ANALYSIS OF REINFORCED CONCRETE COUPLED SHEAR WALLS

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1. Setting the problem

Recent development of reinforced concrete structures was featured by the extended application of wall structures.

Industrialized building systems based on tunnel shutterings and on large slabs, load-bearing walls, precast beam-and-column frameworks, increased storey number and reduced building weights enhanced the importance of stiffening walls.

A field of research on reinforced concrete of increasing actuality is concerned with walls and wall systems, involving ever more researchers.

Fundamental work by CHITTY and BECK opened the line of great many studies on shear walls. Analysis methods follow either that of ROSMANN or of ALBIGES and GOULET. Stresses in the shear wall are determined by assuming homogeneous, crack-free cross sections in stress state I rather than to follow the stress state of structures in cracked condition. In general, these methods examine the horizontal force effects in themselves, assume that the vertical loads can also be considered in themselves, and that the combined effect of both leaves the wall crack-free, and the results determined for stress state I can be simply superimposed.

For theoretically preparing experiments in the scope "Load-Bearing Walls and Systems", a computation method for the uniform handling of r.c. walls in any stress state conform to the education delivered at the Faculty of Architecture has been developed.

2. Initial assumptions

A. *Assumptions on the wall design:*

— High storey number.
— Walls much stiffer than connecting beams ($k_0 > k_g$).
— Within a storey, material and cross section of wall sections and connecting beams are the same.

— Cross section, material and height of wall sections may vary for each storey and so may stiffness. (Walls with identical storeys throughout are termed regular walls.)
— Axes of wall strips are continuous throughout the wall height.

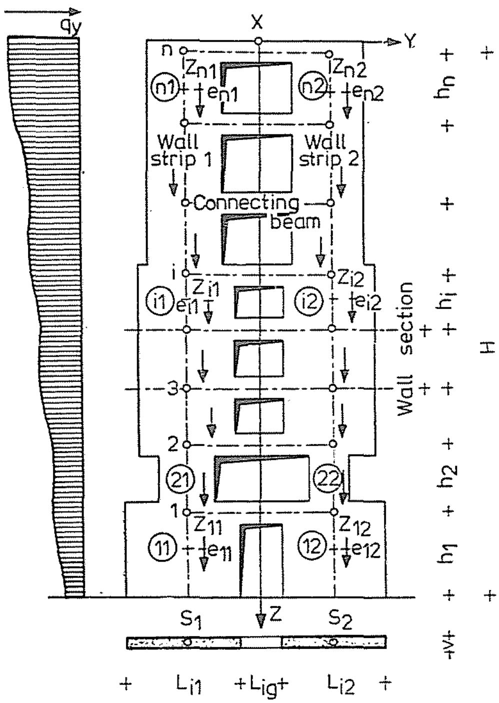

Fig. 1.

B. *Assumptions on the loads*

— All vertical and horizontal forces acting on the wall each storey are transferred by the floors. (Also wall dead loads are assumed to act on each storey.)
— Forces acting on the wall each storey are of identical direction and distribution but their values may be different.

C. *Assumptions on stress state and deformations*

— Interaction of two wall strips considered as cantilever clamped at the bottom is provided by the connecting beams.
— Floors are considered as plates infinitely stiff in their plane, with bending stiffnesses much lower in the normal plane.

— Axes of the two wall strips making up the wall system remain parallel after loading deformations, identity of displacements being provided by floors considered as infinitely stiff in their plane.
— The law of plane cross sections is valid for individual connecting beams and wall strips but cannot be assumed valid for the entire wall.
— The wall undergoes elastic deformations.
— Axial deformations of connecting beams are negligible.
— In general, also the shear deformations of connecting beams and wall sections are negligible. This effect can be approximated by the reduction of the moment of inertia:

$$I_p = \frac{I_g}{1 + 2(1+\mu)\left(\frac{m}{L}\right)^2}.$$

— Reinforced concrete units comply with assumptions and material characteristics of Hungarian Standard Specifications MSZ 15021/71.

