the most heavily loaded section, under the effect of first and second order actions. Subsequently, in the field of metallic structures, Ayrton and Perry (1886a; 1886b), deepened the role of geometric imperfections, dealing with the study of metallic elements, characterised by initial cosinusoidal geometric imperfections. On the basis of this result, Maquoi and Rondal, studied the characteristics of the imperfections in real struts (Maquoi and Rondal, 1978; Rondal and Maquoi, 1979), reaching the formulation, which today, is the basis of the verification formula used in Eurocode 3 (CEN, 2005b) and in the Technical Standards for Construction (NTC) (Ministero delle Infrastrutture e dei Trasporti, 2018). This formulation

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has also been adopted by Eurocode 5 and the Italian NTC technical standards for the verification of slender timber struts. The method proposed here, could be applied in the design of compressed elements, belonging to flat trusses or belonging to spatial trusses. In the presentation of the proposed method, we have chosen to refer to the study of elements with doubly simmetric sections characterized by a generic form. This allows a simpler exposition, and a more versatile application of the proposed method. In particular, the exposition will be divided into three parts: in the first part it is assumed that the ratio between the main dimensions of the section is fixed a priori. In the second part, it will be assumed instead that the design of the element is carried out by fixing at will the dimension with respect to which the section assumes its greatest inertia.

Finally, in the third part, it will be assumed that the design is carried out by fixing a priori the dimension with respect to which the section assumes its lowest inertia. The validation of the method will be carried out by reporting the results obtained from several calculation cases, in which all the main quantities that are significant in the problem are varied. The method validation will be conducted by performing numerous examples, with reference to sections with common shapes in practice. Timber as a construction material could be successfully used in residential construction, as demonstrated by the construction of a recent 14-storey building in Norway, which is the tallest structure made of such a material to date (Malo et al., 2016). Furthermore, timber turns out to be a sustainable material and suitable for reducing seismic risk. This is essentially due to the low density of this material and its significant mechanical resistance.

2 Proposed analytical method (Case 1)

2.1 Design of slender struts, with a generic doubly symmetric cross section, and a fixed ratio between the principal dimensions (Case 1)

Let us consider a doubly simmetric reference section S_0, characterized by a generic shape (Fig. 1). Let x_0 be the "strong" axis, with respect to which the reference section assumes its maximum inertia. Let y_0 be the "weak" axis, with respect to which the reference section assumes its minimum radius of inertia i_{y0}. We suppose that x_0 and y_0 are simmetry axis and principal axis for S_0. The origin of (x_0,y_0) coordinate system is conicident with the centroid G_0 of the section. We shall denote by h_{X0} the maximum dimension that the reference section assumes in the x_0 direction, and by h_{Y0} the maximum dimension that the reference section assumes in the y_0 direction. Let h_{X0} be the weak side of the reference section and h_{Y0} the strong side of the reference section. Let the area of the reference section be A_0, and let J_{y0} be the moment of inertia that the reference section assumes with respect to its weak axis. For the reference section, we define the "shape ratio" of the section as the quantity

$$\varphi = \frac{h_{Y0}}{h_{X0}}. \quad (1)$$

The reference cross section S_0, is introduced to define the cross section shape, but S_0 does not necessarily satisfy the strength conditions. For this reason, the section S, suitable to resist an assigned axial action N_{Ed}, on an effective length L_0, results from the enlargement of the reference section S_0 (Figs. 1, 2), so that, the coordinates (x,y) of the generic point belonging to the section S, can be deduced, from the coordinates (x_0,y_0) of S_0, by Eq. (2)

$$\begin{aligned} x &= w_X \times x_0 \\ y &= w_Y \times y_0, \end{aligned} \quad (2)$$

where w_X and w_Y are two scale factors. Also for the section S, the (x,y) axis are simmetry axis and principal axis. The origin of the (x,y) coodinate system is coincident with

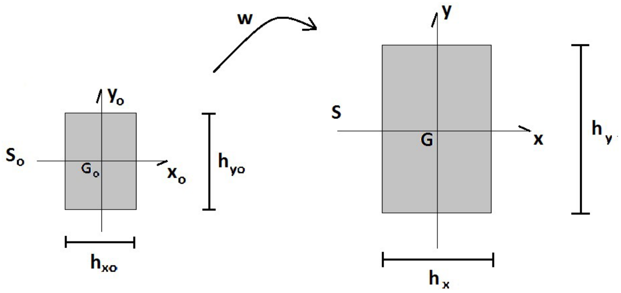

Fig. 1 Column cross section

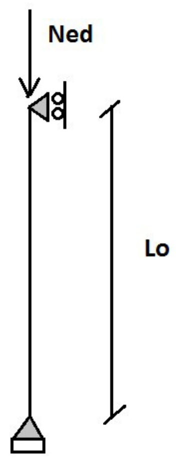

Fig. 2 Column static schematization

the centroid G of the S section. With the transformation of Eq. (2), we obtain

$$h_X = w_X \times h_{X0}$$
$$h_Y = w_Y \times h_{Y0}. \tag{3}$$

If, in the transition from the reference cross-section S_0 to the actual cross-section S, it is desired that the shape ratio remains unchanged, it will be necessary to impose that

$$\frac{h_Y}{h_X} = \frac{h_{Y0}}{h_{X0}} \tag{4}$$

by the use of Eq. (3), the result is then

$$\frac{w_Y \times h_{Y0}}{w_X \times h_{X0}} = \frac{h_{Y0}}{h_{X0}} \tag{5}$$

hence

$$w_X = w_Y \overset{def}{=} w \tag{6}$$

by the same equality, Eq. (2) can also be rewritten as follows

$$x = w \times x_0$$
$$y = w \times y_0 \tag{7}$$

and Eq. (3) can be rewritten as follows

$$h_X = w \times h_{X0}$$
$$h_Y = w \times h_{Y0}. \tag{8}$$

At this point, it is necessary to determine which value must be assigned to the scale factor w, so that section S can satisfy the resistance condition required by the standards Eurocode 5 (CEN, 2005a) and NTC 2018 (Ministero delle Infrastrutture e dei Trasporti, 2018). In this sense, in the passage from section S_0 to section S, the area of the section will vary according to Eq. (9)

$$A = \iint dxdy = \iint d(wx_0)\,d(wy_0) = w^2 \iint dx_0 dy_0 = w^2 A_0 \tag{9}$$

while the minimum moment of inertia of the section will vary according to Eq. (10)

$$J_{\min} = \iint x^2 \times dxdy = \iint w^2 x_0^2 \times d(wx_0)\,d(wy_0) = w^4 \iint x_0^2 \times dx_0 dy_0 = w^4 J_{y0} \tag{10}$$

