SAFETY AGAINST LOSS OF STATIC EQUILIBRIUM

GY. DEÁK—L. BAINTNER

Department of Strength of Materials and Structures,
Technical University, Budapest, H-1521

(Received: September 1st, 1983)

1. The role of "static equilibrium" in structural design

Certain structures or members may be displaced (overturned, slipped, floated) from their position without failure of the solid connections, their equilibrium is due to permanent loads or to the resulting friction. National standards and international recommendations discuss the stability of such structures as a special case of the ultimate load capacity, considering the structures and the soil as rigid bodies; certain specifications impose special safety factors for loads — exceeding those in strength analyses.

This approach is reprehensible from several aspects:

— displacement of structures is usually preceded by failure or yield of materials over a part of the contacting surfaces;

— exceeding load effects due to structural deformation and to constructional inaccuracy affects the stability and strength of the structure in a similar manner;

— often permanent loads and solid connections provide together for structural equilibrium.

This latter circumstance and the resulting contradictions are illustrated by the following example. The structure in Fig. 1 is subject to wind load F_W. Overturning of the structure is prevented by force R_S acting in the steel anchorage, and by permanent load R_G. (In this case the letter symbol R points to the resisting role of the permanent load.) The respective safety factors are γ_W, γ_S and γ_{RG}. The diagram complying with four different specifications shows the variation of load-resistance ratio depending on the shares of the solid connection and of the permanent load in the resistance. In the CEB-FIP Recommendation [1] and in the Hungarian Standard [3], where special safety factors are prescribed for the analysis of the "static equilibrium", this limit case is considered as an outstanding singular point. This can easily result in the following absurdity: for example, the stability analysis of a structure, proving the permanent load to be insufficient in itself, requires a solid anchorage too, to sustain stability; but when this connection has to be designed in compliance with safety factors specified for strength analysis, it may turn out for the connection to be needless (it seems to be compressed).

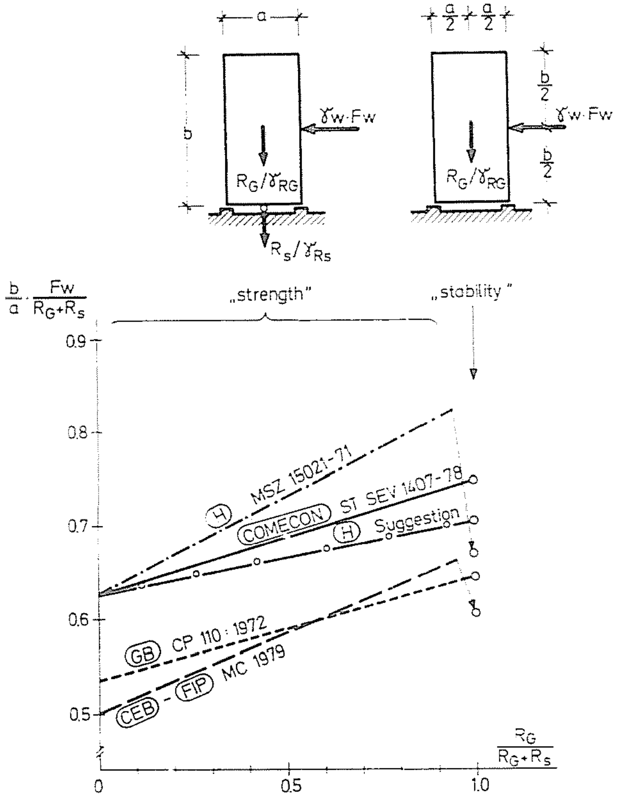

Fig. 1. Safety factors of "strength" and of "stability" in different specifications

Introduction of the concept "static equilibrium", and its distinguished treatment had been imposed by the method of "permissible stresses", failing to provide for an adequate structural safety in equilibrium analyses where material strengths had no role at all or only subordinate. Then it was replaced by the method of "permissible loads" applying safety factors to actions. This procedure pervaded the professional mind to a degree to be preserved in some specifications even after the advent of the limit state analysis method where it is needless and meaningless.