3. Calculation method

3.1 *Determination of stresses in a shear wall by the compatibility method*

a) *Fundamentals of the compatibility method*

— cutting the structure leads to statically determined primary beams affected, beside loads, by unknown constraints;

— primary beam deformations due to unknown constraints and loads can be determined according to the elementary laws of the strength of materials (e.g. by the Mohr method);

— unknown constraints can be determined from deformational equations written for the points of cutting.

b) *Equations of the compatibility method for a shear wall*

— A shear wall of n storeys can be reduced to a problem of hyperstaticity of n redundancies. To this effect, axial deformation of connecting beams will be neglected (hence $N_g \approx 0$) and the structure assumed to be cut by a vertical plane passing through the moment zero points of the connecting beams. Then a single unknown constraint will develop at each storey, viz. n shear forces Q_i acting at the points of cutting.

The n unknowns will be determined by n deformational equations of the compatibility method: compatibility equations for the cutting places at any storey.

The applied method starts from the assumption that the deformed wall strip cross sections remain normal to the parallel curved axes of the wall strips. This results in the relative shifting of the two wall strip cross sections, originally in the same horizontal plane, and so, of the halves of the two primary beams at the fictitious cutting place a_i^0.

No rupture is possible in the real structure. The unknown constraints, the connecting beam forces are responsible for the continuity, the interaction

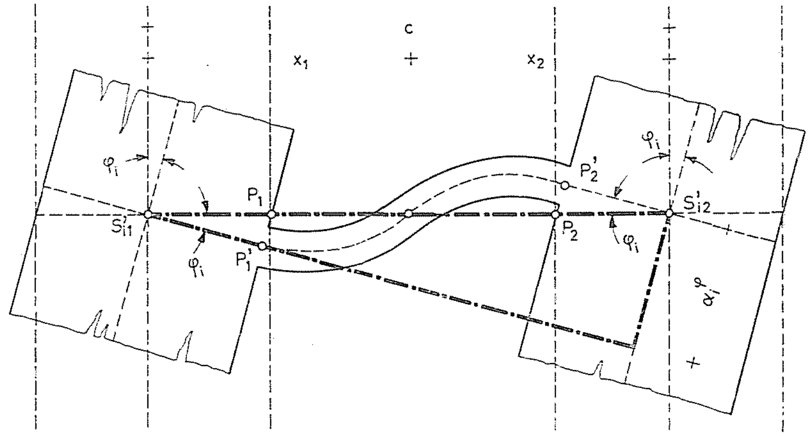

Fig. 2.

of the structure. Thus, the opposite deformation a_i^Q due to unknown connecting beam forces must equal the calculated relative displacement of primary beams:

$$a_i^{Q*} = a_i^0 .$$

This deformation equation written for all storeys yields a set of n equations for determining the unknown forces acting on the n-storey hyperstatic wall of n redundancies.

c) *Stresses in a coupled shear wall*

Once the set of equations has been solved, stresses in the coupled shear wall can be determined for the statically determinate cantilevered beam, subject to loads and the already known connecting beam forces according to the laws of elementary statics.

Wall strip stresses are:

$$M_i = M_i^0 - M_i^Q$$
$$N_i = N_i^0 \pm N_i^Q$$
$$T_i = T_i^0 .$$

Bending moments in the two wall strips are distributed according to stiffness ratios:

$$M_{i1} = \beta_1 \cdot M_i ; \qquad M_{i2} = \beta_2 \cdot M_i$$

where

$$\beta_1 = \frac{k_1}{k_1 + k_2} ; \qquad \beta_2 = \frac{k_2}{k_1 + k_2} .$$

Connecting beam stresses:

$$M_{ig} = M_{ig}^0 \pm Q_i \cdot \frac{L_g}{2}$$

$$T_{ig} = T_{ig}^0 + Q_i .$$

3.2 *Load vector A_n*

Deformations of statically determinate primary beams involved in the set of equations are advisably determined by the Mohr method.

