Geometric Design in Architecture: Examination of Tessellation Configurations in Structural Systems

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Received: 18 June 2023, Accepted: 06 November 2023, Published online: 09 November 2023

Abstract

The tessellation method, which is a mathematical concept used in geometric design, is also used in architecture and engineering, as in many other fields. In this study, it is aimed to investigate the tessellation configurations in structural systems within the scope of geometric design in architecture, to show the effectiveness of geometry in the architectural production in these structures and to highlight the tessellation-structure relationship. The number of studies emphasizing tessellation configurations in structural systems is limited. For this reason, the study will contribute to the literature. The research question of the study is how tessellation configurations are applied in structural systems and how this relationship is established. Depending on this research question, eight architectural buildings in different cities with different tessellation configurations were examined within the scope of this study. Content analysis was performed by looking at the architectural features of these structures, and a comparative analysis framework was created with the data obtained. Inferences were made on geometric and architectural features. As a result, it has been seen that tessellation patterns are used and will continue to be used on a wide scale in structural design.

Keywords

geometric design, geometric shape, tessellation, structure, structural design

1 Introduction

The architecture, engineering, and construction (AEC) industry develops various methods to increase productivity, quality, and qualified construction systems (Takva and İlerisoy, 2023a). In this context, construction projects are turned into complex construction systems day by day. By using integrated systems, it is ensured that structural systems can be handled versatilely (Azhar, 2011; Azhar et al., 2008). Passing long spanning in flexible design logic in structural systems also creates convenience by increasing human capacity. Industrial buildings, shopping and cultural centers, swimming pools, and stadiums are structures that provide long-spans in architectural design where structural elements come to the fore (Kaveh and Talatahari, 2011; Makowski, 1993). Progress has been made in architectural structural design with developments in architectural modeling programs, nanotechnological and strength interventions in building materials, the use of technological and robotic tools in building construction processes (Takva et al., 2023a). Technological combinations in the AEC industry bring structural systems in sustainable architecture to a size that can last for many years (Arayici et al., 2011; Smith and Tardif, 2009). In addition to the durability parameters and risk assessments of the buildings, geometric designs in the structural systems create diversity in the architectural building stock (Sarıcıoğlu and Ayçam, 2021; Takva et al., 2023b). Geometric design features may also change depending on the space requirement, function, area data, and building materials of the building (Bekler et al., 2022; Eastman, 2018; İlerisoy and Tuna, 2018).

There are studies on structural design in the literature. Takva and İlerisoy (2023b) examined the structural performances and costs of planar and vaulted structure systems, which are formed with 8 geometrical layouts, which are most commonly used in tessellation configurations. Grasshopper (Grasshopper) and Sap2000 (Sap2000) programs were used. Laccone et al. (2023) proposes a new high-rise building class method, equipped with an organic-looking and mechanically robust pipe structure, as an alternative to diagrid, hexagrid and random
Voronoi tessellations. Scaramozzino et al. (2020) conducted an extensive investigation of diagrid structural systems. Structural optimization was performed on geometric shapes. The applicability of diagrid systems with structural analyses and seismic evaluations is discussed. Angelucci and Mollaioli (2018) investigated the mechanical responses of structures in high-rise buildings with the Voronoi tessellation inspired by nature. In the study focusing on periodic and non-periodic Voronoi tessellation, numerical and analytical results were compared with static and modal analyses. Rian and Sassone (2014) examined fractal-like tree-shaped structures, which are a mathematical concept. The relationship between geometric shape and structural strength is emphasized. Examples of architectural structures built with tree-like structural systems are presented. Mashhadiali and Kheyroddin (2013) aimed to increase the efficiency of tube-type structures in high-rise buildings by developing a new structural system called hexagrid. For 30, 50, 70, and 90 story buildings, high-rise buildings with four different systems have been designed with an approach based on strength and stiffness to withstand wind load. The effects of maximum lateral displacement of structural elements on engineering and architectural performance in diagrid and hexagrid systems with different geometric configurations were compared. Wonoto et al. (2013) designed a parametric system through origami tessellation and performed finite element analysis. LS-DYNA finite element tool and Grasshopper program were used (Grasshopper). Turrin et al. (2011) combined parametric modeling and genetic algorithms to achieve a performance-oriented process in architectural design. ParaGen software was used to automatically generate alternative geometric design solutions. With this software, which supports parametric design alternatives, the structural performance of a dome and energy analyses of a long-span roof were made.