A number of professionals are inclined to consider loss of the stability as an especially dangerous mode of failure justifying unusually rigorous safety factors. As concerns the cases of "slipping" and "floating", these are unlikely to belong to extremely dangerous modes of failure. Overturning or failure by collapse of an elevated structure (e. g. chimney stack) cannot be sharply distinguished from the aspect of consequences. Neither can overturning of a retaining wall be stated to be more damaging than failure of the ground floor column of a multistorey building.

2. Safety factor of the permanent load favourable for equilibrium

According to the above, in determining safety factor γ_{RG} of force R_G favourable for equilibrium, a failure probability about equal to that in strength analyses has to be specified. This requirement is not absolutely met by safety factors imposed by various specifications for favourable permanent loads in strength analyses. Remind:

— In specifications based on "semiprobabilistic" methods, generally different fractiles are used for determining the extreme value of the unfavourable loads or that of resistance (e. g. $F_{0,95}$ and $R_{0,00135}$). Favourable loads act, however, as "resistance", hence it were not correct to take them as a fractile $F_{0,05}$ in account.

— The "semiprobabilistic" method provides for an about equal, stable failure probability if standard deviations of load and of resistance are about equal. If the two significantly differ — as in the case of e. g. wind load and dead load as resistance — then reckoning with invariable fractiles significantly increases the failure probability. In the extreme case, for a steady (deterministic) value of the resistance, the probability of failure would be equal to that of exceeding F.

Changing of the failure probability will be illustrated according to Hungarian Standard [3] for two extreme cases of resistance due exclusively to solid connection R_S ("strength") and to weight R_G of the structure or certain members ("stability"). Snow load F_S and wind load F_W will be examined as accidental loads affected by rather different safety factors. Combination F_G; R_G corresponds to the case of a structure made of different materials or with different technologies; the weight of certain members acts as load, that of the others as resistance. (Weight of a structure made of uniform material and with uniform technology has to be treated as a single force: either as load or as resistance, depending on its line of action.)

Resistances are considered as of normal distribution; the analysis will be made by assuming both normally distributed loads and those of double exponential distribution.

2.1 *Loads of normal distribution*

The following assumptions have been made in the analysis (Table 1, Fig. 2):

— Basic values of the variable loads (F_W and F_S) are the mean values of maxima during the service life of the structure. Extreme value $F_U = \gamma_F \cdot F$ corresponds to fractile 95%. Hence, $t_F = 1.64$. Safety factors for snow and wind load are $\gamma_S = 1.4$ and $\gamma_W = 1.2$, respectively.

Table 1

Analysis data assuming normal load distribution

	"strength" $R=R_S$			"stability": $R=R_G$		
	snow load F, R_s	wind load F_w, R_s	dead load F_G, R_s	snow load F_s, R_G	wind load F_w, R_G	dead load F_G, R_G
γ_F	1.4	1.2	1.1	1.4	1.2	1.1
γ_{Gm}	—	—	1.05	1.05	1.05	1.05
t_F		1.64			1.64	
γ_R		1.15			1.0	
t_R		3.00			1.64	
s_R/s_F	0.633	1.085	3.979	0.175	0.300	1.100
P	$13 \cdot 10^{-4}$	$5 \cdot 10^{-4}$	$5 \cdot 10^{-4}$	$250 \cdot 10^{-4}$	$180 \cdot 10^{-4}$	$95 \cdot 10^{-4}$
			α'_R		$1.15 \sim 1/0.85$	
	suggested:		t'_R		5.92	
			P'	$34 \cdot 10^{-4}$	$6.1 \cdot 10^{-4}$	$<10^{-6}$

— Basic value of the permanent load F_G is the design load. The mean load is somewhat higher: $F_G \sim 1.05\ F_G$. Extreme values correspond to 5% and 95% fractiles, respectively, thus, $t_G = 1.64$. Safety factors are 1.1 for unfavourable, and 1.0 for favourable loads. (Data refer to compact structural materials, e. g. concrete or steel.)

— Basic and extreme values of steel strength R_S and R_{SU} correspond to 5% and 0.135% fractiles, resp., thus, $t_R = 3.0$. The safety division factor $\gamma_{RS} = 1.15$.

— The case just meeting the load bearing requirement, i. e. $F_U = R_U$, will be considered where the s_R/s_F ratio can be determined from relationships in Fig. 2.