Practically, separate calculation of the two primary beams, hence distribution of the effects (e.g. horizontal forces, eccentric moments) between the two primary beams may be saved.

A so-called "substitution beam" is produced by summing up the two primary beams. Its deformations will directly yield the relative displacement values sought for. (Of course, normal force effects need reckoning with separately for both wall strips or primary beams.)

Numerical values will be obtained for relative displacements due to loads and effects, to be considered load factors as usual in the compatibility method.

Relative deformations of each storey constitute a column vector:

$$A_n = \begin{bmatrix} a_n^0 \\ \vdots \\ a_i^0 \\ \vdots \\ a_2^0 \\ a_1^0 \end{bmatrix} .$$

3.3 *Deformations due to unknown connecting beam forces*

Connected beam forces cause partly wall strip deformations hence relative displacements opposite to the loads, and partly beam displacements:

$$a_i^{Q*} = a_i^Q + a_i^D .$$

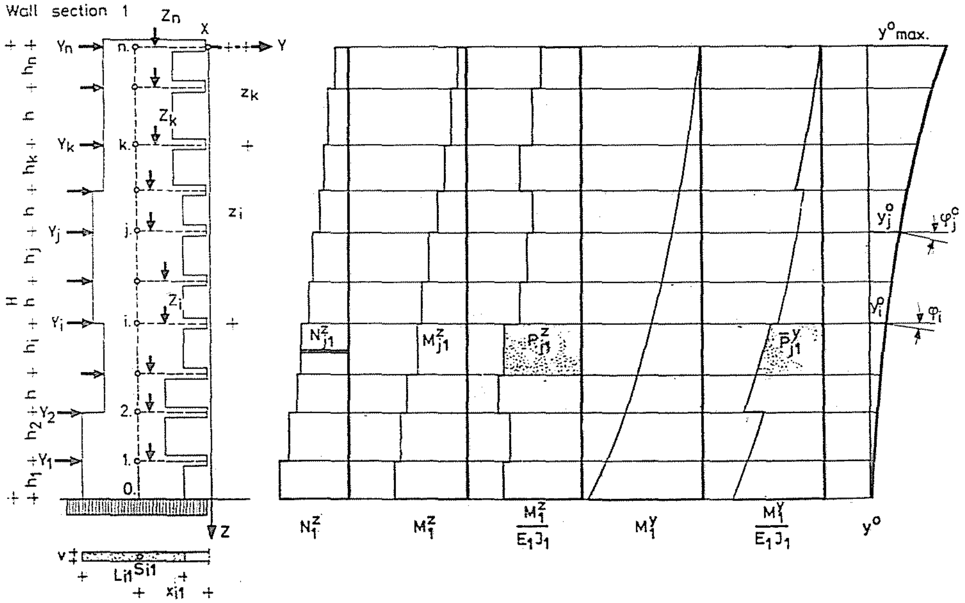

Fig. 3.

3.31 *Relative displacements* will be determined by summing up relative displacements at each storey i due to shear forces Q_k acting at the k-th storey:

$$a_i^Q = \sum_{k=1}^{k=n} a_i^{Q_k} \ .$$

Relative displacement at any storey due to a shear force acting on a given storey is determined by means of unit factors as usual in the compatibility method:

$$a_i^{Q_k} = Q_k \cdot E_{ik} \ .$$

Unit factor E_{ik} means the relative displacement at the i-th storey due to a unit shear force Q_k acting on the k-th storey, the so-called stiffness coefficient of the wall strips.

Summing up the displacement changes at j-th storeys below the tested i-th storey (the wall strip rigidity coefficients) results in the unit factor. A unit force acting on the k-th storey affects the relative displacement of lower storeys alone but leaves them constant above.