The concepts provided by geometry and mathematics are seen in architectural designs and planning. Based on the information obtained from the literature, it is seen that the effects of tessellation configurations have increased due to the development of technology. The tessellation we see in floor coverings and wall decorations in ancient times are also used in new-generation buildings. Tessellation patterns in facade systems and top cover systems can be seen both as a panel material and in load-bearing configurations. When the differing facade systems are examined, it has been determined that the tessellation configurations vary from structure to structure (Table 1). Tessellation module, panel shape, module formation, and mathematical sequence parameters are reviewed. These examples, which are created by the translation, rotation, or reflection of a geometrical form and are widely used, are the result of the tessellation method.

Voronoi tessellations are used in structures such as Beijing National Aquatics Center (Water Cube), Melbourne Recital Center and Alibaba Headquarters. Tessellation configurations are found in many areas. Tessellations in mathematics, geometry and engineering are also found in urban and regional planning, landscape architecture and architecture. There are tessellations in various fields in architecture.

Table 1 Tessellation patterns in facade systems

<table>
<thead>
<tr>
<th>Building</th>
<th>Federation Square</th>
<th>Duo Twin Towers</th>
<th>The Prada Aoyama Store</th>
<th>Aula Medica</th>
<th>Sage Gateshead</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tessellation module</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Panel shape</td>
<td>Triangle</td>
<td>Hexagon</td>
<td>Diamond</td>
<td>Triangle</td>
<td>Rectangle</td>
</tr>
<tr>
<td>Tessellation module formation</td>
<td>Rotation and reflection</td>
<td>Translation</td>
<td>Translation, rotation, and reflection</td>
<td>Translation</td>
<td></td>
</tr>
<tr>
<td>Mathematical sequence</td>
<td>3.3.3.3.3.3.3.3</td>
<td>6.6.6</td>
<td>4.4.4.4</td>
<td>3.3.3.3.3.3</td>
<td>4.4.4.4</td>
</tr>
</tbody>
</table>
as well. Tessellation configurations found in facade systems are also found in structural systems. In structural systems, geometric design can also be achieved by the tessellation method. In this study, tessellations in structural systems in different cities were examined and comparative analysis was carried out. In addition to the structural features of the examined buildings, their geometric features were examined and the structure-geometry relationship was established.

2 Material and method

2-dimensional tessellations are the process of creating a pattern of a plane with geometric shapes. In this pattern, geometric shapes should not be overlapped and should be arranged side by side without leaving any spaces between them (Phillips, 2014). 3-dimensional tessellations are also obtained with 3-dimensional polygons. In addition to regular and semi-regular periodic tessellations, there are Delaunay and Voronoi aperiodic tessellation types. In this study, structural systems with different tessellation patterns were examined. Fig. 1 shows the flowchart of this study.

The existence of different tessellation types enables flexible and versatile designs to emerge. A tessellation pattern is formed by rotation, translation, reflection, and combinations of these movements of geometric shapes (Fig. 2). While a tessellation pattern can only be brought together by translational movement, it may also occur by translational rotational or translational reflection movement. Tessellation configurations occur with the combination of three different movements (Takva, 2021).

While regular tessellations can be obtained using regular triangle, quadrilateral, and hexagon geometric shapes alone, semi-regular tessellations can be created using more than one regular polygon (Fig. 3). By determining a tessellation module, the mathematical sequence is revealed according to how many this module is and how many sided geometric shapes it consists of (Cicerone et al., 2021). In the tessellation module, a mathematical expression is written by counting the geometric shapes counterclockwise. The tessellation module must complete 360 degree (Takva and İlerisoy, 2023b). 6 triangles, 4 squares or rectangles and 3 hexagons can form a tessellation module. In this study, structural systems in which configurations are created with tessellation modules have been investigated. These structures, which are important in terms of geometric design, and the combination of geometry and architecture are seen.

