WORLD SOLAR CHART*

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Introduction

The solar effect is an important aspect of architecture in any climate, whether moderate or tropical. To determine the solar angles, basis of daylight planning, insolation or shading, there are two procedures: mathematical and graphical. The first is tedious and involves a degree of precision that is not required in architecture; but graphical procedures are widely spread. These usually are single latitude systems: each chart appropriate for one geographical latitude only, based upon the idea that the earth is stable and the sun passes above the horizon plane, on which its paths are marked on key dates with relation to standard time and solar altitudes.

The World Solar Chart presents an entirely new feature: it is multi-latitudinal. Instead of 180 charts, seven provide the necessary readings.

The earth's spatial movement i. e. spinning around its axis, declining and revolving along its orbit has been transformed into a few graphs. Readings are considerably simplified and generally applicable. The author hopes that the World Solar Chart will be of value in the development of engineering and architectural activities throughout the world.

Description of the Chart

Solar azimuths, the horizontal sun-angles measured from the true North Point, are presented by black arcuate lines within a semi-circle according to the period of the year, with precise dates. Readings are worked out for twelve months. Each arcuate line is the integrate of those geometric places on the globe's surface where, at the given date, the sun's azimuths are identical. Along the vertical line passing the centre, azimuth readings are marked at sunset and sunrise. The circumference of the semicircle shows azimuths at noon, 180° for the northern and 0° for the southern hemisphere.

* Based on tests made by the Author at the Karthoum University.

Solar altitudes, the sun's vertical elevation above the horizon, in degrees, are read at the circumference of the semicircle, connected to the corresponding azimuths by vertical guide lines.

Solar standard time is along red elliptic lines, and connected to azimuths and altitudes with regard to the globe's declination.

Geographical latitudes are straight lines at right angles to the North-South axis, printed also in red, marked in five degrees graduation on both the northern and southern hemispheres.

Application of the Chart

To read solar altitudes and azimuths, first select the chart appropriate to the requested date and find the latitude corresponding to the examined geographical place.

Now all possible readings are available within the semicircle alongside the specific latitude.

Azimuths are read along the red latitude lines at or between the intersections of the black arcuate azimuth lines according to the selected solar time.

Altitudes are obtained by projecting the azimuth reading up or downwards along the vertical guide lines.

The vertical 'Sunrise-Sunset' line represents the shadow contour of the globe.

The right-hand half of the globe where readings are obtained is that lit by the sun, beyond this line to the left is shadow. If latitude and solar time intersect beyond the Sunrise-Sunset line there are no azimuths and altitudes, since the sun is below the horizon.

Shadow angle protractor

Sun-angles will only give shadow angles directly in special cases when the sun's azimuth is normal to the elevation. To calculate the depth of insolation, components of sun beams in normal plane to the elevation, called vertical shadow angles, are needed. Their calculation or geometrical projection would be time consuming. True solar altitudes can be converted into shadow angles by means of a graph, the Shadow Angle Protractor (Fig. 8).

The diagram is a semicircle. The horizontal angle between azimuth and elevation is marked along the graduated external arc. Vertical shadow angles are read along the graduated internal arc. The concentric semicircles and horizontal lines present altitudes. Vertical and radial lines facilitate reading.