Thus, for a storey below the point of application of the force:

$$i \leq k \qquad E_{ik} = \sum_{\gamma=1}^{\gamma=i} e_j$$

and above the same:

$$i > k \qquad E_{ik} = \sum_{\gamma=1}^{\gamma=k} e_j \,.$$

Remark: In conformity with the Maxwell—Betti theorem of exchangeability for unit factors: deformation at the i-th storey due to a unit force acting on the k-th storey equals that at the k-th storey due to a force acting on the i-th storey.

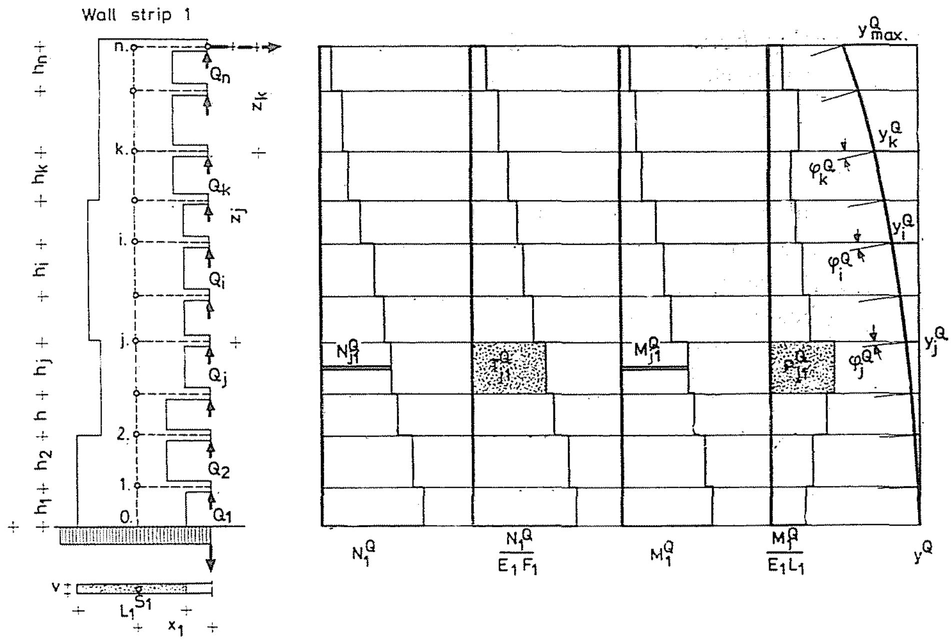

Fig. 4.

Wall strip rigidity coefficient e_j involved in the calculation means the change of relative displacement on an arbitrary j-th storey due to unit force $Q_k = 1$:

$$e_j = e_j^N + e_j^\varphi = \frac{h_j}{E_j}\left(\frac{1}{F_{j1}} + \frac{1}{F_{j2}}\right) + \frac{c^2 \cdot h_j}{E_j \cdot I_{j0}} = \frac{c}{E_j \cdot I_{j0}} \cdot \frac{I_j}{S_j} \cdot h_j \,.$$

Wall strip stiffness matrix E_{nn}. All storeys of an n-storey wall are subject to shear forces causing relative deformations on each storey. Deformations due to unit forces, i.e. the unit factors can be written in a square matrix of n order, symmetrical about the principal diagonal in conformity with the theorem of exchangeability.

$$E_{nn} = \begin{bmatrix} E_{11} & E_{12} \ldots E_{1i} \ldots E_{1n} \\ E_{21} & E_{22} \ldots E_{2i} \ldots E_{2n} \\ \vdots & \vdots \qquad \vdots \qquad \vdots \\ E_{i1} & E_{i2} \ldots E_{ii} \ldots E_{in} \\ \vdots & \vdots \qquad \vdots \qquad \vdots \\ E_{n1} & E_{n2} \ldots E_{ni} \ldots E_{nn} \end{bmatrix}.$$

3.32 *Connecting beam deflections.* Calculation starts from the displacement belonging to unit shear force — the connecting beam unit factors:

$$a_i^D = Q_i \cdot d_i.$$

Relative displacement of the ends of the two elementary cantilevers obtained by cutting the structure at the zero point of connecting beam moment is calculated as the second moment of area of the imaginary diagram *1/EI* of the connecting beam, written for the axis passing through the point of inflection.