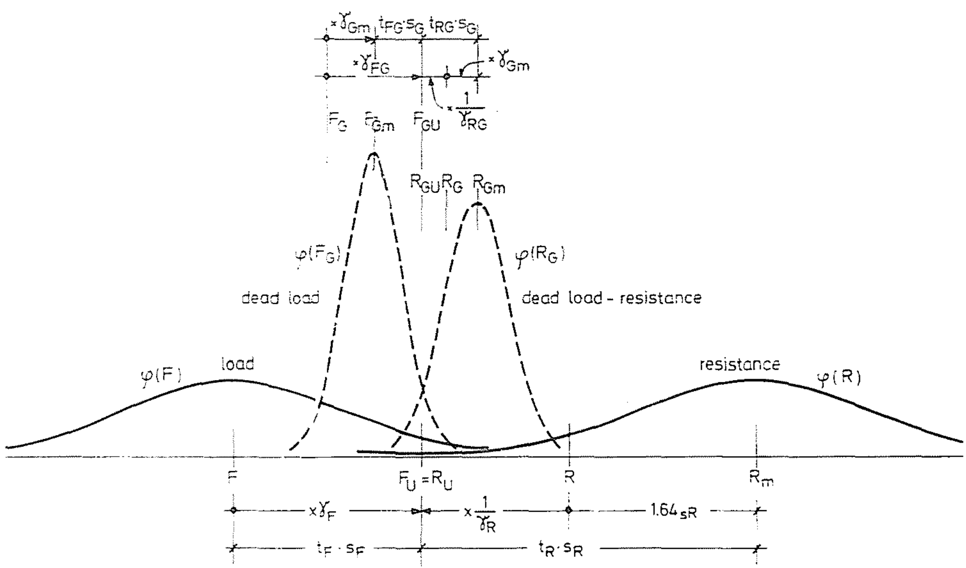

Fig. 2. Statistical distributions of load and of resistance

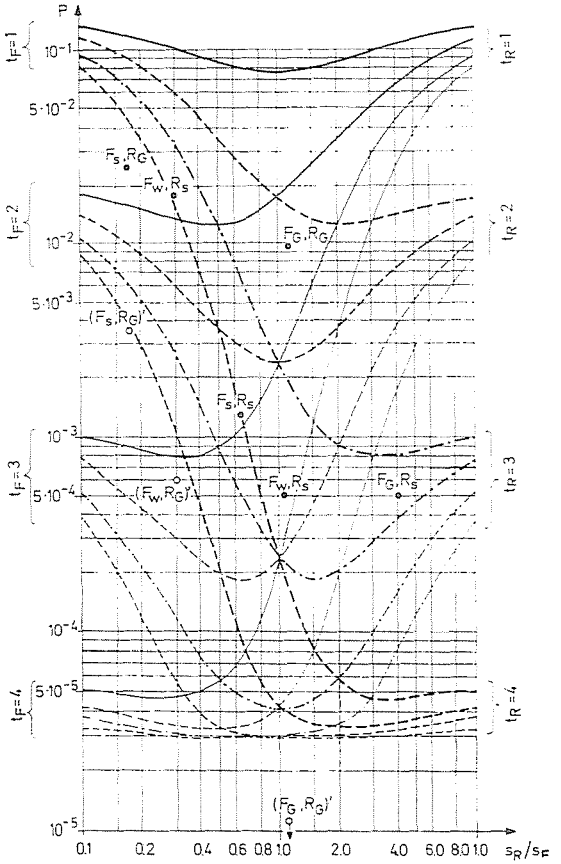

Fig. 3. Course of failure probability for normal load distribution

Density function of the variable load and of the connection strength has been plotted in full line, and that of the dead load acting as load or as resistance in dash line in Fig. 2.

Calculation results are seen in Fig. 3. Probability of the ultimate state to occur has been plotted in ordinate according to

$$P\{F > R\} = \int_{-\infty}^{\infty} \varphi(F) \cdot \int_{-\infty}^{x} \varphi(R)\, dx\, dx$$

for different s_R/s_F ratios of the standard deviations. Curve sets correspond to different fractiles of load and resistance. Perceivably, for s_R/s_F ratios other than 1.00, the failure probability much increases.