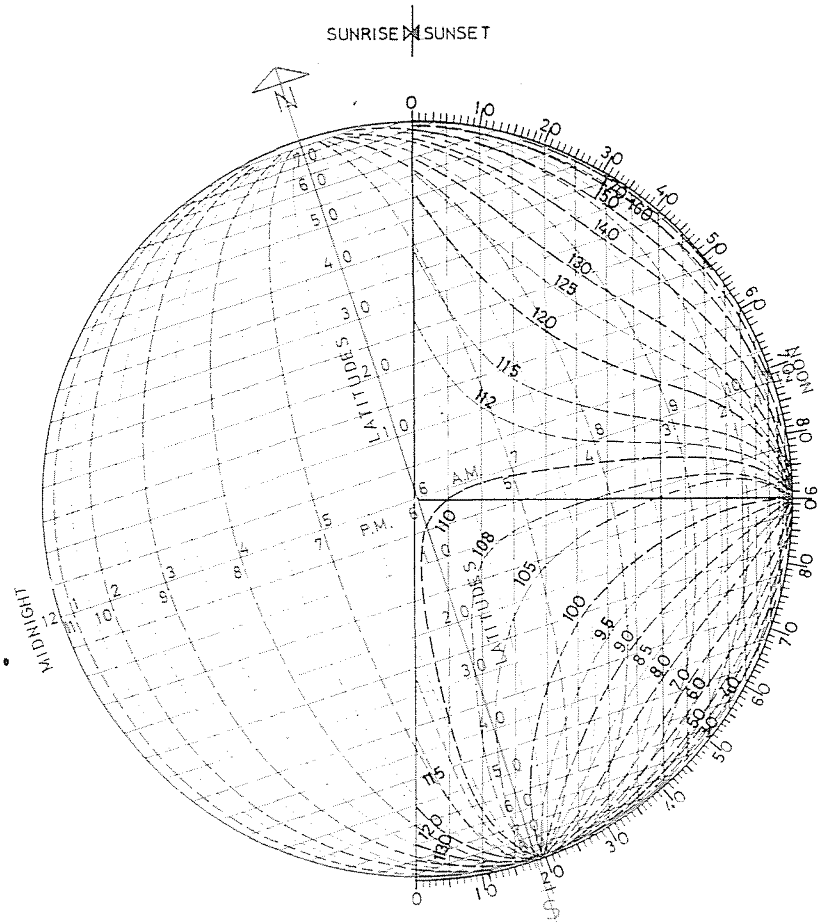

Fig. 1. Jan. 24 and Nov. 19

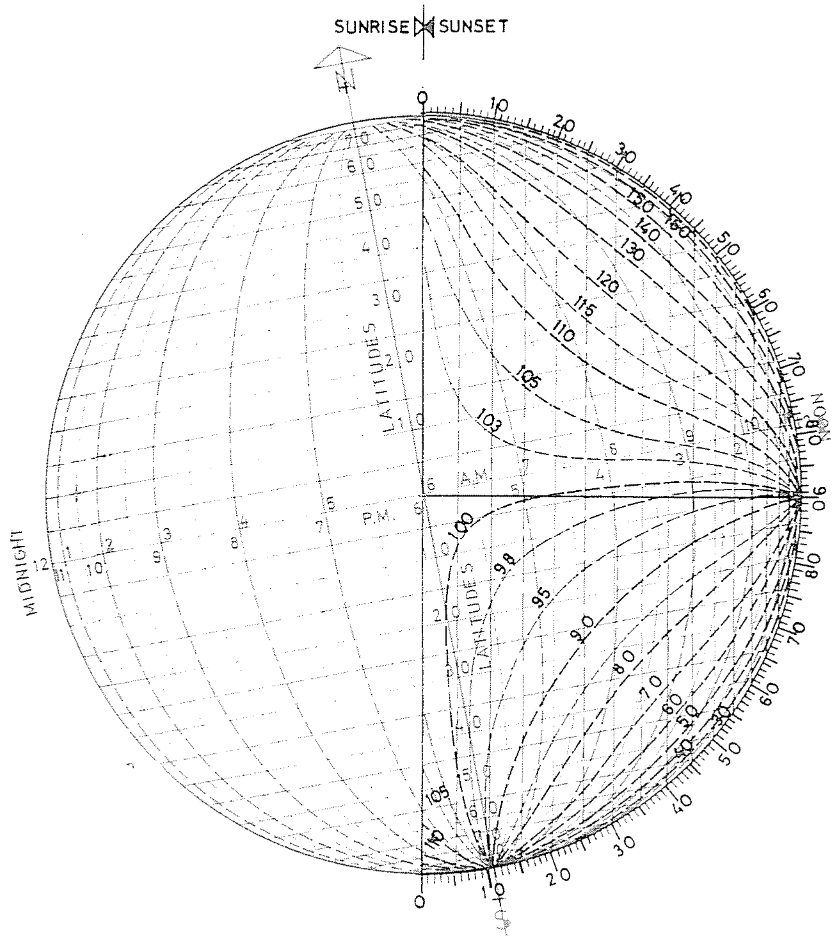

Fig. 2. Febr. 21 and Oct. 22

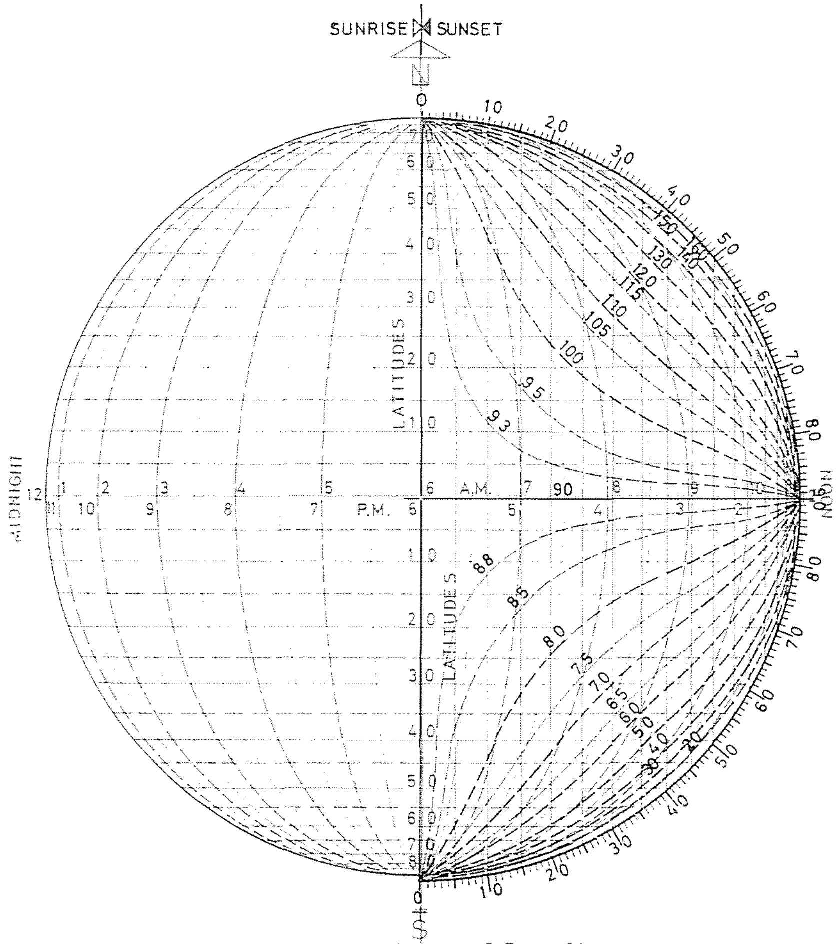

Fig. 3. March. 21 and Sept. 23

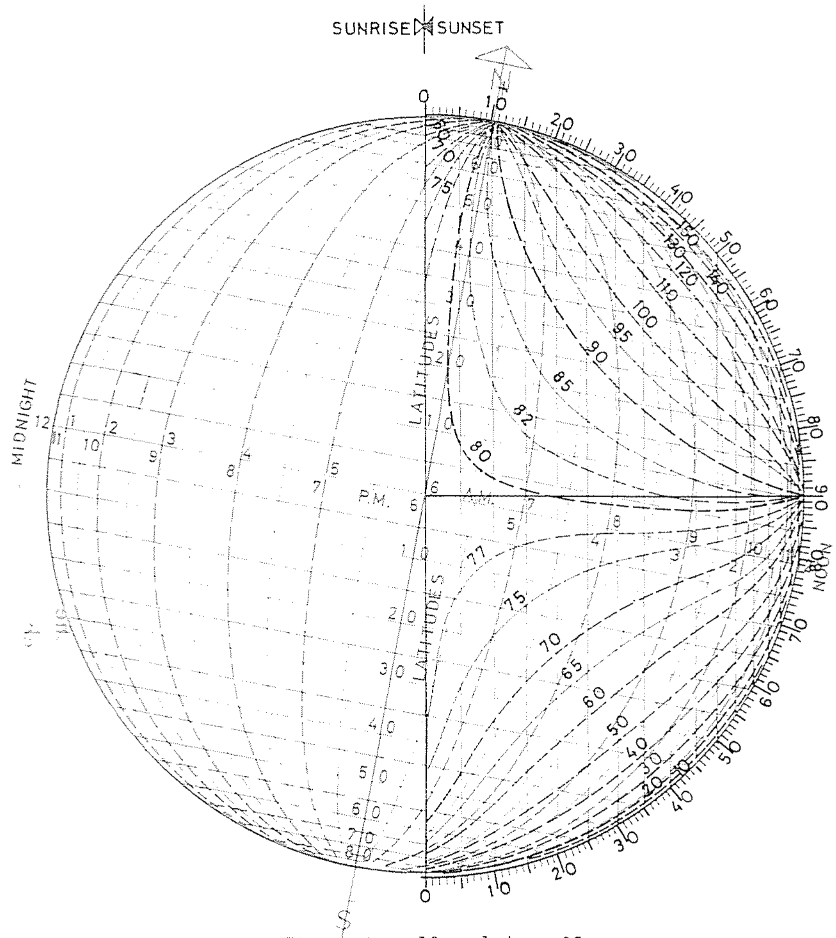

Fig. 4. Apr. 19 and Aug. 25

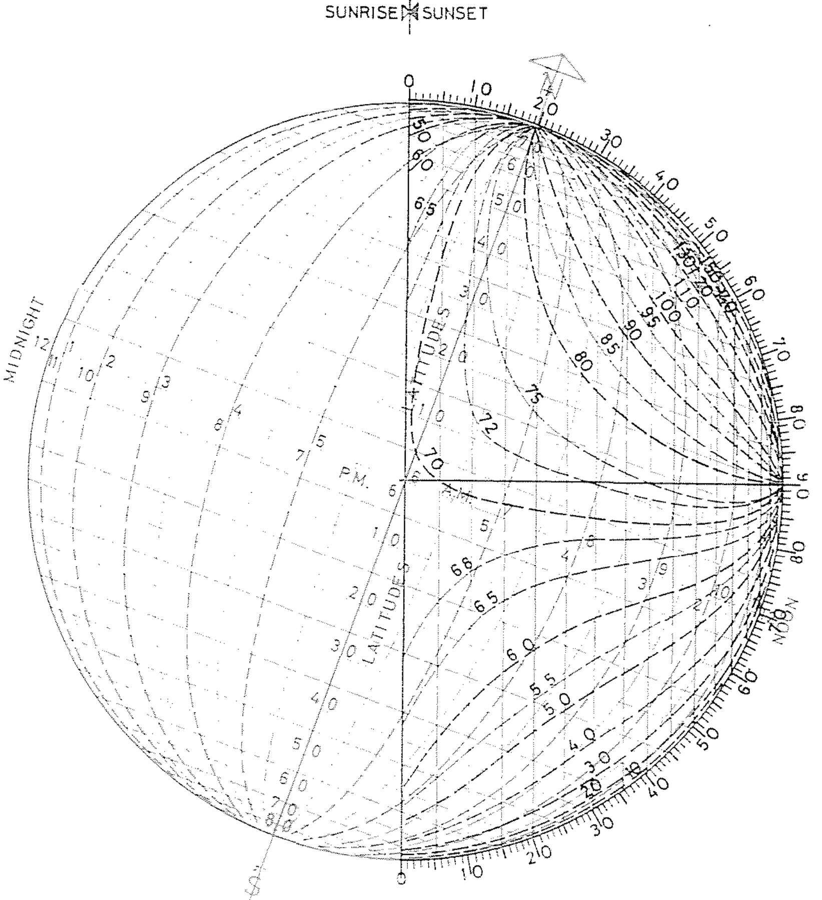

Fig. 5. May 18 and July 27

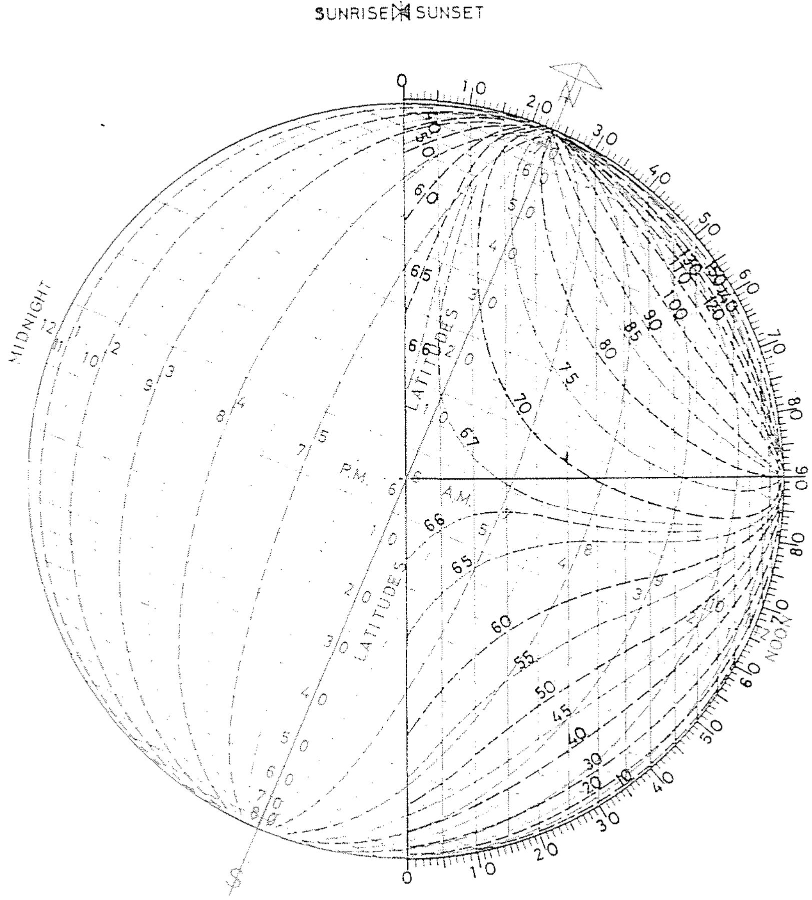

Fig. 6. June 22

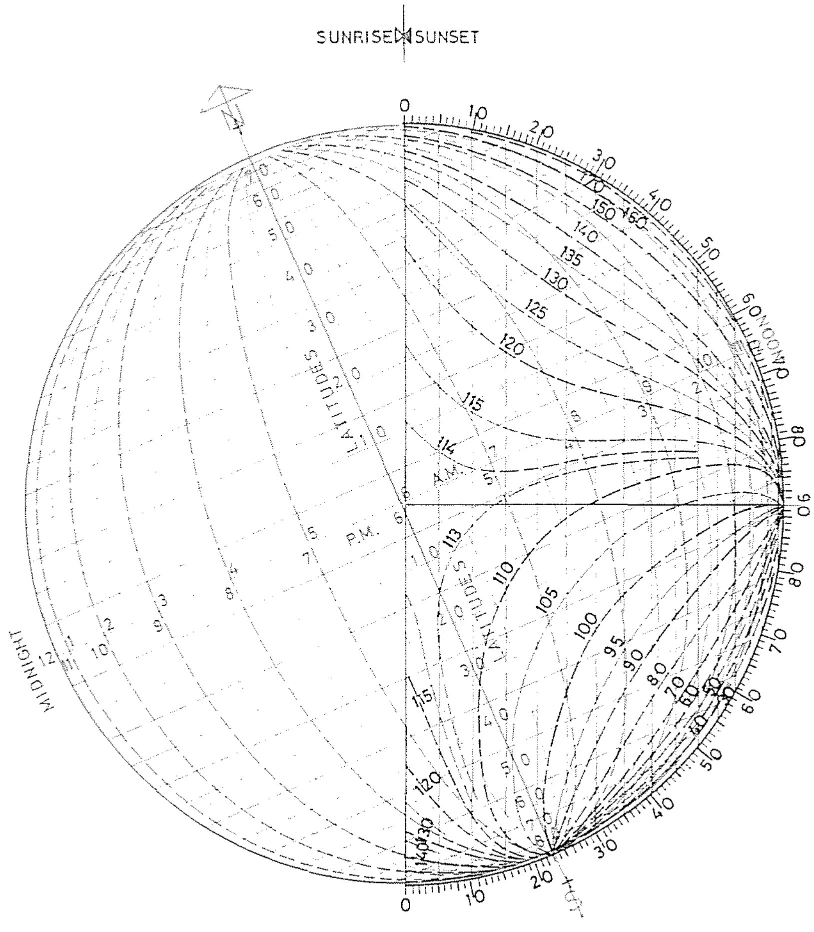

Fig. 7. Dec. 22

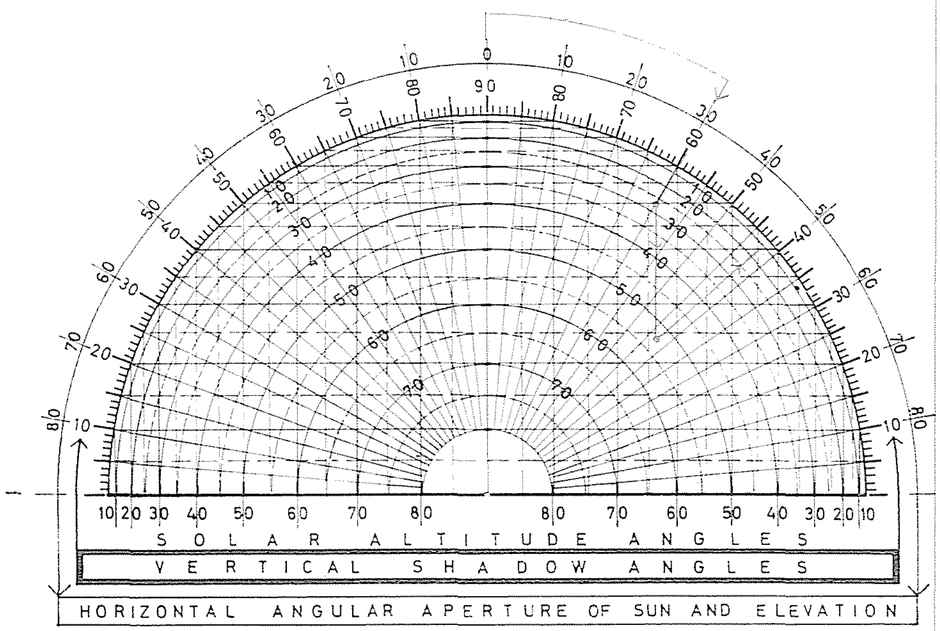

Fig. 8. Shadow angle protractor

Shadow angle projection

Find the horizontal angle of the sun and the elevation along the external arc, then follow the radial line to the intersection with the semicircle corresponding to the sun's altitude. Project this point vertically to the horizontal line representing the same altitude. Connecting this second point of intersection radially to the internal graduation yields the vertical shadow angle.

Example: horizontal angle: 30°, sun altitude; 25°. Follow the red line on the diagram. Read at the circumference the vertical shadow angle: 42°30'.

Correction of solar time and direction of poles