3 Case studies

Geometric decisions and shapes, which are central to the identity of the building, bring freedom and originality to the buildings in cooperation with the load-bearing systems. Many application areas in which tessellation configurations are applied in areas close to architecture are found in the literature. In the field of architecture, there are various systems in which this understanding of geometric order is applied. One of these systems is structure systems. In structural systems, there are examples of structures in which regular and semi-regular tessellation configurations are used. In Section 3, examples of structures in different cities with different tessellation configurations
were examined. In chronologically ordered structures, the focus is on building function, the year of completion of the building, panel shape, tessellation module, tessellation module formation, mathematical sequence, structure shape, structure materials, maximum structure height, and span parameters.

3.1 Union Tank Car
Completed in 1958, the Union Tank Car Company dome is located in Baton Rouge, Louisiana (Quinn and Gengnagel, 2014). Designed by Richard Buckminster Fuller and the Synergetics team (Šiber, 2020). The highest point of the geodesic dome is about 36 meters and has a diameter of about 117 meters. A tunnel that is approximately 6 meters high, 12 meters wide, and 60 meters long forms part of the structure. When the structure was completed in 1958, it was the largest span dome in the world. The dome is approximately in the form of a quarter dome (Table 2). It is made of steel bars resistant to tensile and compressive forces. In the dome, which consists of a two-layer truss structure, the distance between the two layers is approximately 1 meter. All fabrication and sub-assemblies related to the dome were made at the construction site. After the large sections were assembled on the ground, they were anchored in place with mobile cranes. The scaffolding system was not used in the construction of the structure, and the structure was self-supporting. Hexagonal grids were used as geometric shape. The Union Tank Car project was the first dome project to use a computer to calculate geometry (Popko, 2012). The building, which became in need of maintenance after losing its original function, was demolished in 2007.

3.2 Montreal Biosphere
The United States Pavilion, designed by engineer and architect Richard Buckminster Fuller, was created for Expo’67 in Montreal, Canada (Senechal et al., 2013). Built in 1967, the structure is a three-quarter spherical double-layer space truss geodesic dome 62 meters high and 76 meters in diameter (Kolpakov et al., 2022). In a system where steel bars and acrylic panels are used, there are triangular geodesic grids on the outside and hexagonal grids on the inside (Chilton, 2007). Shade elements are also placed in some regions between the triangular geometries. Shade elements, which are automatically controlled by light sensors and adapted to depend on the angle of the sun, protect this structure from exposure to the sun, where a large-span is passed. Automatic shade elements are combined with thermostatically controlled conventional air-handling equipment, minimizing the use of fossil fuels (Massey, 2016). The geodesic dome structure, which can support itself without any columns or walls, provides the use of large areas (Pilarska and Maleska, 2021). The view of the structure is given in Table 3. Today, it functions as a museum where exhibitions and activities are held.

3.3 Neckarsulm Swimming Arena
Completed in 1989, the Aquatoll Swimming complex is located in Neckarsulm, Germany (Malek, 2012). The radius of the gridshell roof cover above the swimming complex was 16.5 meters, and the maximum base diameter was 25.2 meters. The basic grid size is made of 6 × 4 centimeters S235 steel construction material. Twin cables a diameter of 5 millimeters are cross-linked to obtain a shell structure (Schober, 2015). Spherical curved insulating glass panels were used in accordance with the gridshell structure (Berk and Giles, 2017). With the production of glass structural elements in this way, it was easy to configure the nodal points. The design of the nodes has increased the economic efficiency and stiffness of double-curved spatial grid structures. Diagonal cables are effective in the strength of the structure. In the gridshell, all steel bars are of equal length except the bars in the corner. Each rectangular module measures 1 × 1 meter (Schober, 2015). The angle of the steel bars is between 65 and 90 degrees. Structural elements were produced

| Mathematical sequence – 6.6.6 |

Table 2 The Union Tank Car (NeolithicSphere), the formation of the tessellation module, and the mathematical expression of the tessellation pattern
by prefabrication and transported to the construction site (Schlaich and Schober, 1996). The gridshell structure is shown in Table 4.

