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Physical Characteristics Influencing Space Usage and Pedestrian Behaviour in Public Space

Case Study: Városháza Square in Dunaújváros

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Abstract

Analyzing pedestrian movement in urban spaces contributes to understanding how these spaces function. In this study it is revealed by investigating the key questions: (1) From where do the main pedestrian flows come? (2) How does the pedestrian movement depend on the depth of view and spatial situation?

A visual programming platform and a geographic information system software, namely Grasshopper and QGIS, are used for this analysis which has numerous plug-ins for assessing spatial arrangements. Personal observation helps to understand how to do the analysis correctly and, simultaneously validate the result of the software calculations.

The location of the case study, Dunaújváros, is a major Hungarian city, mainly developed during Socialism. Its historical background explains the transformation related to public spaces' usage in the city.

As a result of the analysis, the main features and problems of the Városháza square in Dunaújváros are determined, and possible solutions are offered for its improvement. More importantly, the existing methodologies for public space assessments were developed further by including the effects of pedestrian behaviour, which can be used for evaluating any other public space.

Keywords

post-socialist public space, space usage, pedestrian behaviour, spatial analysis, Dunaújváros, QGIS, Grasshopper

1 Introduction

According to the "Experience of Nature" by Kaplan and Kaplan (1989), space usage is revealed in the concept of affordance. Active exploration is essential in adaptation to the possibilities of space since, by perceiving the different functional properties and affordances, the individual can "use" the environmental elements. Affordances encouraging physical activity include safety from crime and traffic, perceived aesthetics, pedestrian activity, quality upkeep, higher density, mixed-use, connectivity, and accessible destinations.

As part of the design process, urban planners make initial assumptions about how the users of public spaces will react to the properties of the environment while planning and implementing their walking routes. How these assumptions should be made is an extensively researched area regarding public spaces. This research focuses on space usage of a public space expressed in pedestrian behaviour, particularly in pedestrian direction choices in relation to the visibility depth and spatial situation.

1.1 Research methodology

This study aims to develop a methodology for assessing pedestrian behaviour in public spaces, making suggestions for improvement through the case study of Városháza tér in Dunaújváros.

A mixed method study is created to assess the square, including personal observations of pedestrian behaviour and a computer analysis using Rhino 7 and Grasshopper (Robert McNeel & Associates, online) software. The data used in the analysis were collected from OpenStreetMap (OpenStreetMap Foundation, online) and were complemented in QGIS (QGIS Development Team, online) software using Google Maps and on-site measurements.

The idea is that the QGIS-Grasshopper combination can be used to assess the usage of public spaces based on three parameters: accessibility, visibility, and pedestrian movement. During the personal observation, the main directions of pedestrian movement and the number of people moving along them were counted. These measurements

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are supposed to validate the result of using the software and to develop the strategy for public space assessment.

1.1.1 Grasshopper schemes

For the accessibility assessment, an isochrone map is used, which indicates the area pedestrians can reach. Next, the functional schemes within this area showing the context are built in Grasshopper to assume the main pedestrian flows. The visibility graph analysis (Turner et al., 2001) is used for visibility analysis.

An agent-based simulation was conducted in Grasshopper for the pedestrian direction analysis using the PedSim (Parametric Design Studio, online(a)) plug-in.

1.2 Case study

Dunaújváros, the first Hungarian New Town, was formed using the same principles applied to all new Soviet cities of that time. Christina Lenart (2013) summarized the typical characteristics of such socialist cities:

1. In socialist countries, the ratio of public space to the total urban area was much greater than in capitalist states (Engel, 2006). While in Western cities, this share was only 1:3; it was 3:4 in Eastern and Central Europe. This ratio for capitalist cities is much bigger in the city centre, while socialist public spaces were spread all over the cities (Stanilov, 2007).
2. The scale of socialist public spaces reflects the power of government rather than the residents' needs. Such public spaces were often used as a big stage for different political activities, such as festivals and parades meant to show the power of the communist system (Engel, 2006).