The readings of the charts are calculated for solar time, which is related to the sun's position on the hemisphere. Local Standard Time (LST) — according to the geographical location — is based on solar time with a one-hour graduation, and in most cases, will not exactly correspond to the solar time. For accurate azimuth and altitude readings time correction may become necessary.

The globe revolves by 360° within 24 hours. Each 15° angular rotation is the equivalent of one hour and each 1° of four minutes. LST is determined along the multiple of t' 15° longitudes with a one-hour graduation, from a zero point at Greenwich, Longitude 0°, Solar Time Noon. To establish Local Standard Time, going eastward or westward one hour is to be added or subtracted for each 15°, respectively.

This method is applicable to find the Solar Time with correction of LST. If the place in question is not located exactly on any of the multiples of 15° longitudes, for each 1° of deviation add or subtract four minutes to the east or to the west, respectively.

Note that correction will be required if a summer-winter daylight saving system is in force.

Sometimes correction may be necessary for the polar direction. The compass points to the magnetic pole, seldom coincident with the actual geographic pole. The deviation of the magnetic and geographic poles is marked on maps, and it could be considered by adding or subtracting it from the magnetic polar direction. For the Sudan the deviation is about 1°, so its neglect does not cause noteworthy error.

Examples of application

Find solar azimuth and altitude in Khartoum at the time of the equinoxes 21st March, 23rd September at 9 a. m.

Khartoum is on latitude 16° north, longitude 32°30' round. The latitude and the solar time line intersect between the arcuate azimuth lines of 105° and 110°, giving by interpolation 106°. Projecting the point of intersection upwards or downwards parallel to the guide lines to the circumference gives an altitude reading of 43°. These are values not corrected for the local time. Khartoum is 2°30' to the East from the longitude of 30°, where the actual Local Standard Time coincides with the solar time, and is GMT + 2 hours. As each 1° longitude equals 4 minutes in time, the local time must be corrected by the total amount