### 3.4 DZ Bank
Located in Berlin, Germany, DZ Bank consists of a free-form double-curved gridshell structure (Table 5). Steel bars and glass building materials are used in the structure (Falk and Van Buelow, 2009). Designed by Frank Gehry and his team and completed in May 2001, the building has a floor area of 20,000 square meters (Myerson and Ross, 2003). In a commercial sense, it is a mixed-use building with a residential function consisting of 39 flats, in addition to the Berlin headquarters of DZ Bank. After the main entrance, the high-volume foyer highlights the building’s gridshell structure of curved glass and steel. Office spaces are arranged around the atrium and benefit from the natural light provided by the gridshell. The conference hall, which is covered with stainless steel outside and wood inside, is located at the center of the project (ArchDaily (a)). The roof cover is formed from 4 × 6 cm stainless steel bars. The nodes are in the form of stars (Schober, 2015).

### 3.5 New Milan Trade Fair
The project, which was started in 2002 and completed in 2005, is located in Milan, Italy. It was designed by Massimiliano and Doriana Fuksas. The cover construction, consisting of free-form surfaces, spreads an area of approximately 50,000 square meters (Adriaenssens et al., 2014). The structure system, which is 1300 meters long and varying between 32 and 41 meters wide, connects the trade fair grounds. The structural parts resting on the ground are likened to the trunk of the trees and volcanoes (Schober, 2015). The roof structure was divided into twelve independent sections. Looking at the entire structural system, it is seen that it consists of approximately 16,000 nodal points and 41,000 structural elements. Approximately 180 columns were used to carry the roof structure (Stephan et al., 2004). The structure, which has a complex system, is made of steel and glass building materials (Table 6). The structural system includes variations such as craters, waves, hills, and sand dunes found in nature (Fuchs, 2006). The length of the steel bars used ranged from 80 centimeters to 3 meters. A structural system was created with an S355 steel construction material (Schlaich et al., 2005).

### 3.6 Haesley Nine Bridges Golf Clubhouse
The wooden gridshell structure, which was under construction between 2006 and 2009, was completed in 2010 and is located in South Korea. It was designed by Shigeru Ban Architects and Kyeongsik Yoon (Chilton and Tang, 2016). The project, which has spaces such as restaurants, conference halls, and spa areas, includes multifunctional uses. The three-storey structure consists of a truss system consisting of single and double curvature laminated wooden beams (Correa et al., 2019). There are 21 tree-like column structures on a 9 × 9 meter square grid and support the 36 × 72 meter roof. The free-form roof geometry was developed using a parametric model. Considering the nodes in detail in the gridshell structure, which consists of triangles and hexagons and is based on traditional Korean weaving geometry, has brought along a complex system (Table 7). The maximum length of a structural element was taken as 11 meters and 3500 structural elements were used. Five different element types were included in the project. Additional insulation materials have been applied to the wooden structures against heat and humidity (Jeska and Pascha, 2014). The roof structure was prefabricated and assembled with a crane. Lighting and ventilation are
provided with the gaps between the geometric system (Chilton and Tang, 2016).

3.7 Dalian Sports Center Stadium
Located in the city of Dalian in China, the stadium has a capacity of 61,000 people (Xue and Mason, 2019). A sports complex built as the port city of Dalian was chosen to host the 12th national games in China. The Dalian stadium, a large indoor arena, two secondary athletics stadiums, two small football stadiums, baseball stadium, tennis courts, hotel, and office buildings are located in this complex. The stadium was designed by Nadel. The structure in an elliptic geometric shape is covered with 2,745 Ethylene tetrafluoroethylene (ETFE) construction material, which is also used in the Allianz Arena (StadiumDB). In the Dalian stadium, the ETFE polymer was used in three different colors, white, gray, and blue (Hu et al., 2020). The structure of the stadium consists of a large-span truss system. There is a tessellation pattern in the pneumatic membrane on the structure (Table 8). 68 main and hoop trusses form the main structure of the structural system. The maximum length of the cantilever in the structure was 37 meters (Ren et al. 2016). The structural span of the stadium is 293 × 320 meters. Rectangular and circular cross-section structural elements were used (Li et al., 2022).