For the effective section S, we then have

$$i_{\min} = \sqrt{\frac{J_{\min}}{A}} = \sqrt{\frac{w^4 J_{y0}}{w^2 A_0}} = w\sqrt{\frac{J_{y0}}{A_0}} = w \times i_{y0} \tag{11}$$

therefore

$$\lambda = \frac{L_0}{i_{\min}} = \frac{L_0}{w \times i_{y0}} \tag{12}$$

from which

$$w = \frac{L_0}{\lambda \times i_{y0}}. \tag{13}$$

By reason of Eq. (9), it is then

$$A = \frac{L_0^2}{i_{y0}^2}\frac{A_0}{\lambda^2}. \tag{14}$$

According to the standards Eurocode 5 (CEN, 2005a) and NTC 2018 (Ministero delle Infrastrutture e dei Trasporti, 2018), section S will only be suitable for resisting the load N_{Ed} if the following condition is met

$$\frac{N_{Ed}}{A} \le k_{crit,c} \times f_{c,0,d}. \tag{15}$$

For the application of Eq. (15), it is necessary to evaluate the Eulerian slenderness

$$\lambda_1 = \pi\sqrt{\frac{E_{0.05}}{f_{c,0,k}}} \tag{16}$$

and

$$\lambda_{rel,c} = \frac{\lambda}{\lambda_1}. \tag{17}$$

If $\lambda_{rel,c} \ge 0.3$ turns out

$$k_{crit,c} = \frac{1}{k + \sqrt{k^2 - \lambda_{rel,c}^2}} \tag{18}$$

with

$$k = 0.5\left[1 + \beta_c\left(\lambda_{rel,c} - 0.3\right) + \lambda_{rel,c}^2\right] \tag{19}$$

otherwise if $\lambda_{rel,c} \le 0.3$ is $k_{crit,c} = 1$. In the previous relationships, β_c it is 0.1 for elements made of glued laminated timber, while its value is 0.2 for elements made of solid timber. In the limit situation, Eq. (15) becomes

$$\frac{N_{Ed}}{A} = k_{crit,c} \times f_{c,0,d} \tag{20}$$

using Eq. (14), Eq. (20) is transformed into the following

$$\frac{N_{Ed} i_{y0}^2 \lambda^2}{L_0^2 A_0} = k_{crit,c} \times f_{c,0,d} \tag{21}$$

separating the variables, we obtain

$$\frac{N_{Ed} i_{y0}^2}{f_{c,0,d} L_0^2 A_0} = \frac{k_{crit,c}}{\lambda^2} \tag{22}$$

from Eq. (17), we then obtains

$$\lambda = \lambda_{rel,c} \times \lambda_1. \tag{23}$$

By substituting Eq. (23) into Eq. (22), we obtain

$$\frac{N_{Ed} i_{y0}^2}{f_{c,0,d} L_0^2 A_0} = \frac{k_{crit,c}}{\lambda_{rel,c}^2 \lambda_1^2} \tag{24}$$

from which

$$\frac{k_{crit,c}}{\lambda_{rel,c}^2} = \frac{\pi^2}{\left(A_0/i_{y0}^2\right)} \frac{N_{Ed}}{f_{c,0,d} L_0^2} \frac{E_{0.05}}{f_{c,0,k}} \tag{25}$$

by inserting the section coefficient

$$\alpha_\varphi = \frac{A_0}{i_{y0}^2} \tag{26}$$

and since

$$f_{c,0,d} = k_{mod} \frac{f_{c,0,k}}{\gamma_M} \tag{27}$$

we obtain

$$\frac{k_{crit,c}\left(\lambda_{rel,c}\right)}{\lambda_{rel,c}^2} = \frac{\pi^2}{\alpha_\varphi} \frac{\gamma_M N_{Ed}}{k_{mod} L_0^2} \frac{E_{0.05}}{f_{c,0,k}^2}, \tag{28}$$

then introducing the function

$$\Gamma_{sl\varphi}\left(\lambda_{rel,c}\right) = \frac{k_{crit,c}\left(\lambda_{rel,c}\right)}{\lambda_{rel,c}^2} \tag{29}$$

and the coefficient

$$G_{sl\varphi} = \frac{\pi^2}{\alpha_\varphi} \frac{\gamma_M N_{Ed}}{k_{mod} L_0^2} \frac{E_{0.05}}{f_{c,0,k}^2}, \tag{30}$$

Eq. (28) can be rewritten as follows

$$\Gamma_{sl\varphi}\left(\lambda_{rel,c}\right) = G_{sl\varphi}. \tag{31}$$

The function $\Gamma_{sl\varphi}$, associates with the generic value of relative slenderness $\lambda_{rel,c}$, the corresponding value $\Gamma_{sl\varphi}$. In solving the design problem, it is important to determine a function that can interpolate, with sufficient precision, the values assumed by the inverse function $\Gamma_{sl\varphi}^{-1}$. Analysing the results provided by the analysis of 4860 interpolating functions, it was concluded that the function

$$\lambda_{rel,c} = a + b_1 \ln\left(c\Gamma_{sl\varphi}\right) + b_2 \ln^2\left(c\Gamma_{sl\varphi}\right) + b_3 \ln^3\left(c\Gamma_{sl\varphi}\right) + b_4 \ln^4\left(c\Gamma_{sl\varphi}\right) + b_5 \ln^5\left(c\Gamma_{sl\varphi}\right) \tag{32}$$

can interpolate the values assumed by the inverse function $\Gamma_{sl\varphi}^{-1}$ with a considerable degree of accuracy. By virtue of Eq. (31), it is also possible to pose

$$\lambda_{rel,c} = a + b_1 \ln\left(cG_{sl\varphi}\right) + b_2 \ln^2\left(cG_{sl\varphi}\right) + b_3 \ln^3\left(cG_{sl\varphi}\right) + b_4 \ln^4\left(cG_{sl\varphi}\right) + b_5 \ln^5\left(cG_{sl\varphi}\right) \tag{33}$$

or finally

$$\lambda_{rel,c} = a + b_1\psi + b_2\psi^2 + b_3\psi^3 + b_4\psi^4 + b_5\psi^5 \tag{34}$$

in which it is placed

$$\psi = \ln\left(cG_{sl\varphi}\right). \tag{35}$$

In Eq. (34), for elements made of glued laminated timber, the numerical values of the coefficients a, c, b_1, b_2, b_3, b_4, b_5, can be approximated by Eqs. (36)–(42):