According to Stanilov (2007), under the communist regime, the public space of the urban centre was given ideological meaning with its monumentality making it a tool of propaganda. Nevertheless, apart from this, in socialist cities, many unproperly maintained and deserted open spaces formed most urban landscapes. Under these circumstances, social interaction could be the only function of public spaces. In Western cities, such functionality is more substantial and complemented by commercial activities.

Overall, socialist public spaces were primarily intended for organised and ceremonial usage than for everyday life (Kissfazekas, 2015). Some functionalities like trading were not possible back then. The question about these spaces is if they can find use in post-socialist cities. Political and economic shifts resulted in public squares created in the Soviet times losing their symbolic significance showing that they were not actually meant for public life.

The public buildings that no longer had their functional background became empty, degrading spaces near them. According to Kissfazekas (2013), the public spaces of Dunaújváros require changes.

Városháza square in Dunaújváros is located in the city centre at the intersection of Dózsa György road and Vasmű road.

Before the refurbishment of Városháza square in 2013, a street open to traffic crossed the square. Bird's-eye views of the square from 1984 and 2022 are shown in Figs. 1 and 2. While the road is present in the first image, it is already a pedestrian zone in the second picture. The current state of the square is investigated in this research.

1.3 Space usage of Városháza square

Jan Gehl (2011:pp.9–12) discusses three types of outdoor activities in his works. Necessary activities include daily activities, like going to work, waiting for a tram or shopping, which can happen in all weather conditions. Optional activities happen if a person wishes to do them and has time and space, like walking, sunbathing, or resting in a park. They are most likely to occur during good weather. In contrast, social activities mean any human interaction. It is not necessarily direct contact (like speaking), but seeing or hearing someone on the street also falls into this category.

Personal observation showed that all types of activities are present in Városháza square. However, most people simply crossed the square, indicating that necessary activities prevailed. Thus, the square is a good case study for investigating pedestrian movement. The functional scheme

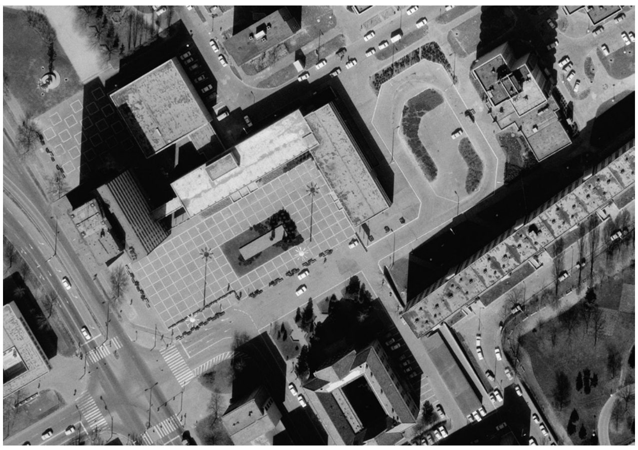

Fig. 1 Bird's-eye view on the Városháza square from 1984 (Source: fentről.hu, online; Lechner Nonprofit Kft. / License number: 99772)

Fig. 2 Bird's-eye view on the Városháza square from 2022 (Source: Microsoft Bing, online. Microsoft product screen shot(s) reprinted with permission from Microsoft Corporation)

of public buildings around the Városháza square is shown in Fig. 3 and the photo of the square is presented in Fig. 4.

2 Accessibility analysis using isochrone map

For the pedestrian movement analysis in the square, it is necessary to understand how it fits into the pedestrian routes of the city. The pedestrian accessibility area appears in Fig. 5. People can pass this zone to reach where they live, work or leisure areas like parks or green zones. The isochrone map used is an aid in explaining the main pedestrian flows.

An isochrone is a line connecting a starting position with a point that can be reached in a given timeframe at a certain speed (Desai, 2008). Isochrone maps contain lines leading to all possible achievable points in any direction.