3.8 Swatch Headquarters
Located in Biel, Switzerland, the structure was designed by Shigeru Ban Architects and opened in October 2019. The four-storey office building has serpentine floor plans and the building height ranges from 8 to 26 meters. The building area of 11,000 square meters consists of a concrete core and floors. The roof cover is constructed of timber gridshell and 2,800 diamond geometries, each roughly 2 × 2 meters in size (Table 9). The gridshell has ETFE and glass coatings. The building shell is free-form shaped and supported by 4,500 glue-laminated timber beam segments (Stehling et al., 2020). 7,700 timber pieces were used (ArchDaily (b)). These beams were manufactured using special mounting techniques. Prefabricated gridshell elements were combined with on-site assembly. Optimizations were made in computer programs in a gridshell structure designed parametrically (Stehling et al., 2020).

4 Findings and results
Eight structures in which tessellation patterns differ were compared by considering features such as tessellation module, structure shape and materials, and span. By looking at the parameters in Table 10, the relationships between the structures were observed. Structural systems have different panel sections and consist of different panel shapes. Different tessellation configurations were seen in the examined building samples. The most used geometric shapes in terms of the tessellation module are triangle and hexagon. Triangular and hexagonal geometries are used in the structures of Union Tank Car, Montreal Bioshpere, DZ Bank, New Milan Trade Fair, and Haesley Nine Bridges Golf Clubhouse. Considering that a regular hexagon geometry is formed by the combination of 6 equilateral triangles, it is observed that the triangle geometry is one of the basic patterns in the creation of tessellation configurations. Notably, after triangle and hexagon geometric shape, a rectangular geometric shape is applied in structures. In the structure examples, it was seen that the quadrilateral geometry was obtained with rectangular and trapezoidal shapes. Rectangular tessellation modules have been applied at the Neckarsulm Swimming Arena, Dalian Sports Center Stadium, and Swatch Headquarters.

Table 7 The Haesley Nine Bridges Golf Clubhouse (Chilton and Tang, 2016), the formation of the tessellation module, and the mathematical expression of the tessellation pattern

Table 8 The Dalian Sports Center Stadium (StadiumDB), the formation of the tessellation module, and the mathematical expression of the tessellation pattern

Table 9 The Swatch Headquarters (ArchDaily (b)), the formation of the tessellation module, and the mathematical expression of the tessellation pattern
Table 10 Properties of structures to which tessellation configurations are applied