$$a = \sqrt{8e - 6\pi - 2\ln 2} \tag{36}$$

$$c = \frac{2\left(3+\pi\right)^2}{9\pi} \tag{37}$$

$$b_1 = \frac{6}{7\pi} - \frac{\pi}{5} \tag{38}$$

$$b_2 = \frac{1}{3 - e + 7e^2} \tag{39}$$

$$b_3 = -\frac{3}{100e\pi} \tag{40}$$

$$b_4 = \frac{e^\pi}{\pi^{e+3} \times e^{e+2/\pi}} \tag{41}$$

$$b_5 = \frac{2\pi^2 - 5\pi - 4}{5\left(1 + \pi + 7\pi^2\right)} \tag{42}$$

while, for elements made of solid timber, the coefficients a, c, b_1, b_2, b_3, b_4, b_5, can be approximated by the following other analytical expressions (Eqs. (43)–(49)):

$$a = \frac{3}{5} \tag{43}$$

$$c = e^\pi + \ln\pi - 13\ln\left(2\pi\right) \tag{44}$$

$$b_1 = 8 - \frac{8}{\pi} + \frac{1}{\sqrt{\pi}} - 2\pi \tag{45}$$

$$b_2 = \frac{e^2 - 4}{5\left(3 - e + 3e^2\right)} \tag{46}$$

$$b_3 = \frac{\pi - 3}{8\pi} \tag{47}$$

$$b_4 = \frac{2\pi^2 - \pi - 16}{\pi\left(1 + 42\pi\right)} \tag{48}$$

$$b_5 = -\frac{1}{2} \frac{e^{1+1/e}}{e^{e+1/\pi} \pi^{1+2e}} \sin\left(2e\pi\right). \tag{49}$$

For glued laminated timber elements, Eqs. (32)–(34) are valid for values of $G_{sl\varphi}$ that meet the condition $0.001569 < G_{sl\varphi} < 11.111$. On the other hand, for elements made of solid timber, Eqs. (32)–(34) are valid for values of $G_{sl\varphi}$ which comply with the condition $0.001539 < G_{sl\varphi} < 11.111$. Equations (36)–(49) were deduced by performing numerous attempts. Having deduced, through Eq. (34), the relative slenderness that the element must have, the absolute slenderness is calculated using Eq. (23). At this point, by means of Eq. (13), the scale factor w is evaluated. Finally, by means of Eq. (8), we evaluate the effective dimensions that it is necessary to assign to section S, in order to allow it to resist the design load. Although the cross-sections most commonly used in the construction of slender struts are of a few different types (square, rectangular, circular), it has been decided to refer to a doubly simmetric cross-section with a generic shape in order to avoid having to adapt the demonstration each time, to the different inertia characteristics of the section. Thus, by means of a single demonstration, it was possible to summarise the various frequent cases in practical applications. Before continuing, it should be noted that the function $\Gamma_{sl\varphi}^{-1}$, is decreasing as the values of the coefficient $G_{sl\varphi}$ increase. This means that the dominant load combination will be the one with the highest value of $G_{sl\varphi}$. From one load combination to another, in addition to the value of the axial action N_{Ed}, the value of the coefficient k_{mod} may also change. In this way, the dominant load combination will be the one with the highest value of the modified axial action.

$$N_{Ed}^{*} = \frac{N_{Ed}}{k_{mod}} \tag{50}$$

At this point, it is important to express the value that the section coefficient must assume for the main types of section used in practice.

2.1.1 Rectangular sections with fixed side ratio or square sections

Using the millimetre as the unit of length, consider a reference section S_0, having a rectangular shape, with base h_{X0} = 1 mm (conventionally unitary), and with height $h_{Y0} = \varphi\, h_{X0}$ or $h_{Y0} = \varphi$. In this way, the shape ratio of the reference section S_0 is

$$\frac{h_{Y0}}{h_{X0}} = \varphi \geq 1 \tag{51}$$

$$\frac{h_Y}{h_X} = \frac{wh_{Y0}}{wh_{X0}} = \varphi \tag{52}$$

for the reference section, therefore results in

$$A_0 = \varphi \tag{53}$$

$$J_{y0} = \frac{\varphi}{12} \tag{54}$$

$$i_{y0} = \sqrt{\frac{J_{y0}}{A_0}} = \frac{1}{2\sqrt{3}} \tag{55}$$

hence

$$\alpha_{\varphi} = \frac{A_0}{i_{y0}^2} = 12\varphi \tag{56}$$

to design elements with a square cross-section, one should set $\varphi = 1$.

2.1.2 Circular sections

Consider a circular reference section S_0 with a diameter D_0 = 1 mm (conventionally unitary). For this section, the result is

$$A_0 = \frac{\pi}{4} \tag{57}$$

$$J_{y0} = \frac{\pi}{64} \tag{58}$$

$$i_{y0} = \sqrt{\frac{J_{y0}}{A_0}} = \frac{1}{4} \tag{59}$$

hence

$$\alpha_{\varphi} = \frac{A_0}{i_{y0}^2} = 4\pi \; . \tag{60}$$

2.2 Numerical example

A strut with a square cross-section ($\varphi = 1$) with a effective length L_0 = 4618 mm is designed. The service class is 1. Use glued laminated timber with the following mechanical properties: $f_{c,0,k}$ = 24 MPa, $E_{0.05}$ = 9600 MPa. The element have to resist to an axial load N_{Ed} = 5.68 kN. The load duration class shall be "permanent". In accordance with NTC 2018 (Ministero delle Infrastrutture e dei Trasporti, 2018), It is assumed $\gamma_M = 1.45$ and $k_{mod} = 0.6$ furthermore:

$$G_{sl\varphi} = \frac{\pi^2}{12\varphi} \frac{\gamma_M N_{Ed}}{k_{\text{mod}} L_0^2} \frac{E_{0.05}}{f_{c,0,k}^2} = 0.00882$$

$$\left(0.001569 < G_{sl\varphi} < 11.111\right)$$

$$\psi = \ln\left[\frac{2(3+\pi)^2}{9\pi} \times G_{sl\varphi}\right] = -3.749$$

$$\lambda_{rel,c} = a + b_1\psi + b_2\psi^2 + b_3\psi^3 + b_4\psi^4 + b_5\psi^5 = 3.183$$