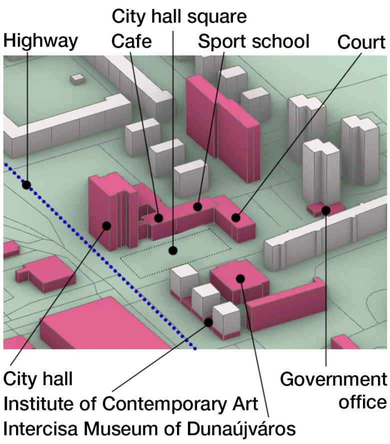

Fig. 3 Functional scheme of public buildings around the Városháza square (Adapted from OpenStreetMap Foundation (online) modified in QGIS and Grasshopper (QGIS Development Team, online) by the author)

Fig. 4 Photo of Városháza square from the highway (Fesenko, 2022a)

Fig. 5 Pedestrian accessibility: Isochrone map with the centre at Városháza square for a 10-minute walk at the speed of 4 km/h. The location of Városháza square is marked with a black dot. The pedestrian accessibility area is highlighted (Adapted from OpenStreetMap Foundation (online) modified in QGIS and Grasshopper (QGIS Development Team, online) by the author)

Fig. 5 shows all points accessed from the centre of Városháza square in 10 minutes at a speed of 4 km/h. This timeframe and speed are chosen because it can be considered a relatively easy walk for average pedestrians. The endpoints of all possible routes are connected, forming the perimeter of the green area. The green area represents the points pedestrians can access.

The isochrone map was built in Grasshopper with the help of a plug-in called Shortest Walk (Food4Rhino: Apps for Rhino and Grasshopper, online(b)). Data from OpenStreetMap is loaded with the plug-in named Elk (Food4Rhino: Apps for Rhino and Grasshopper, online(a); OpenStreetMap Foundation, online). The road network is presented as a graph, allowing us to quickly calculate the shortest distance between any pair of points on the map.

Figs. 6 and 7 show the isochrone map with buildings serving public and residential functions, respectively. Fig. 8 shows the city's green zones, including the large park near the Danube, with many visitors. It is assumed that people will cross Városháza square from all sides, and the pedestrian flows are approximately the same size in all directions.

Fig. 6 Public buildings and buildings with public functions (highlighted in pink) on the ground floor within the 10-minute area (Adapted from OpenStreetMap Foundation (online) modified in QGIS and Grasshopper (QGIS Development Team, online) by the author)

Fig. 7 Residential buildings (highlighted in blue) within the 10-minute area (Adapted from OpenStreetMap Foundation (online) modified in QGIS and Grasshopper (QGIS Development Team, online) by the author)

Fig. 8 Leisure areas, including parks, squares, and green zones, are highlighted green (Adapted from OpenStreetMap Foundation (online) modified in QGIS and Grasshopper (QGIS Development Team, online) by the author)

The data on buildings' positions, functions and heights were taken from OpenStreetMap and modified, relying on information from Google Maps in QGIS (OpenStreetMap Foundation, online). Finally, they were set up in the Grasshopper model.

3 Visibility analysis using the isovist method

Visibility is an essential quality of a public space connected to a sense of security, which is a significant factor influencing approach-avoidance decisions (Mehta, 2014).

This parameter is investigated in the frames of Space Syntax (Hillier and Hanson, 1984). Space Syntax is a term that unifies many techniques and theories developed for analysing spatial layouts and human activity patterns in buildings and urban areas.

Its main idea is that spaces consist of discrete components related to human behaviour and can be represented as graphs showing mutual connectivity. Three main concepts are used for describing spaces: axial line (represents a road like a street, an alley, or a boulevard), convex space (an area on which all points are visible to each other) and isovist (a field of view from a given point) (Benedikt, 1979).