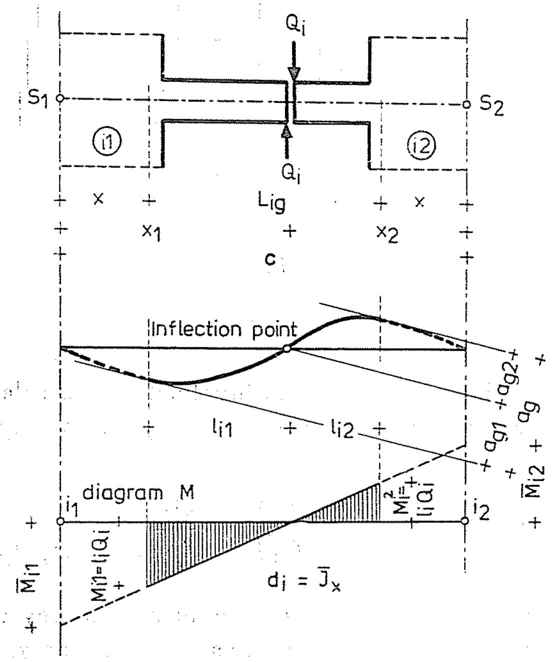

Fig. 5.

In conformity with published test results, the zero point of moments can be assumed at the halving point for all connecting beams.

Accordingly, unit factor of the crack-free connecting beams of constant cross-section is:

$$d_i = \frac{L_g^3}{12\, E_i \cdot I_{gi}} .$$

Shear deformation of connecting beams can be approximated by modifying their moment of inertia.

3.33 *Wall stiffness matrix K.* Relative displacements at the cutting place due to wall strip and connecting beam deflections imposed by a unit shear force, the so-called unit factors can be comprised in a single matrix:

$$K_{nn} = D_{nn} + E_{nn} = \begin{bmatrix} d_1 + E_{11} & E_{12} \ldots & E_{1i} \ldots & E_{1n} \\ E_{21} & d_2 + E_{22} & E_{2i} \ldots & E_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ E_{i1} & E_{i2} \ldots & d_i + E_{ii} \ldots & E_{in} \\ \vdots & \vdots & \vdots & \vdots \\ E_{n1} & E_{n2} \ldots & E_{ni} \ldots & d_n + E_{nn} \end{bmatrix} .$$

3.4 *Equation system of the connecting beam forces*

Using notations as before, compatibility of the coupled shear wall can be written in the matrix equation:

$$K_{nn} \cdot Q_n = A_n^0 .$$

This concise equation system written in matrix form has the following features:

a) Right-hand side of any equation is the non-zero numerical value of the relative displacements of the two primary beams, due to loads.

b) Left-hand side of all equations includes *all* unknown shear forces.

c) Coefficients of the unknown shear forces constitute a matrix symmetrical about the main diagonal.

d) Coefficients involved in the set of equations of the preliminary calculation, and constant values of relative displacements due to loads can be determined by elementary statical methods. In conformity with the initial assumptions of calculation, the method can be applied for cross sections and material characteristics varying each storey.

e) The suggested method permits to take characteristics of cracked structural members according to stress state II into consideration.