$$\lambda_1 = \pi\sqrt{\frac{E_{0.05}}{f_{c,0,k}}} = 62.831$$

$$\lambda = \lambda_{rel,c} \times \lambda_1 = 200$$

$$i_{y0} = \frac{1}{2\sqrt{3}}$$

$$w = \frac{L_0}{\lambda \times i_{y0}} = 80$$

$$h_X = w \times h_{X0} = w \times 1 = 80 \text{ mm}$$

$$h_Y = w \times h_{Y0} = w \times \varphi = 80 \text{ mm}.$$

If we carry out the strength verification, we have

$$A = h_X h_Y = 6400 \text{ mm}^2$$

$$J_{\min} = \frac{h_Y h_x^3}{12} = 3413333.\overline{3} \text{ mm}^4$$

$$i_{\min} = \sqrt{\frac{J_{\min}}{A}} = 23.094 \text{ mm}$$

$$\lambda = \frac{L_0}{i_{\min}} = 199.965$$

$$\lambda_{rel,c} = \frac{\lambda}{\lambda_1} = 3.182$$

$$k = 0.5\left[1 + \beta_c\left(\lambda_{rel,c} - 0.3\right) + \lambda_{rel,c}^2\right] = 5.708$$

$$k_{crit,c} = \frac{1}{k + \sqrt{k^2 - \lambda_{rel,c}^2}} = 0.0957$$

$$f_{c,0,d} = k_{mod}\frac{f_{c,0,k}}{\gamma_M} = 9.931 \text{ MPa}$$

$$N_{Rd} = k_{crit,c} \times f_{c,0,d} A = 6.083 \text{ kN} > N_{Ed}.$$

In particular, if we define "Element Utilization Factor" as the following quantity

$$\text{EUF}[\%] = 100 \times \frac{N_{Ed}}{N_{Rd}} = 93.36\%.$$

This means that, in this example, the proposed method uses the section to 93.36% of its possibilities. The strength N_{Rd} is evaluated according to the current standards Eurocode 5 (CEN, 2005a) and NTC 2018 (Ministero delle Infrastrutture e dei Trasporti, 2018), for this reason the Element Utilization Factor represents a direct comparison between the results obtained from the presented method and the stability check procedure provided by the technical standards Eurocode 5 (CEN, 2005a) and NTC 2018 (Ministero delle Infrastrutture e dei Trasporti, 2018). Before examining the other design cases, in the following section we will validate the relationships set out in this section by presenting several design examples in tabular form. The results obtained in this calculation example are also shown in the first row of Table 1.

2.3 Numerical applications (case 1)

The accuracy of the above relationships is now put to the test by carrying out several design examples. In Tables 1 and 2, the dimensions h_X and h_Y that need to be assigned to rectangular sections in order to resist the assigned axial action N_{Ed} are deduced. The material characteristics are assumed to be known, and the form ratio of the section is assumed to be assigned. It is also assumed that the effective length of the elements is assigned. In particular, Table 1 refers to glued laminated timber elements for service class 1 and permanent loads, while Table 2 refers to the case of elements made of solid timber, service class 2, and long term actions. In Tables 3 and 4, the diameter D, which must be assigned to a circular section in order to resist the assigned axial action N_{Ed}, is deduced. The characteristics of the materials and the effective length of the element are assumed to be known. In particular, Table 3 refers to the case of elements made of glued laminated timber, service class 3 and medium term loads, while Table 4 refers to the case of elements made of solid timber, service class 1, and short term actions. Tables 1–6 show the slenderness of the newly designed elements is indicated and the quality of the design obtained is measured using the Element Utilization Factor EUF.

3 Proposed analytical method (Case 2)

3.1 Design of slender struts with a generic doubly symmetric cross-section and fixed strong side (Case 2)

In this case, let us suppose that, in the transition from section S_0 to section S, the coordinates of the points of the section are transformed in accordance with Eq. (61):

$$x = w \times x_0$$
$$y = y_0. \quad (61)$$

Also in this case, (x_0,y_0) are the simmetry axis and principal axis for S_0. The origin of the (x_0,y_0) coordinate system is conicident with the centroid G_0 of the section S_0. For the section S, the (x,y) axis are simmetry axis and

Table 1 Rectangular section with fixed side ratio, glued laminated timber, service class 1, load duration class "permanent", $k_{mod} = 0.6$, $\gamma_M = 1.45$

φ	$f_{c,0,k}$ [MPa]	$E_{0.05}$ [MPa]	L_0 [mm]	N_{Ed} [kN]	h_Y [mm]	h_X [mm]	λ	EUF [%]
1	24	9600	4618	5.68	80	80	200	93.36
1	24	9600	4156	6.98	80	80	180	93.28
1	24	9600	3695	8.84	80	80	160	93.85
1	24	9600	3233	11.64	80	80	140	95.28
1	24	9600	2771	16.03	80	80	120	97.45
1	24	9600	2309	23.13	80	80	100	99.55
1	24	9600	1847	34.14	80	80	80	98.51
1	24	9600	1385	48.46	80	80	60	94.87
1	24	9600	923	60.08	80	80	40	99.68
1	24	9600	461	61.49	80	80	20	96.93
2	26	10100	5773	18.7	200	100	200	93.46
2	26	10100	5196	22.97	200	100	180	93.35
2	26	10100	4618	29.08	200	100	160	93.81
2	26	10100	4041	38.23	200	100	140	95.09
2	26	10100	3464	52.66	200	100	120	97.27
2	26	10100	2886	76.15	200	100	100	99.48
2	26	10100	2309	112.97	200	100	80	98.73
2	26	10100	1732	161.82	200	100	60	94.87
2	26	10100	1154	202.73	200	100	40	99.59
2	26	10100	577	207.19	200	100	20	96.53
3	28	10500	6928	42.03	360	120	200	93.5
3	28	10500	6235	51.61	360	120	180	93.33
3	28	10500	5542	65.26	360	120	160	93.69
3	28	10500	4849	85.75	360	120	140	94.88
3	28	10500	4156	118.15	360	120	120	97.01
3	28	10500	3464	171.09	360	120	100	99.35
3	28	10500	2771	255.54	360	120	80	98.96
3	28	10500	2078	370.21	360	120	60	94.9
3	28	10500	1385	469.53	360	120	40	99.46
3	28	10500	692	480.06	360	120	20	96.21
4	32	11800	8082	85.8	560	140	200	93.55
4	32	11800	7274	105.3	560	140	180	93.33
4	32	11800	6466	133.08	560	140	160	93.65
4	32	11800	5658	174.75	560	140	140	94.78
4	32	11800	4849	240.79	560	140	120	96.89
4	32	11800	4041	349.1	560	140	100	99.28
4	32	11800	3233	522.86	560	140	80	99.07
4	32	11800	2424	761.64	560	140	60	94.93
4	32	11800	1616	971.68	560	140	40	99.39
4	32	11800	808	993.96	560	140	20	96.08