of 2,5 × 4 = 10 minutes. Thus the solar time is 9 h 10 a. m. The corrected reading now will be azimuth 108°, altitude 45°. Since the deviation of the longitude is 2°30' only, the difference in time is trifling, but in other cases it may be significant and the correction will not be negligible.

Construction of Chart

The construction has been approached from a realistic viewpoint, considering the Sun-Earth relation in their spatial movement (Fig. 9).

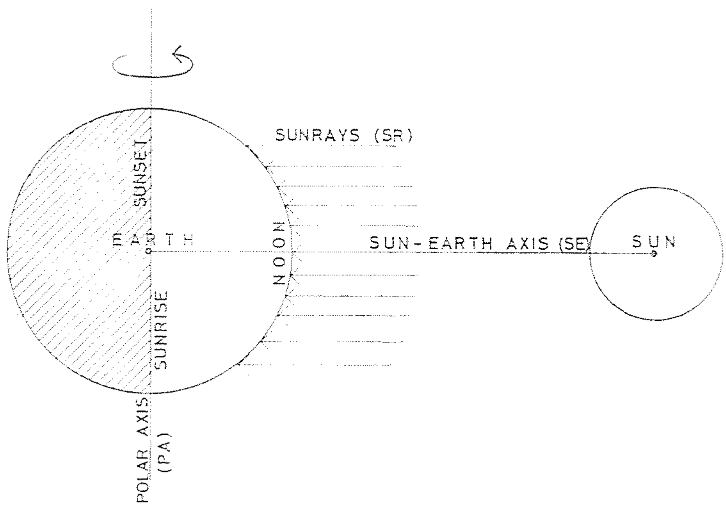

Fig. 9. Sun and earth relation at equinoxes

Construction of solar altitudes

For the start the simplest relation of sun and earth was chosen. This is the time of equinoxes, 21st March, 23rd September, when the declination is zero, the solar axis PA is vertical, sun-earth axis SE, a theoretical line connecting centres of sun and earth, is parallel to the equator and the latitudes. Sun beams are parallel to SE. PA in this case represents the shadow contour, i. e. the sunrise-sunset line. The contour of the globe on the right is the line of solar noon. The earth is reduced to a sphere. The horizon plane at any point of the globe is a tangent plane.