Points for different matches of load and resistance show — as obvious also from Table 1 — the failure probability in "strength" analysis not to significantly differ for different loads. The failure probability is, however, greater by an order of magnitude in "stability" analysis if the 5% quantile of dead load acting as resistance is reckoned with, in conformity with the Hungarian Standard for strength analysis. Thus, safety factor γ_{RG} is justly specified so as to yield a much lower fractile of dead load. Points marked ( )′ in Fig. 3 refer to division factor $\gamma_{RG} = 1.15 \sim 1/0.85$ ($t_{RG} \sim 6.0$). Although there is very low probability for still lower dead loads to occur, this is that where the failure probability about equals that in the strength analysis. The situation is particularly favourable for wind loads, the primary cause of instability. As a matter of fact, the safety is excessive for the case $(F_G: R_G)'$ (its point would lie outside the diagram), but such a contribution is infrequent in design problems.

2.2 *Double exponential load distribution*

Experience shows the distribution of timely maxima of variable loads to be other than normal as a rule. Different other functions are found in publications for such cases, including distribution function

$$\Phi(x) = \exp\left[-\exp\left(-\frac{x-\alpha}{\beta}\right)\right]$$

also considered in our investigations. The distribution function itself and the density function obtained from it by derivation are seen in Fig. 4, with some significant relationships. Basic data and calculation results have been compiled in Table 2 and in Fig. 5.

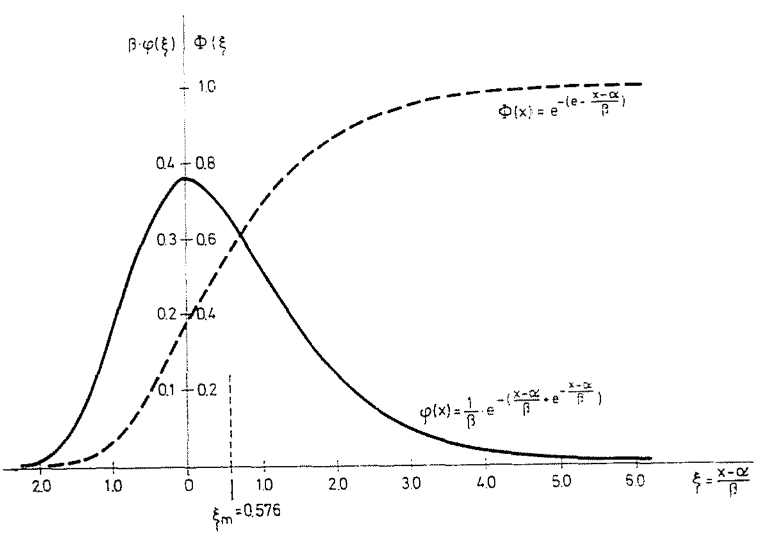

Fig. 4. Double exponential distribution

Table 2

Analysis data assuming double exponential load distribution

	"strength"		"stability"	
	snow load F_S, R_S	wind load F_W, R_S	snow load F_S, R_G	wind load F_W, R_G
γ_F	1.4	1.2	1.4	1.2
α_F/F	0.904	0.952	0.904	0.952
β_F/F	0.167	0.084	0.167	0.084
t_F	2.97		2.97	
γ_R	1.15		1.0	
γ_{Gm}	—		1.05	
t_R	3.00		1.64	
s_R/β_F	0.924	1.584	0.256	0.438
P	$48 \cdot 10^{-4}$	$14 \cdot 10^{-4}$	$320 \cdot 10^{-4}$	$250 \cdot 10^{-4}$
suggested: γ'_R			$1.15 \sim 1/0.85$	
suggested: t'_R			5.92	
suggested: P'			$105 \cdot 10^{-4}$	$35 \cdot 10^{-4}$