Table 2 Rectangular section with fixed side ratio, solid timber, service class 2, load duration class "long term actions", $k_{mod} = 0.7$, $\gamma_M = 1.5$

φ	$f_{c,0,k}$ [MPa]	$E_{0.05}$ [MPa]	L_0 [mm]	N_{Ed} [kN]	h_Y [mm]	h_X [mm]	λ	EUF [%]
1	18	7000	5773	7.29	100	100	200	96.01
1	18	7000	5196	8.95	100	100	180	96.18
1	18	7000	4618	11.29	100	100	160	96.73
1	18	7000	4041	14.69	100	100	140	97.63
1	18	7000	3464	19.86	100	100	120	98.87
1	18	7000	2886	27.97	100	100	100	99.83
1	18	7000	2309	40.51	100	100	80	99.02
1	18	7000	1732	57.94	100	100	60	96.78
1	18	7000	1154	75.27	100	100	40	99.5
1	18	7000	577	81.6	100	100	20	97.63
2	21	7900	5773	16.47	200	100	200	96
2	21	7900	5196	20.23	200	100	180	96.2
2	21	7900	4618	25.51	200	100	160	96.7
2	21	7900	4041	33.19	200	100	140	97.55
2	21	7900	3464	44.89	200	100	120	98.77
2	21	7900	2886	63.34	200	100	100	99.8
2	21	7900	2309	92.19	200	100	80	99.17
2	21	7900	1732	132.98	200	100	60	96.82
2	21	7900	1154	174.6	200	100	40	99.4
2	21	7900	577	189.77	200	100	20	97.42
3	25	11000	6928	49.36	360	120	200	96.16
3	25	11000	6235	60.68	360	120	180	96.5
3	25	11000	5542	76.54	360	120	160	97.16
3	25	11000	4849	99.58	360	120	140	98.17
3	25	11000	4156	134.22	360	120	120	99.33
3	25	11000	3464	187.12	360	120	100	99.87
3	25	11000	2771	265.76	360	120	80	98.44
3	25	11000	2078	368.21	360	120	60	96.84
3	25	11000	1385	460.28	360	120	40	99.77
3	25	11000	692	499.46	360	120	20	99.16
4	26	10000	5196	33.76	360	90	200	96.04
4	26	10000	4676	41.48	360	90	180	96.25
4	26	10000	4156	52.29	360	90	160	96.74
4	26	10000	3637	68.03	360	90	140	97.63
4	26	10000	3117	92.02	360	90	120	98.86
4	26	10000	2598	129.58	360	90	100	99.84
4	26	10000	2078	188.06	360	90	80	99.08
4	26	10000	1558	269.83	360	90	60	96.79
4	26	10000	1039	351.54	360	90	40	99.47
4	26	10000	519	381.51	360	90	20	97.57

principal axis. The origin of the (x,y) coodinate system is coincident with the centroid G of the S section. From Eq. (61), the Eq. (62) is deduced

$$\begin{aligned} h_X &= w \times h_{X0} \\ h_Y &= h_{Y0}. \end{aligned} \quad (62)$$

Table 3 Circular section, glued laminated timber, service class 3, load duration class "medium term loads", $k_{mod} = 0.65$, $\gamma_M = 1.45$

$f_{c,0,k}$ [MPa]	$E_{0.05}$ [MPa]	L_0 [mm]	N_{Ed} [kN]	D [mm]	λ	EUF [%]
24	9600	2000	1.208	40	200	93.28
24	9600	1800	1.485	40	180	93.33
24	9600	1600	1.881	40	160	93.88
24	9600	1400	2.475	40	140	95.25
24	9600	1200	3.41	40	120	97.47
24	9600	1000	4.919	40	100	99.56
24	9600	800	7.255	40	80	98.47
24	9600	600	10.304	40	60	94.87
24	9600	400	12.778	40	40	99.68
24	9600	200	13.071	40	20	96.87
26	10100	4000	5.09	80	200	93.45
26	10100	3600	6.25	80	180	93.29
26	10100	3200	7.91	80	160	93.75
26	10100	2800	10.4	80	140	95.03
26	10100	2400	14.33	80	120	97.22
26	10100	2000	20.72	80	100	99.46
26	10100	1600	30.75	80	80	98.73
26	10100	1200	44.05	80	60	94.85
26	10100	800	55.19	80	40	99.59
26	10100	400	56.4	80	20	96.51
28	10500	6000	11.92	120	200	93.5
28	10500	5400	14.63	120	180	93.3
28	10500	4800	18.5	120	160	93.66
28	10500	4200	24.31	120	140	94.87
28	10500	3600	33.49	120	120	97
28	10500	3000	48.52	120	100	99.35
28	10500	2400	72.46	120	80	98.96
28	10500	1800	104.97	120	60	94.9
28	10500	1200	133.15	120	40	99.45
28	10500	600	136.11	120	20	96.19
32	11800	8000	23.83	160	200	93.54
32	11800	7200	29.25	160	180	93.33
32	11800	6400	36.97	160	160	93.65
32	11800	5600	48.55	160	140	94.79
32	11800	4800	66.87	160	120	96.88
32	11800	4000	96.96	160	100	99.27
32	11800	3200	145.25	160	80	99.07
32	11800	2400	211.53	160	60	94.93
32	11800	1600	269.93	160	40	99.38
32	11800	800	276.13	160	20	96.07

Table 4 Circular section, solid timber, service class 1, load duration class "short term actions", $k_{mod} = 0.9$, $\gamma_M = 1.5$