The sun's altitude is an angle included between the horizon and the sun's rays, as presented in Fig. 10 for noon. Tangent plane (horizon plane) of the sphere is normal to the radius R. Consequently, $90° - \alpha = \beta$, where α is the angle between SE and R for the tested point, and β the solar altitude.

Conclusion: solar altitude at noon corresponds to the complementary angle of the latitude of the examined geographical spot. The geometrical definition of solar altitude by day partly differs from the above. In this case the selected point K is between PA and solar noon rather than on any particular line (Fig.

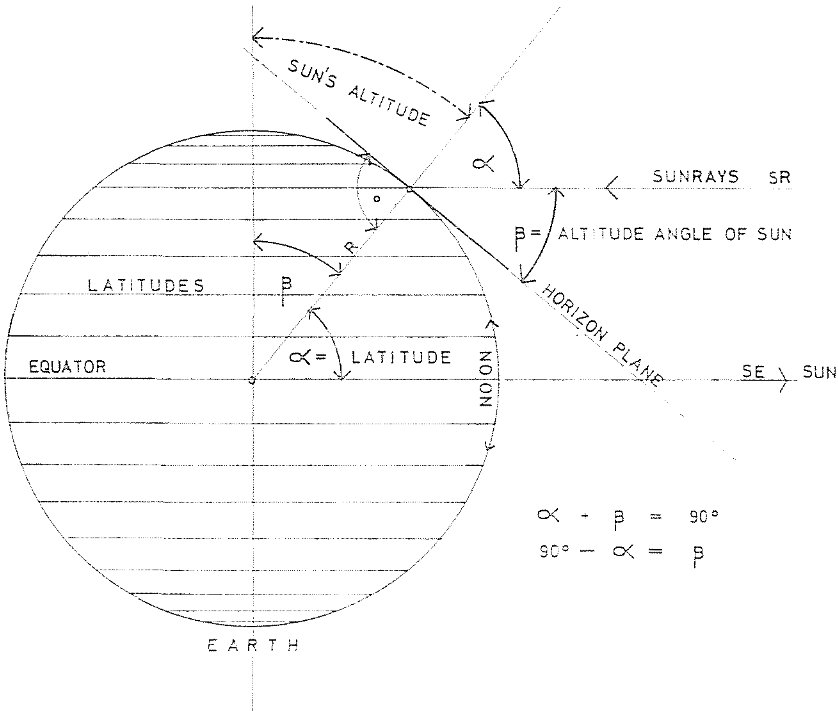

Fig. 10. Solar altitude definition at equinoxes at noon

11). Further, this point is on the latitude circle and on a great circle GC of the sphere passing through the SE axis. Consequently, this circle and the radius R' joining K are in one plane provided by R'—GC. Therefore the tangent plane of K is normal to the R'—GC plane.

The solar altitude at K is defined by its tangent t' constructed by means of affinity. K appears now in plan and elevation on one ellipse — on the image of GC. The affin point Ka is on the circle, affin axis is SE. Tangent t projected from Ka defines a point on SE wherefrom tangent t' can be drawn to K, including with the tangent plane the *solar altitude* β at K. But it is readable neither in plan nor in elevation in reality, since it is an ordinary spatial position. Rotating the plane R'—t'—GC—SR into horizontal (plan) or vertical (elevation) position, R—t—SR, β becomes measurable.

The integrate of tangents from one external point to a sphere is a conical surface with a small circle SC on the sphere as base line (Fig. 12). These tangents