<table>
<thead>
<tr>
<th>Building function</th>
<th>Union Tank Car</th>
<th>Montreal Biosphere</th>
<th>Neckarsulm Swimming Arena</th>
<th>DZ Bank</th>
<th>New Milan Trade Fair</th>
<th>Haesley Nine Bridges Golf Clubhouse</th>
<th>Dalian Sports Center Stadium</th>
<th>Swatch Headquarters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Trading</td>
<td>Social, cultural and symbolical</td>
<td>Social and trading</td>
<td>Trading and residential</td>
<td>Social, cultural, and trading</td>
<td>Trading and transport</td>
<td>Social and trading</td>
<td>Management office</td>
</tr>
<tr>
<td>Panel shape</td>
<td>Hexagon</td>
<td>Triangle and hexagon</td>
<td>Rectangle</td>
<td>Triangle</td>
<td>Triangle</td>
<td>Triangle and hexagon</td>
<td>Trapezoid</td>
<td>Diamond</td>
</tr>
<tr>
<td>Tessellation module formation</td>
<td>Translation</td>
<td>Translation, rotation, and reflection</td>
<td>Translation</td>
<td>Rotation and reflection</td>
<td>Translation, rotation, and reflection</td>
<td>Translation and reflection</td>
<td>Rotation and reflection</td>
<td>Translation</td>
</tr>
<tr>
<td>Mathematical sequence</td>
<td>6.6.6</td>
<td>3.3.3.3.3.3 and 6.6.6</td>
<td>4.4.4.4</td>
<td>3.3.3.3.3</td>
<td>3.3.3.3.3</td>
<td>3.6.3.6</td>
<td>4.4.4.4</td>
<td>4.4.4.4</td>
</tr>
<tr>
<td>Structure shape</td>
<td>Double-layered one-quarter geodesic sphere</td>
<td>Double-layered three-quarter geodesic sphere</td>
<td>Spherical double-curved gridshell structure</td>
<td>Free-form double-curved gridshell structure</td>
<td>Free-form double-curved gridshell structure</td>
<td>Elliptic geometric shape</td>
<td>Free-form curved gridshell structure</td>
<td></td>
</tr>
<tr>
<td>Structure materials</td>
<td>Steel bars and cable trusses</td>
<td>Steel bars and acrylic cells</td>
<td>Steel bars, twin cables, and spherical curved insulating glass panels</td>
<td>Stainless steel bars, tempered and laminated glass, and cable trusses</td>
<td>Stainless steel bars and laminated glass</td>
<td>Glued laminated timber (spruce)</td>
<td>Ethylene tetrafluoroethylene (ETFE), grid metal panel, main trusses, and hoop trusses</td>
<td>Glue-laminated timber beams, ETFE, and glass</td>
</tr>
<tr>
<td>Maximum structure height (meter)</td>
<td>36</td>
<td>62</td>
<td>5.75</td>
<td>Not found</td>
<td>37</td>
<td>13.6</td>
<td>Not found</td>
<td>26</td>
</tr>
<tr>
<td>Maximum span (meter)</td>
<td>117</td>
<td>76</td>
<td>25.2</td>
<td>20</td>
<td>41</td>
<td>9</td>
<td>Maximum span: 320, maximum length of the cantilever: 37</td>
<td>35</td>
</tr>
</tbody>
</table>
Looking at the mathematical sequence in the tessellation modules, it was seen that the structures were mostly derived from triangular geometry. The use of hexagons, obtained from the combination of 6 triangles, supports triangles as the basic geometric shape. When looking at the structural forms in buildings, it is seen that the use of curved and parametric surfaces for large-spans is more common. It was found that curvilinear surfaces were applied in all of the examined structures. It has been determined that there are large-span constructions mostly consisting of geodesic domes among the circular surface structures. The Union Tank Car and Montreal Biosphère structures are circular surface structures and were built with the largest spans among the examples studied. In these structures, a geodesic sphere has been applied. When the building samples are analyzed, it can be stated that the curvilinear structure systems formed by triangle, hexagon, or a combination of triangle-hexagon are more preferred. Considering the tessellation configurations, all but a structure were formed with regular tessellation patterns. The only structural system obtained with semi-regular tessellation is the Haesley Nine Bridges Golf Clubhouse. Another feature that distinguishes this structure from others is that it is the structure with the least spans. As the structures listed chronologically approach the present, changes are also observed in the structural systems. It is seen that combinations of these geometries and different geometries (trapezoid) are used instead of the basic geometric shapes of triangle, rectangle, and hexagon. In addition to changing their geometric configurations, their structural shapes change. In the structures examined, gridshell systems have replaced the geodesic domes. Looking at the materials of the structural systems, it has been observed that construction materials such as steel, cable truss, glass, wood, and ETFE are used. While it is seen that steel and cable trusses are applied in the structural systems where complex and large-spans are passed, it is seen that durable wooden building material also participate in the building production process.

5 Conclusion

Geometric shapes are brought together in a set of mathematical rules to form an architectural design. The tessellation method, which is a concept of mathematics and geometry, is seen in many fields and in the fields of architecture and engineering. Tessellations, in which the relationship between geometry and architecture are integrated, come to the fore in various parts of architecture. Tessellation configurations are encountered in I arrangements, floor coverings, and wall, ceiling decorations. Tessellation patterns, which date back centuries and have a deep-rooted history, are developing from 2-dimensional planar dimensions to 3-dimensional complex systems. In the architectural structures examined, 3-dimensional structural systems were also focused on. Each of these structures has a complex construction system in its own architectural dimension. At this point, it was determined that in the structures examined, wooden construction material was predominantly used in addition to steel and glass structural elements. With the tessellation configurations created, long-spans have been passed, and progress has been made in the architectural design and the engineering dimension. Considering the information obtained, it is thought that the examples of buildings in which geometry is integrated into architecture will increase in future studies. The structures examined form the basis of the structures to be built in the future. As a result of the developing systems, it is possible to see the systems that will create complex geometries by integrating different tessellation types in the structures to be built in the future.

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