$f_{c,0,k}$ [MPa]	$E_{0.05}$ [MPa]	L_0 [mm]	N_{Ed} [kN]	D [mm]	λ	EUF [%]
18	7000	2000	1.178	40	200	96.04
18	7000	1800	1.447	40	180	96.25
18	7000	1600	1.824	40	160	96.76
18	7000	1400	2.374	40	140	97.67
18	7000	1200	3.209	40	120	98.88
18	7000	1000	4.518	40	100	99.85
18	7000	800	6.544	40	80	99.03
18	7000	600	9.361	40	60	96.78
18	7000	400	12.159	40	40	99.5
18	7000	200	13.183	40	20	97.63
21	7900	4000	5.323	80	200	96.03
21	7900	3600	6.53	80	180	96.1
21	7900	3200	8.24	80	160	96.69
21	7900	2800	10.72	80	140	97.53
21	7900	2400	14.5	80	120	98.74
21	7900	2000	20.46	80	100	99.81
21	7900	1600	29.78	80	80	99.16
21	7900	1200	42.96	80	60	96.8
21	7900	800	56.4	80	40	99.39
21	7900	400	61.31	80	20	97.41
25	11000	6000	16.61	120	200	96.14
25	11000	5400	20.42	120	180	96.49
25	11000	4800	25.75	120	160	97.13
25	11000	4200	33.5	120	140	98.14
25	11000	3600	45.16	120	120	99.33
25	11000	3000	62.98	120	100	99.87
25	11000	2400	89.44	120	80	98.44
25	11000	1800	123.91	120	60	96.83
25	11000	1200	154.91	120	40	99.77
25	11000	600	168.03	120	20	99.12
26	10000	8000	26.93	160	200	96.02
26	10000	7200	33.09	160	180	96.25
26	10000	6400	41.71	160	160	96.76
26	10000	5600	54.27	160	140	97.63
26	10000	4800	73.39	160	120	98.86
26	10000	4000	103.38	160	100	99.84
26	10000	3200	149.99	160	80	99.08
26	10000	2400	215.17	160	60	96.79
26	10000	1600	280.46	160	40	99.47
26	10000	800	304.32	160	20	97.55

In this case, when moving from section S_0 to section S, the area of the section will vary according to Eq. (63):

$$A = \iint dxdy = \iint d\left(wx_0\right)d\left(y_0\right) = w\iint dx_0dy_0 = wA_0 \quad (63)$$

Table 5 Rectangular section, strong side fixed, glued laminated timber, service class 2, load duration class "instantaneous action", $k_{mod} = 1.1$, $\gamma_M = 1.45$

$f_{c,0,k}$ [MPa]	$E_{0.05}$ [MPa]	h_Y [mm]	L_0 [mm]	N_{Ed} [kN]	h_X [mm]	λ	EUF [%]
24	9600	160	5773	27.56	100	200	98.85
24	9600	160	5196	33.81	100	180	98.62
24	9600	160	4618	42.26	100	160	97.86
24	9600	160	4041	54.93	100	140	98.09
24	9600	160	3464	74.78	100	120	99.2
24	9600	160	2886	105.2	100	100	98.77
24	9600	160	2309	158.3	100	80	99.68
24	9600	160	1732	229.68	100	60	98.14
24	9600	160	1154	275.71	100	40	99.8
24	9600	160	577	287.99	100	20	99.05
26	10100	200	6928	43.48	120	200	98.78
26	10100	200	6235	53.44	120	180	98.71
26	10100	200	5542	66.77	120	160	97.92
26	10100	200	4849	86.67	120	140	97.98
26	10100	200	4156	118.13	120	120	99.14
26	10100	200	3464	166.36	120	100	98.82
26	10100	200	2771	250.66	120	80	99.58
26	10100	200	2078	368.68	120	60	98.23
26	10100	200	1385	446.73	120	40	99.75
26	10100	200	692	467.3	120	20	98.96
28	10500	240	8082	63.27	140	200	98.69
28	10500	240	7274	77.91	140	180	98.8
28	10500	240	6466	97.35	140	160	98.02
28	10500	240	5658	126.11	140	140	97.88
28	10500	240	4849	172	140	120	99.06
28	10500	240	4041	242.88	140	100	98.89
28	10500	240	3233	366.13	140	80	99.45
28	10500	240	2424	547.2	140	60	98.36
28	10500	240	1616	671.02	140	40	99.68
28	10500	240	808	703.34	140	20	98.86
32	11800	280	9237	94.76	160	200	98.63
32	11800	280	8313	116.82	160	180	98.84
32	11800	280	7390	146	160	160	98.08
32	11800	280	6466	189	160	140	97.85
32	11800	280	5542	257.74	160	120	99.01
32	11800	280	4618	364.49	160	100	98.93
32	11800	280	3695	549.45	160	80	99.38
32	11800	280	2771	827.11	160	60	98.43
32	11800	280	1847	1020.59	160	40	99.65
32	11800	280	923	1071.06	160	20	98.82

Table 6 Rectangular section with strong side fixed, solid timber, service class 3, load duration class "permanent action", $k_{mod} = 0.5$, $\gamma_M = 1.5$

$f_{c,0,k}$ [MPa]	$E_{0.05}$ [MPa]	h_Y [mm]	L_0 [mm]	N_{Ed} [kN]	h_X [mm]	λ	EUF [%]
18	7000	120	4618	5.05	80	200	96.98
18	7000	120	4156	6.22	80	180	97.44
18	7000	120	3695	7.82	80	160	97.74
18	7000	120	3233	10.17	80	140	98.58
18	7000	120	2771	13.73	80	120	99.66
18	7000	120	2309	18.8	80	100	97.87
18	7000	120	1847	26.97	80	80	96.12
18	7000	120	1385	40.67	80	60	99.03
18	7000	120	923	51.87	80	40	99.99
18	7000	120	461	56.72	80	20	98.96
21	7900	140	5773	8.32	100	200	96.99
21	7900	140	5196	10.24	100	180	97.38
21	7900	140	4618	12.89	100	160	97.72
21	7900	140	4041	16.75	100	140	98.46
21	7900	140	3464	22.64	100	120	99.63
21	7900	140	2886	31.13	100	100	98.1
21	7900	140	2309	44.64	100	80	96.04
21	7900	140	1732	67.93	100	60	98.92
21	7900	140	1154	87.81	100	40	99.98
21	7900	140	577	96.22	100	20	98.79
25	11000	160	6928	15.86	120	200	97.33
25	11000	160	6235	19.49	120	180	97.63
25	11000	160	5542	24.52	120	160	98.05
25	11000	160	4849	31.95	120	140	99.22
25	11000	160	4156	42.65	120	120	99.42
25	11000	160	3464	57.66	120	100	96.94
25	11000	160	2771	82.83	120	80	96.65
25	11000	160	2078	119.87	120	60	99.31
25	11000	160	1385	146.36	120	40	99.93
25	11000	160	692	159.57	120	20	99.8
26	10000	180	8082	18.95	140	200	97.02
26	10000	180	7274	23.34	140	180	97.49
26	10000	180	6466	29.35	140	160	97.77
26	10000	180	5658	38.15	140	140	98.56
26	10000	180	4849	51.54	140	120	99.68
26	10000	180	4041	70.64	140	100	97.96
26	10000	180	3233	101.3	140	80	96.09
26	10000	180	2424	153.29	140	60	99
26	10000	180	1616	196.34	140	40	99.99
26	10000	180	808	214.81	140	20	98.89

while the minimum moment of inertia of the section will vary according to Eq. (64):

$$J_{\min} = \iint x^2 \times dxdy = \iint w^2 x_0^2 \times d\left(wx_0\right) d\left(y_0\right) = w^3 \iint x_0^2 \times dx_0 dy_0 = w^3 J_{y0}. \quad (64)$$