Visibility estimation can be done not just by isovists alone; visibility graph analysis is the method that extends the isovist's definition by building a graph of intervisible locations in space. Every node of such a graph represents a point location, and an edge between them is assigned if there is no obstacle from one point to the other.

The result can be presented on a map with the help of various tools; one is the DecodingSpaces plugin for Grasshopper software (DeCoding Spaces, online). The area of interest is split into square segments, each representing a single point that counts as a node in the graph. After computing mutual visibility between nodes, the squares get painted in colours ranging from red to blue. From red points, less space in the investigated territory is visible, while from blue points, visibility is the best. The visibility graph analysis scheme for Városháza square is shown in Fig. 9.

4 Personal observation of pedestrian behaviour

An on-site observation was carried out to register the main pedestrian flows in Városháza square. To ensure sufficient data, it was decided to conduct on-site observations in three different time frames: on a weekday from 11 AM to

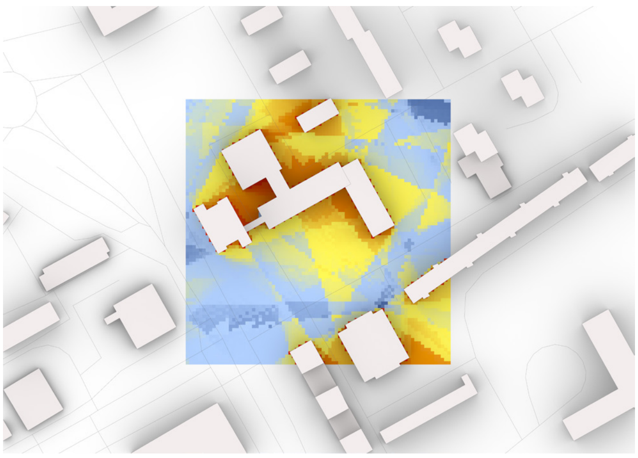

Fig. 9 Visibility graph analysis scheme from the centre of Városháza square. The visibility area is represented in a gradient from red to blue. The red area has the smallest number of intervisible zones, and the blue is the opposite (Adapted from OpenStreetMap Foundation (online) modified in QGIS and Grasshopper (QGIS Development Team, online) by the author)

12 PM and from 5 PM to 6 PM and on a weekend day from 3 PM to 4 PM.

During the personal observation, the main pedestrian flows were registered. There were two points on the square from where the observation was conducted. They are marked as A and B in Fig. 10.

Table 1 shows the observation results. The first two columns illustrate the pedestrian flows registered during the observation. The numbers are taken from Fig. 10. Column "Start Gate" has all the gates from which pedestrians can enter the square in ascending order. Column "End Gate" contains the numbers of all the gates reached by pedestrians at least once from the entrance on the same row of Table 1 in ascending order. Three other columns refer to the time frames during which the observations were made. The numbers in these columns represent the number of times a pedestrian reached from the entrance to the exit point labelled by the numbers on the same line. A dash instead of a number indicates that a particular pair of gates were not used in that time frame.

For example, the first line states that from entrance point 1 to exit point 3 on Friday, 29 April 2022, two people walked along between 11 AM and 12 PM. In two other time frames, nobody used this pair of gates.

The two last rows represent the sum of pedestrians registered on the square during each time frame. The first sum is the total number of people. The second sum counts the number of pedestrians who walked through the pairs