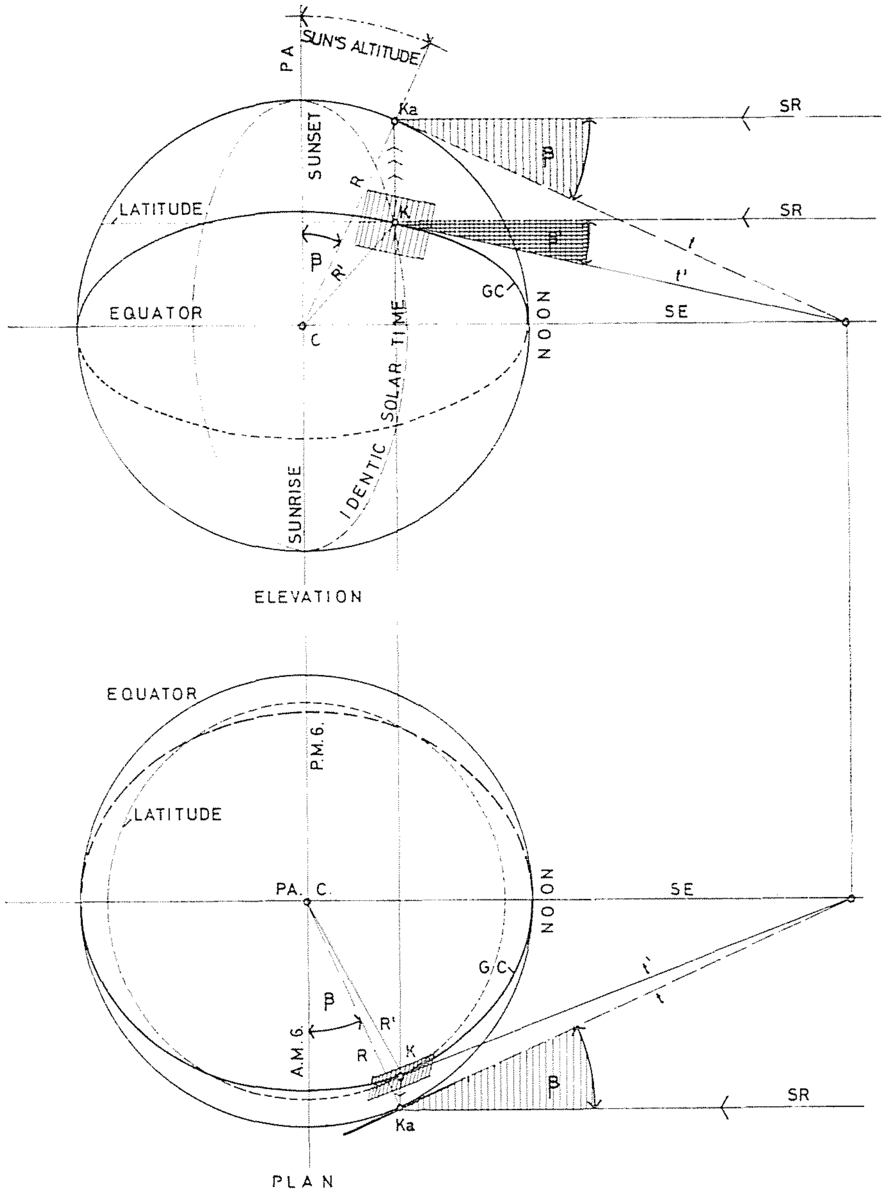

Fig. 11. Solar altitude definition at equinoxes at any time of the day

define also horizon planes along SC. Sun beams SR form cylinder SC, with constant β' between the tangents t' and SR.

Real β between t—SR occurs only at two points, where SC intersects the circumference of the sphere (Fig. 13). In our case, the external point from where the tangents are constructed is on the SE axis. The bases SC of any of

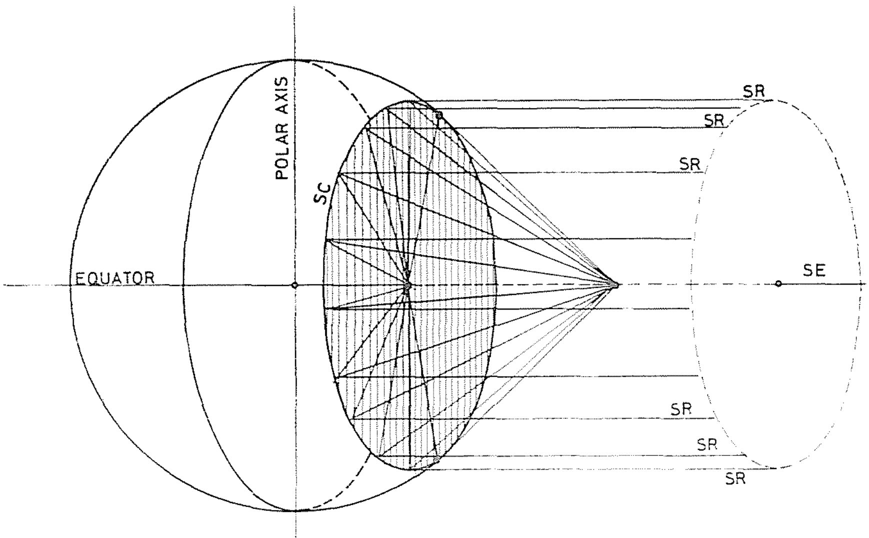

Fig. 12. Solar altitude is constant along a circle of the sphere normal to sun-earth axis

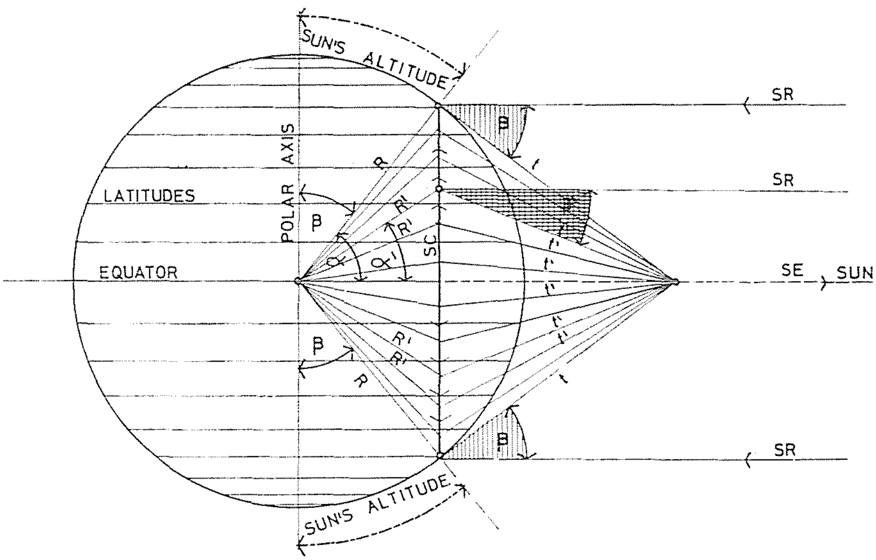

Fig. 13. Solar altitude definition of any geographical spot by day at equinoxes

these cones are normal to SE and appear as lines perpendicular to SE in orthogonal projection. SE is parallel to SR, so $\beta' = \beta$ and

$$\alpha + \beta' = \alpha + \beta = 90^\circ$$

thus:

$$90^\circ - \alpha = \beta.$$

Thus, at equinox, the solar altitude β at any point of the globe can be directly measured as the angle included between the $\overline{PA}$ axis and the projection of that point on the perimeter of the sphere normal to SE.