f) In case of "regular" walls — where wall strips are assumed to be crack-free — calculation work is reduced, and factoring out the wall strip stiffness coefficient e identical for all storey, the wall stiffness matrix $\mathbf{K}$ simplifies into:

$$\mathbf{K}_{nn} = e \begin{bmatrix} d_1+1 & 1 & \dots & 1 & \dots & 1 \\ 1 & d_2+2 & \dots & 2 & \dots & 2 \\ \vdots & \vdots & & \vdots & & \vdots \\ 1 & 2 & \dots & d_i+i & \dots & i \\ \vdots & \vdots & & \vdots & & \vdots \\ 1 & 2 & \dots & i & \dots & d_n+n \end{bmatrix}.$$

g) "Manual" determination of constants and solution of the equations is rather tedious — imposing computer methods. This method appears practically prone to computerization, determination of coefficients and solution of the equation system with several unknowns easy to handle, and so is the preparation of computation and the use of outputs. Computation of an average wall of 10 to 20 storeys takes a running time of about 1 to 3 min on the computer ODRA 1204 of the Technical University, Budapest, together with the detailed printing of starting data and outputs.

4. Iterative application of the compatibility method

4.1 *Effect of initial cracking*

Stiffness of units is affected by cracking, entraining the variation of deformations under load as well as of the relevant terms of the stiffness matrix of the structure.

The previously described method had been applied for the iterative analysis of load cases beyond the cracking load.

4.2 *Iterative analysis of a coupled r.c. shear wall in stress state II by the compatibility method*

Analysis of r.c. structures consists in verifying a given structure of given material, dimensions and reinforcement.

Step 1. *Computation of the ultimate stress*

In conformity with specifications in force, ultimate stresses in connecting beams and wall strips at cracking and at failure are determined. These will be the boundary conditions. It is advisable to determine eccentricity limits of forces and of normal force eccentricities for parapets and for wall sections, respectively:

$$Q_{crack}; \quad e_{1H\,crack}; \quad e_{2H\,crack}.$$

Rather than strains, however, practically stresses are often directly compared, like in the following.

Step 2. Calculation of strains in stress state I

Assuming exemptness of cracks, stresses will be determined by simultaneously considering *all effects*

$$Q_i^{(1)}; \quad N_{i1}^{(1)}; \quad N_{i2}^{(1)}; \quad M_{i1}^{(1)}; \quad M_{i2}^{(1)}; \quad \sigma_{bi\,\max}^{(1)}.$$

- For $\sigma_{i\,\max}^{(1)} \leq \sigma_{hH}$, ultimate cracking stress values are never exceeded, determined stresses correspond to the real state of stresses in the structure and can thus be considered as *final results* of calculation.
- For $\sigma_i^{(1)} > \sigma_{hH}$, strains in stress state I in one or more structural members exceed the ultimate cracking stress value — the structure is considered as *cracked*, and the calculation has to be repeated, taking the stiffness variation of the cracked members into consideration.

Step 3. Calculation of strains in stress state II

- Stiffness of structural members found to be cracked according to the calculation assuming stress state I has to be determined again with the assumption of stress state II.
- Analysis by the compatibility method will be repeated, taking the change of the stiffness matrix of the wall strip into consideration by replacing the stiffness values of the cracked members. In case of stress state II/A (cracked connecting beams), only terms in the main diagonal of the matrix will

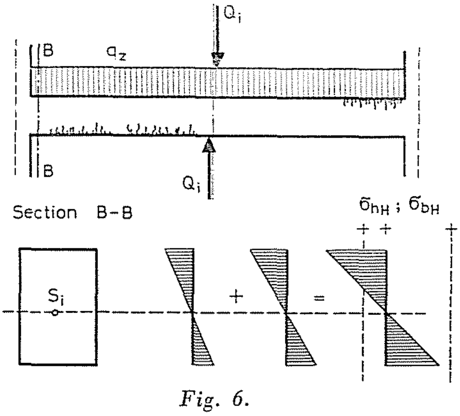

Fig. 6.

change, while in stress state II/C, also terms outside the main diagonal vary.