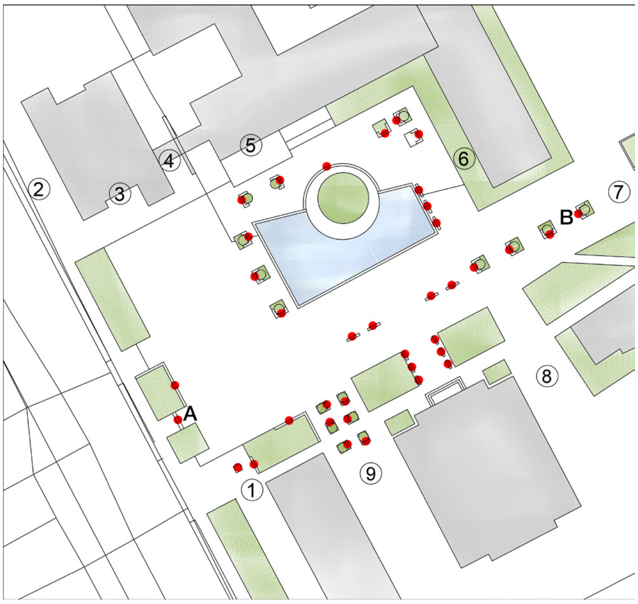

Fig. 10 The resultant Városháza square scheme, showing the main gates – entrances and exits of the pedestrian flows (1–9), and places to stay – mostly benches (red dots) (Adapted from OpenStreetMap Foundation (online) modified by the author)

Table 1 Personal observation results

Start Gate	End Gate	29 April 2022 11:00–12:00	29 April 2022 17:00–18:00	08 May 2022 15:00–16:00
	3	2	–	–
	4	8	14	–
1	6	7	–	–
	7	25	74 – max	42
	8	2	–	–
	4	6	–	–
	5	–	–	6
2	7	21	66	27
	9	24	28	80 – max
3	–	–	–	–
	1	25	4	–
4	2	2	–	–
	9	18	6	36
5	9	–	–	6
6	1	2	2	–
	1	25	18	64
7	2	20	16	34
	8	–	–	8
	1	2	–	–
8	2	–	–	6
	7	21	24	63
	2	45 – max	25	42
	3	8	–	–
9	4	25	23	15
	7	40	27	65
Sum1		328	327	494
Sum2 (over 10)		289	315	468

of gates used at least ten times during a single observation period (highlighted in yellow). These results are used in the Grasshopper simulation later in this study.

Limitations and results of the personal observation:

- Ways 1–2 and 2–1 were not considered, as these paths do not cross the square.
- Approximately 20 people were sitting on benches throughout the observation. They were not included in the overall count because there was no information about how they had entered the square.
- One of the methodology's difficulties was properly registering pedestrian movement. Nevertheless, the overall tendency can be tracked from the presented results.
- Some parents with children moved around the square on bicycles, kick scooters, or just running. They used all the free space but they did not enter the corner between Gates 5 and 6 (between the sports centre and the court). The other pedestrians also avoided this corner.

- Table 1 shows that public buildings around the square are rarely used. Even the café (Gate 5 in Fig. 10) had a very small number of visitors. At the same time, all the benches were occupied.

5 Agent-based simulation of pedestrian behaviour in Grasshopper using the PedSim plugin

An agent-based simulation is a method that investigates how the system is affected by the behaviour of decentralized agents. The Grasshopper plug-in PedSim (Parametric Design Studio, 2019) was used for the pedestrian simulation, where the agents were defined as pedestrians. PedSim has already been used for many architectural projects at different scales, from building interiors to public spaces (Parametric Design Studio, online).

The plugin simulates only a particular set of rules that do not reflect a pedestrian's full and accurate behaviour. For example, in the plugin, a pedestrian always follows the shortest route, but an actual person does not always move along the most optimal path. Also, the readability of the space is not considered. For example, specific landmarks greatly influence pedestrians' behaviour and route choices, but this is not considered in the plug-in. In addition, the terrain is also ignored. All movement happens on the ideal two-dimensional plane.

The rules of pedestrian behaviour in Pedsim are the following:

- People interact with each other and obstacles with the help of physical forces and follow the basic rules of dynamics, i.e., the mass of pedestrians, acceleration, speed of movement, and position in space are taken into account.
- Pedestrians are represented as a circle in the plan.
- In a simulation, target force, obstacle repulsion, person repulsion and anticipatory collision avoidance force are calculated.