The aim is to determine which value must be assigned to the scaling factor w in order for the section S to satisfy the strength verification required by the standards Eurocode 5 (CEN, 2005a) and NTC 2018 (Ministero delle Infrastrutture e dei Trasporti, 2018). For the effective section S, we have

$$i_{\min} = \sqrt{\frac{J_{\min}}{A}} = \sqrt{\frac{w^3 J_{y0}}{wA_0}} = w\sqrt{\frac{J_{y0}}{A_0}} = w \times i_{y0} \tag{65}$$

additionally

$$\lambda = \frac{L_0}{i_{\min}} = \frac{L_0}{w \times i_{y0}} \tag{66}$$

hence

$$w = \frac{L_0}{\lambda \times i_{y0}} \tag{67}$$

for Eq. (63), then results

$$A = \frac{L_0}{i_{y0}} \frac{A_0}{\lambda} . \tag{68}$$

By substituting Eq. (68) into Eq. (20) and separating the variables, we obtain

$$\frac{k_{crit,c}}{\lambda} = \frac{N_{Ed} i_{y0}}{f_{c,0,d} A_0 L_0} , \tag{69}$$

then using Eq. (23) we obtain

$$\frac{k_{crit,c}}{\lambda_{rel,c} \times \lambda_1} = \frac{N_{Ed} i_{y0}}{f_{c,0,d} A_0 L_0} \tag{70}$$

or even

$$\frac{k_{crit,c}}{\lambda_{rel,c}} = \frac{N_{Ed} i_{y0}}{f_{c,0,d} A_0 L_0} \lambda_1 . \tag{71}$$

Equation (71), by virtue of Eq. (16), is equivalent to

$$\frac{k_{crit,c}}{\lambda_{rel,c}} = \pi \frac{N_{Ed} i_{y0}}{f_{c,0,d} A_0 L_0} \sqrt{\frac{E_{0.05}}{f_{c,0,k}}} \tag{72}$$

or also

$$\frac{k_{crit,c}}{\lambda_{rel,c}} = \frac{\pi}{\alpha_f} \frac{N_{Ed}}{f_{c,0,d} L_0} \sqrt{\frac{E_{0.05}}{f_{c,0,k}}} , \tag{73}$$

where the section coefficient was introduced

$$\alpha_f = \frac{A_0}{i_{y0}} , \tag{74}$$

furthermore, being

$$f_{c,0,d} = k_{mod} \frac{f_{c,0,k}}{\gamma_M} , \tag{75}$$

Eq. (73) can also be rewritten as follows

$$\frac{k_{crit,c}}{\lambda_{rel,c}} = \frac{\pi}{\alpha_f} \frac{\gamma_M N_{Ed}}{k_{mod} L_0} \sqrt{\frac{E_{0.05}}{f_{c,0,k}^3}} . \tag{76}$$

Equation (76) can then be transformed into Eq. (77):

$$\Gamma_{slf}\left(\lambda_{rel,c}\right) = G_{slf} \tag{77}$$

in which are introduced, the function

$$\Gamma_{slf}\left(\lambda_{rel,c}\right) = \frac{k_{crit,c}}{\lambda_{rel,c}} \tag{78}$$

and the coefficient

$$G_{slf} = \frac{\pi}{\alpha_f} \frac{\gamma_M N_{Ed}}{k_{mod} L_0} \sqrt{\frac{E_{0.05}}{f_{c,0,k}^3}} . \tag{79}$$

The function Γ_{slf}, associates the corresponding value Γ_{slf} with the generic value of relative slenderness $\lambda_{rel,c}$. In solving the design problem, it is important to determine a function that can approximate, with sufficient accuracy, the values of the inverse function Γ_{slf}^{-1}. Analysing the results provided by 4884 interpolating functions, it was concluded that the function

$$\begin{aligned}\lambda_{rel,c} = a - b_1 \times e^{-\Gamma_{slf}/c_1} + b_2 \times e^{-\Gamma_{slf}/c_2} + b_3 \times e^{-\Gamma_{slf}/c_3} \\ + b_4 \times e^{-\Gamma_{slf}/c_4}\end{aligned} \tag{80}$$

can interpolate with a satisfactory degree of accuracy the values assumed by the inverse function Γ_{slf}^{-1}. Furthermore, for Eq. (77), we can also write

$$\begin{aligned}\lambda_{rel,c} = a - b_1 \times e^{-G_{slf}/c_1} + b_2 \times e^{-G_{slf}/c_2} + b_3 \times e^{-G_{slf}/c_3} \\ + b_4 \times e^{-G_{slf}/c_4}\end{aligned} \tag{81}$$

In Eq. (81), for elements made of glued laminated timber, the numerical values of the coefficients a, b_1, b_2, b_3, b_4, c_1, c_2, c_3, c_4, can be approximated by Eqs. (82)–(90):

$$a = \frac{e^{e-\pi}}{\pi} \tag{82}$$

$$b_1 = -\sqrt{\frac{7}{31}}\pi \tag{83}$$

$$b_2 = \ln\left(9 + 3\pi - 4e\right) \tag{84}$$

$$b_3 = e^{\pi+e} \times \frac{\pi^2 e^{4/e}}{\pi^{3e}} \tag{85}$$

$$b_4 = 1 + e - \frac{1}{e} \tag{86}$$

$$c_1 = 2\pi - \frac{16}{\pi} \tag{87}$$

$$c_2 = \frac{3}{14\pi^2} \tag{88}$$

$$c_3 = \frac{3}{e} - 1 \tag{89}$$

$$c_4 = \frac{1}{7\pi^3} \tag{90}$$

while, for elements made of solid timber, the coefficients $a, b_1, b_2, b_3, b_4, c_1, c_2, c_3, c_4$, can be approximated by these other analytical expressions:

$$a = \frac{1}{e^2}\sqrt{\frac{7}{10}\pi \ln \pi} \tag{91}$$

$$b_1 = -\frac{7\pi^2}{50} \tag{92}$$

$$b_2 = \frac{7}{5} \tag{93}$$

$$b_3 = \ln\left(\frac{3\pi^2}{2} - 3e\right) \tag{94}$$

$$b_4 = \ln\left(10e - 4 - \sqrt{2}\right) \tag{95}$$

$$c_1 = \frac{8\pi - 9\sqrt{\pi} - 2}{6} \tag{96}$$

$$c_2 = e^2 \times \frac{\pi^{1+2e}}{e^{3e+\pi}} \sin\left(e\pi\right) \tag{97}$$

$$c_3 = \frac{3}{14\pi^2} \tag{98}$$

$$c_4 = \frac{8}{15\pi^4}\ . \tag{99}$$

For glued laminated timber elements, the Eqs. (80) and (81) apply for values of G_{slf}, which comply with the condition $0.007846 < G_{slf} < 3.333$. On the other hand, for elements made of solid timber, Eqs. (80) and (81) apply for values of G_{slf} which comply with the condition $0.007698 < G_{slf} < 3.333$. Equations (82)–(99) were deduced by performing numerous attempts. Having deduced the slenderness of the element through Eq. (81), the scale factor w is evaluated through Eq. (67). Finally, by means of Eq. (62), the effective dimensions which must be assigned to section S in order for it to resist the design action are evaluated. Before continuing, it should be noted that the function Γ_{slf}^{-1}, is decreasing as the values of the coefficient G_{slf} increase. This means that the dominant load combination will be the one with the highest value of G_{slf}. From one load combination to another, in addition to the value of the axial action N_{Ed}, the value of the coefficient k_{mod} may also change. In this way, the dominant load combination will be the one with the highest value of the modified axial load.

$$N_{Ed}^* = \frac{N_{Ed}}{k_{mod}}$$

At this point, it is important to express the value that the section coefficient must assume for the main types of section used in practice.

3.1.1 Rectangular sections with strong side fixed

Using the millimetre as the unit of length, consider a reference section S_0, having a rectangular shape, with base h_{X0} = 1 mm (conventionally unitary), and with height $h_{Y0} = h_Y$ fixed. For the reference section, the result is then

$$A_0 = h_Y \tag{100}$$

$$J_{y0} = \frac{h_Y}{12} \tag{101}$$

$$i_{y0} = \sqrt{\frac{J_{y0}}{A_0}} = \frac{1}{2\sqrt{3}}\ , \tag{102}$$

therefore

$$\alpha_f = \frac{A_0}{i_{y0}} = 2\sqrt{3}h_Y\ . \tag{103}$$

3.2 Numerical applications (case 2)

The accuracy of the above relationships is now tested by carrying out several design examples. In Tables 5 and 6, the dimension h_X, which needs to be assigned to the sections in order to make them able to resist to the assigned axial action N_{Ed}, is calculated. The dimension h_Y of the section is assumed to be known. The characteristics of the materials and the effective length of the elements are also assumed to be known. In particular, Table 5, refers to glued laminated timber elements, service class 2 and instantaneous actions. Table 6, refers instead to the case of elements made of solid timber, service class 3, and permanent loads. In Tables 5 and 6, the slenderness of the newly designed elements is also indicated, and the quality of the design obtained is measured through the Element Utilization Factor EUF.

4 Design of slender struts with a generic doubly symmetric section and a fixed weak side (Case 3)

In this case, suppose that, in the transition from section S_0 to section S, the coordinates of the points of the section are transformed according to Eq. (104):

$$\begin{aligned} x &= x_0 \\ y &= w \times y_0. \end{aligned} \tag{104}$$

Also in this case, (x_0,y_0) are simmetry axis and principal axis for S_0. The origin of (x_0,y_0) coordinate system is conicident with the centroid G_0 of the section S_0. For the section S, the (x,y) axis are simmetry axis and principal axis. The origin of the (x,y) coodinate system is coincident with the centroid G of the S section. From Eq. (104), we then obtain Eq. (105):

$$\begin{aligned} h_X &= h_{X0} \\ h_Y &= w \times h_{Y0}. \end{aligned} \tag{105}$$

In this case, when moving from section S_0 to section S, the area of the section will vary according to Eq. (106):

$$A = \iint dxdy = \iint d(x_0)\,d(wy_0) = w\iint dx_0 dy_0 = wA_0. \tag{106}$$

In addition, the minimum moment of inertia of the section will vary according to Eq. (107):

$$\begin{aligned} J_{\min} &= \iint x^2 \times dxdy = \iint x_0^2 \times d(x_0)\,d(wy_0) \\ &= w\iint x_0^2 \times dx_0 dy_0 = wJ_{y0}. \end{aligned} \tag{107}$$

Also in this case, we want to determine the dimensions that must be assigned to the section, so that the element can satisfy the strength verification required by the standards Eurocode 5 (CEN, 2005a) and NTC 2018 (Ministero delle Infrastrutture e dei Trasporti, 2018). For the effective section S, we have

$$i_{\min} = \sqrt{\frac{J_{\min}}{A}} = \sqrt{\frac{wJ_{y0}}{wA_0}} = i_{y0} \tag{108}$$

also

$$\lambda = \frac{L_0}{i_{\min}} = \frac{L_0}{i_{y0}} \tag{109}$$

by virtue of Eq. (20) and with reference to Eq. (17)–(19), the sectional area can then be determined as follows:

$$A = \frac{N_{Ed}}{k_{crit,c} \times f_{c,0,d}}, \tag{110}$$

from which

$$wA_0 = \frac{N_{Ed}}{k_{crit,c} \times f_{c,0,d}} \tag{111}$$

and therefore

$$w = \frac{N_{Ed}}{k_{crit,c} f_{c,0,d} A_0}. \tag{112}$$

The correlations we have just obtained are obtained directly from the relations provided by the standards Eurocode 5 (CEN, 2005a) and NTC 2018 (Ministero delle Infrastrutture e dei Trasporti, 2018). It is therefore not necessary to validate them, as they are already consistent with the standards in force.

5 Conclusions

The method presented here, allows to design, in a simple and fast way, slender timber elements subjected to simple compression. The proposed method is in line with Eurocode 5 and the Italian Technical Standards NTC 2018. The validation of the method, has been implemented by performing the design and verification of 240 elements, characterized by different sectional shapes, different materials, different slenderness, different service classes and different load duration classes. In the calculation examples shown, the minimum EUF was 93.28% while the maximum EUF was 99.99%. The method is therefore very precise and always in favour of safety. On average, it leads to elements in which the design action is 97.53% of the resisting action. This accuracy is obtained without the use of complex calculations and this allows a very simple implementation of the proposed method.

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