— Repeated calculation results show wall strip strain to be redistributed:

$$Q_i^{(2)};\quad N_{i1}^{(2)};\quad N_{i2}^{(2)};\quad M_{i1}^{(2)};\quad M_{i2}^{(2)};\quad \sigma_i^{(2)}.$$

Since stiffness of, hence stresses in cracked members decrease compared to the crack-free state, strains in members assumed to be still crack-free are growing.

— For $\sigma_{i\,\max}^{(2)} \leqq \sigma_{hH}$, strains from a repeated calculation taking cracking of members into consideration do not exceed the ultimate cracking stress, the analysis is considered as complete, this step of computation outputs the structure stresses.

— For $\sigma_i^{(2)} > \sigma_{hH}$, strain redistribution results in stresses in other members to exceed the ultimate cracking stress. The calculation has to be repeated by taking the stiffness change of members considered in previous calculations as cracked into consideration.

Step 4. Repeated strain calculation in stress state II

— Stiffness of structural members (connecting beams or wall strips) considered as cracked in the previous calculation is determined assuming stress state II.

— The calculation is repeated by altering the stiffness matrix of the wall, changing the stiffness of all members already found to be cracked.

— Provided other members appear to be cracked, *the calculation has to be repeated until all member stiffnesses have been considered according to the state of stresses involved in the analysis.*

— Before ending the calculation, the structure has to be examined for not to have exceeded the elastic range.

— For $\sigma_{i\,\max} \leqq \sigma_{bH}$; $\sigma_{a\,\max} \leqq \sigma_{aH}$, plastification has started in no cross section of the structure, our assumption was correct, and stress values determined by assuming stress state II can be considered as real.

— For $\sigma_{i\,\max} > \sigma_{bH}$ or $\sigma_{a\,\max} > \sigma_{aH}$, stresses determined assuming stress state II induced steel yield or concrete plastification in certain members.

5. Redistribution of internal forces in stress state II

5.1 *Stress state* II/A

According both to our investigation results and to examination of existing walls, connecting beams are the most likely to crack. This stress state II/A is featured by

a) *Cracking of connecting beams expected in particular beside wall strips.* Cracks are vertical or nearly. Beam developing the maximum shear force is cracked first, then other connecting beams follow, depending on the stress redistribution.

b) *Loss of stiffness of cracked connecting beams.* Stiffness of cracked connecting beams is determined according to Hungarian Standard MSZ 15022/1—71; in some cases the calculated stiffness values (e.g. of T-sections, under-reinforced slabs, etc.) may drop to one-fifth to one-seventh of that in crack-free state.

c) *Loss of shear forces in cracked connecting beams.* Since for a given deformation, shear forces are directly proportional to the connecting beam stiffness, they decrease in cracked and less stiff connecting beams. Shear forces in beams adjacent to the connecting beam(s) that cracked the first are growing and so the cracking stress is exceeded in these connecting beams crack-free under stresses determined in the elastic range.

Stress redistribution in stress state II/A is seen in Fig. 7 for the special case where the wall is acted upon exactly by the load causing a cracking shear force only in the connecting beam under the maximum stress, in conformity with the elastic analysis (diagram 1).

Determining, however, the stiffness of the beam on storey 4 acted upon by the maximum shear force according to stress state II, then shear forces follow diagram 2, cracking stress is exceeded also at storey 5 and its consideration involves further stress redistribution (diagram 3).