In PedSim (Parametric Design Studio, online(b)), gate is defined as an object on Map that can generate or absorb people depending on whether it is a Starting Gate or a Destination Gate. Target is a point people can visit on their way to Destination Gate. An Obstacle is a closed polyline people must move around to avoid hitting it.

6 Pedestrian simulation using the example of Városháza square in Dunaújváros

To analyze Városháza square, we made three simulations, one for each time frame using the personal observation results presented in Table 1.

We used the data shown in Fig. 11 as a basis for the simulation scheme. Entrances and exits 1–9 from Table 1 and Fig. 11 were defined as Gates. Starting Gates were entrances from the first column, and Destination Gates from the second column were exits. Benches and the monument close to Gate 1 were Targets, marked with red dots in Fig. 11. We defined all the buildings, the greenery, the benches and the fountain as Obstacles People must not cross. For the simulation, the number of pedestrians from the bottom row of Table 1 is considered.

The limitations and results of the agent-based simulation were the following:

- Only those directions the pedestrians used more frequently were taken into account.
- The movement of children running or riding bicycles around the square was not simulated. However, they were not counted during the on-site observation either.
- All three simulation schemes in Fig. 11 show similar patterns, so the different times of the observations are not relevant from the point of view of the primary research. Therefore, a summary scheme was created, considering all possible directions, including routes 1–2 and 2–1, between the gates from Fig. 10, excluding the buildings' entrances (Gates 3,5,6) as they were not frequently used in reality. The resultant scheme is shown in Fig. 12.

6.1 The outcomes from the pedestrian simulation

The simulation results were much like those seen during the on-site observation. Therefore, Grasshopper simulation helps analyse pedestrian behaviour in public spaces.

The corner between Gates 5 and 6 (Fig. 13) is identified as unused in the pedestrian simulation scheme (Fig. 12) and by personal observations. It is also highlighted in red in the visibility graph analysis scheme (Fig. 9). Thus, based on all investigations, it is the least preferable location on the square. Consequently, visibility depth results seem related to actual space usage in the case of Városháza square.

During the personal observations, nobody entered the buildings of the sports centre and the court in the unused corner.

Adding new gates to that part of the square could change the circulation of pedestrians. Therefore, one possible way to bring more people to this part of the public space is to rethink the usage of the public buildings around Városháza square.

After socialism, many public buildings lost their social and economic base and became empty. Consequently, according to Kissfazekas (2013:p.191): "*...spaces around*