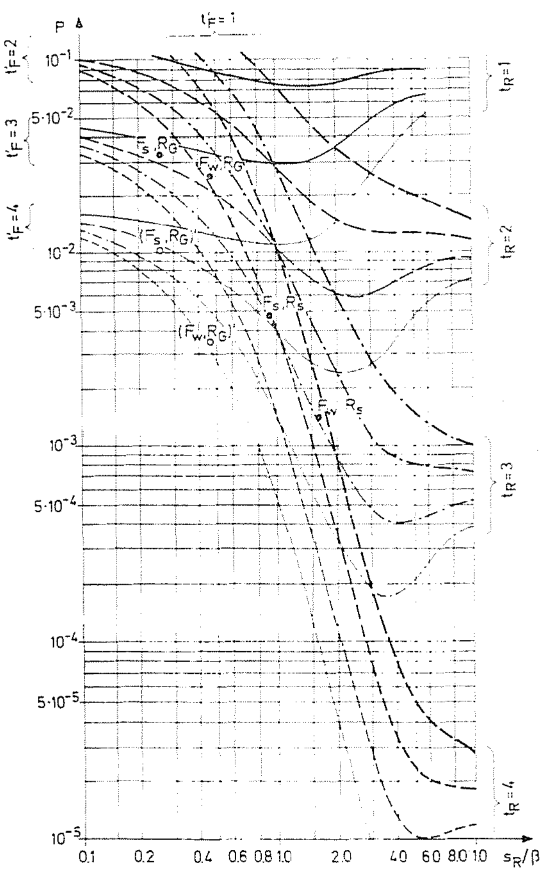

Fig. 5. Course of failure probability for double exponential load distribution

For identical safety factors and pertaining fractiles, the exponential distribution is seen to somewhat increase the failure probability compared to a load of normal distribution. Here also, failure probabilities differ by orders of magnitude, depending on whether the equilibrium is due to a solid connection or to a permanent load, but here modification of the safety factor of the favourable permanent load is less efficient: failure probability belonging to points $(F_s; R_G)$ and $(F_W; R_G)$ determined from factor $\gamma_{RG} = 1.15$ exceeds that in "strength" analysis.

Remind that in most of practical structures, the equilibrium is due to the complex of permanent load and solid connections, reducing the failure probability, namely the term for the resistance comprises the sum of two independent random variables.

3. Conclusions

3.1 The "static equilibrium" of structures has to be analysed as a limit case in strength analysis rather than as a separate requirement. To provide for the continuity of transition, elastic and plastic properties of structural materials, additional effects due to deformations of structure and soil and to constructional inaccuracies have to be identically treated, and equal safety factors have to be applied for favourable permanent loads, irrespective of the shares of permanent load and of solid connections in the resistance of the structure.

3.2 The safety factor of permanent loads favourable for the equilibrium, hence acting as a resistance, has to be stated so as to yield about equal failure probabilities in "stability" and in "strength" analyses. Remind that the failure probability much increases even for identical safety factors if standard deviations of load and resistance significantly differ. It is suggested to apply a division factor of 1.15 (a multiplication factor of 0.85) for the dead load favourable for the equilibrium of compact structural materials, rather than the actual one of 1.00 in Hungarian design codes.

Summary

Analysis of "static equilibrium" is a limit case of "strength" analysis: resistance of the structure is due exclusively or mostly to permanent loads. Neither in this analysis can the structure be considered as a rigid body, the requirement for an increased safety is generally unjustified, neither special safety factors are needed. To have a failure probability at the same level as in strength analyses, the favourable permanent loads should be taken into account with the same fractile as that for the resistance of solid connections. At the same time, however, it has to be kept in mind not to let the relatively low standard deviation of favourable permanent loads increase the failure probability. For Hungarian design codes, a safety division factor of 1.15 may be suggested for the dead load of compact structural materials if it is convenient for the analysis.

References

1. Common Unified Rules for Different Types of Construction and Material. CEB-FIP Recommendation, 1978.
2. Static Design of Load-Bearing Construction of Buildings. General Requirements of Loads and Effects. COMECON ST SEV 1407—78.
3. Design of Load-Bearing Structures of Constructions. Loads on, and Special Requirements for, Building Structures.* Hungarian Standard MSz 15021/1—71.
4. Handbook on the Unified Code for Structural Concrete. CP 110:1972.

Prof. Dr. György DEÁK
Dr. László BAINTNER
H-1521, Budapest

* In Hungarian