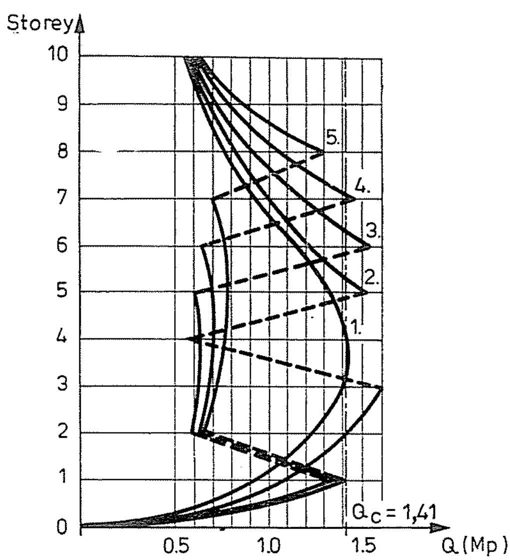

Fig. 7.

d) *Loss of maximum shear force in connecting beams in stress state* II/A. After the calculation following the cracking process has been completed, shear force values corresponding to our assumptions will be obtained. Stresses in originally less loaded connecting beams are seen to grow because of the redistribution of internal forces, compared to values for the elastic range, connecting beams have a more uniform contribution to the state of stresses. This process involves the reduction and the relocation of the shear force maximum. Cracking is followed by the relocation of the maximum shear force to a lower storey, because of lesser stiffnesses to be accounted for.

e) *Drop of interaction between two parts of the coupled shear wall.* Related to stresses determined in crack-free condition, shear forces take up a lesser part of moments due to loads and effects in stress state II/A:

$$(M_i^Q)_{\mathrm{II}} < (M_i^Q)_{\mathrm{I}}.$$

The share of bending moments developed in wall strips being $M_i = M_i^0 - M_i^Q$, wall strips have an increased share of moments in stress state II/A:

$$(M_i)_{\mathrm{II}} > (M_i)_{\mathrm{I}}\,.$$

Thus, wall strip stresses lie between values calculated for a solid wall and those for two independent wall cantilevers.

f) From the above it is clear that in case of a coupled shear wall in stress state II/A, assumption of a crack-free condition involves an error on the safe side for the connecting beam, and on *the unsafe side for the wall strips.*

5.2 *Stress state* II/B

Cracking of wall strips is detrimental to stiffness; deformation and shear forces are increased; stresses transferred to the cracked wall strips decrease, although (especially for load bearing walls) to a lesser degree.

5.3 *Stress state* II/C

In case of multistorey structures, a calculation taking the critical load arrangement into consideration may yield cracking of both wall strips and connecting beams. In conformity with the above, this entrains of course redistribution of internal forces. No generally valid relationships characterizing the procedure could be found to now, since cracking of the two elements: wall strip and connecting beam, affects oppositely the redistribution of internal forces.

Notations

H	total wall height
n	storey number
h	storey height
k_0	wall section stiffness

k_g stiffness of connecting beams
β stiffness ratio
I_g second-order moment of area of connecting beams about the x-axis
I_p I_g reduced by the effect of shear deformation
M_g beam depth
L_g span between coupled walls
μ Poisson's ratio
Q shear force in connecting beams
N axial force in coupled walls
M bending moment in walls
a relative deformation
A_n load vector, column vector of relative deformations
E_{ik} unit factor of wall strip
E_{nn} stiffness matrix of wall strips
d_i unit factor of connecting beam
I_x second moment of area of the imaginary $1/EI$ diagram of the connecting beams
K_{nn} wall stiffness matrix
e wall section stiffness factor
σ_i concrete stress
σ_{hH} ultimate tensile stress in the concrete

Summary

Theoretical preparation of an experiment series on r.c. structures planned at the Department of Strength of Materials and Structures, Technical University, Budapest, is described. To this aim, a calculation method has been developed for the uniform handling of the r.c. wall in its different stress states.

The compatibility method is suggested for the analysis of stresses in a coupled shear r.c. wall applied on a discrete model. The method lends itself for the analysis of irregular walls (of variable height, cross section, material quality or loads). Hence, variations in the stiffness of structural members cracked or plasticized can be reckoned with by the iterative application of the compatibility method. Finally, on the basis of solutions obtained by the suggested method, redistribution of internal forces in stress state II is considered.

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* In Hungarian.