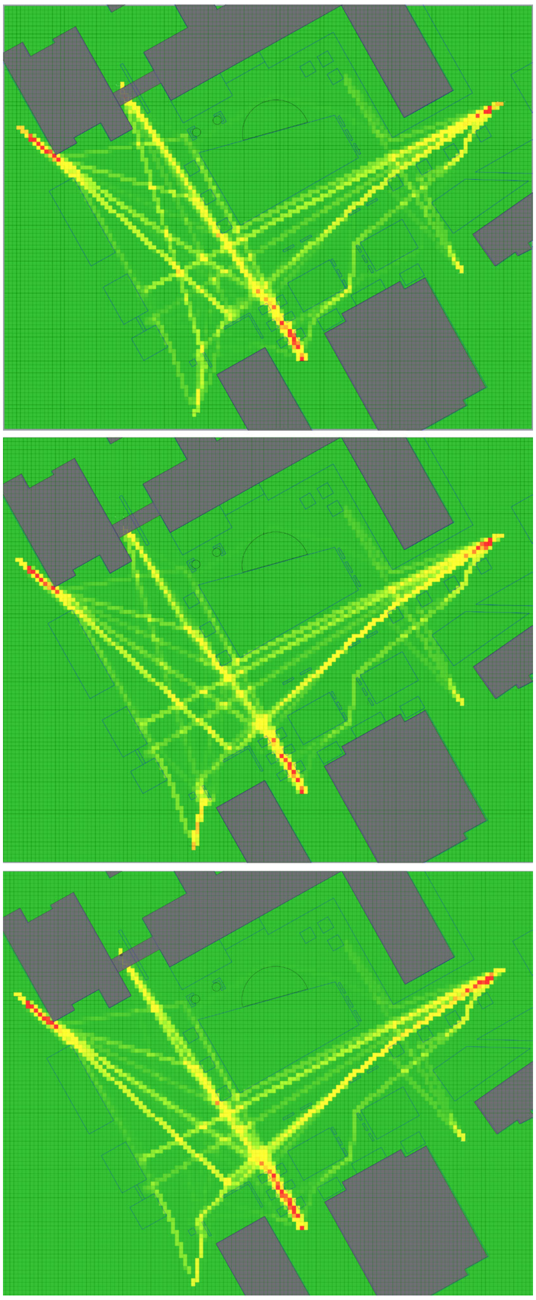

Fig. 11 Agent-based simulation of pedestrian behaviour schemes done in Grasshopper using PedSim plug-in (Parametric Design Studio, online(b)). The images are shown in the order of time frames of the on-site observation they represent from top to bottom: Friday, 29 April 2022, 11 AM – 12 PM, Friday, 29 April 2022, 5 PM – 6 PM, Sunday, 08 May 2022, 3 PM – 4 PM. Yellow lines represent pedestrian flows. Places, where this colour is more intense are more crossed by people. Red indicates that a particular position on the map is the most visited (Adapted from OpenStreetMap (online) modified in Grasshopper with PedSim (Parametric Design Studio, online(b)) by the author)

them lost their original meaning and got into a miserable state. The absence of urban context made spaces in the public realm no longer viable".

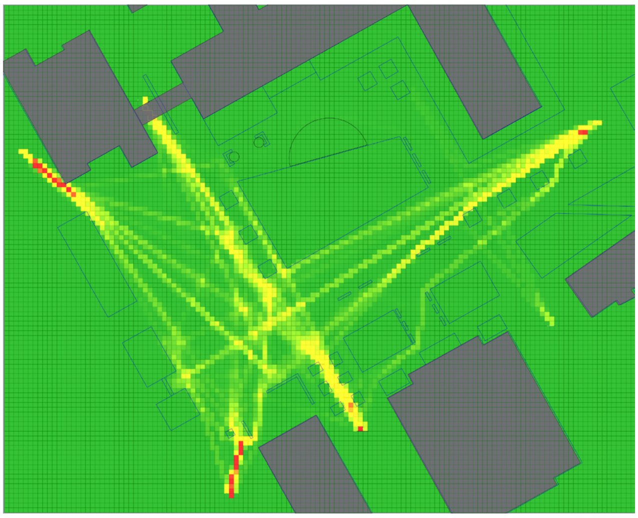

Fig. 12 The resultant pedestrian simulation scheme (Adapted from OpenStreetMap Foundation (online) modified in Grasshopper with PedSim (Parametric Design Studio, online(b)) by the author)

Fig. 13 The photo of the unused corner between the court and the sports school (Fesenko, 2022b)

7 Conclusion

The primary outcome of this research is the assessment methodology used for capturing the physical characteristics influencing space usage and pedestrian behaviour in public spaces. Based on the personal observations and computational analysis results, we observed that the latter is enough for pedestrian movement evaluations in the case of our case study. As part of this research, it was shown that the main flows of people crossing a square could be defined with the help of isochrone maps, made in Grasshopper with data derived from OpenStreetMap and modified in QGIS (OpenStreetMap Foundation, online; QGIS Development Team, online). Unused areas can be determined with the help of the isovist schemes in Grasshopper, showing the depth of view and pedestrian directions analysis carried out by agent-based simulation using the PedSim plugin in the same software.

In this study, we used a methodology to identify the fundamental public space usage issues of the current state of Városháza square in Dunaújváros. With the method, we detected a problematic corner of the square and offered suggestions for improvement. The applied methodology can also be used for examining and making data-based suggestions for other public spaces.

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