At equinox, the globe is in a particular position, namely its solar axis is vertical. At any other day of the year this is inclined (Fig. 14) by the angle of declination.

To define sun's altitude, tangents are constructed from the SE axis, coincident at equinox with the earth's horizontal axis. Otherwise, rotation of the

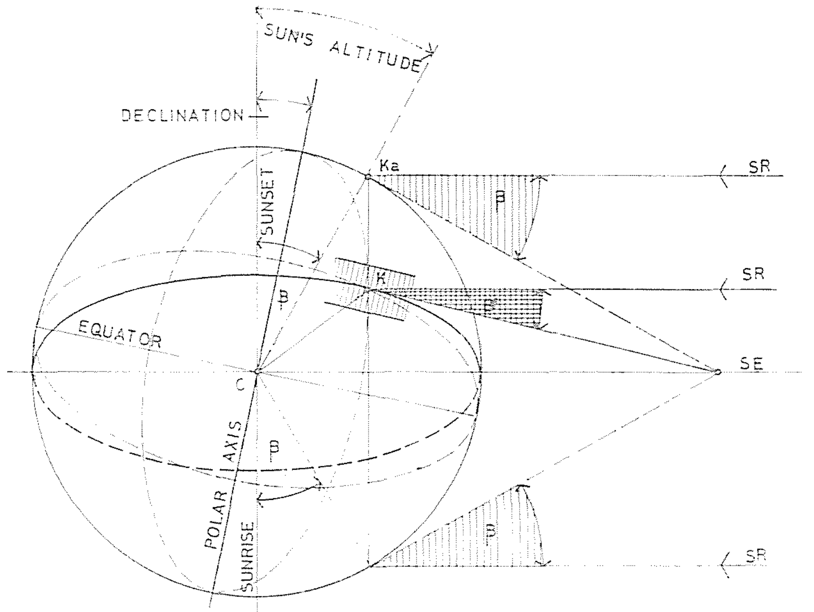

Fig. 14. Solar altitude definition at any time of the day or year

globe axis leaves the sun's position and the earth's center inaffected and the SE axis remains the same. So the solar altitude is constructed as before. For identical points on the globe surface, solar altitudes remain constant in spite of the declination shifting latitudes and solar time lines (Fig. 15). Solar altitudes β significantly vary with declination since the geographical spot determined for equinox is shifted to another spot with another but constant altitude β.

Thus, the *multi-latitude method* gives precise altitude readings over the globe's surface at any day of the year, by projecting the point of intersection of the latitudes and solar time lines perpendicularly to the SE axis on the perimeter of the circle representing the earth. Solar altitudes are to be measured from a perpendicular line passing the earth's centre. This rule is valid regardless of the declination of the earth.

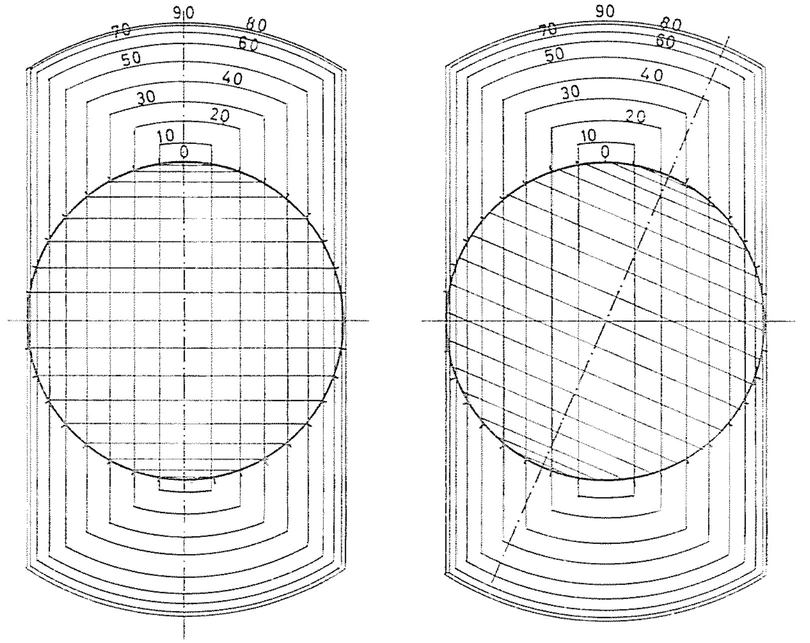

Fig. 15. Altitude readings are constant on the surface of the sphere for any declination

Construction of solar azimuth

The azimuth angle of the sun is measured between two theoretical lines, the pole direction and the image of sun's ray on the horizon plane.

Geometrically, the polar direction is a theoretical line parallel to the horizon plane from a geographical spot directed to any of the poles. This line does not pass the pole, but intersects the polar axis PA. At any point of the earth, the north or south direction is a tangent projected from the polar axis to that point.

The sun's horizontal image is constructed by drawing a tangent from the SE axis to that point. The azimuth angle of the sun is included between the two tangents drawn from the PA and SE axes.

For the projection of the north or south direction of a surface, realize it to be on a longitude (Fig. 16*a*), — a great circle of the globe — appearing (except two cases) as an ellipse in orthogonal projection.

Affin point Ka is on that circle, the contour circle of the globe. The tangent *tp* to Ka intersects polar axis PA at a point from where tangent *tp'* to point K can be drawn.

The method of projection is the same on any day (Fig. 16*b*), since PA follows the declination of the earth.

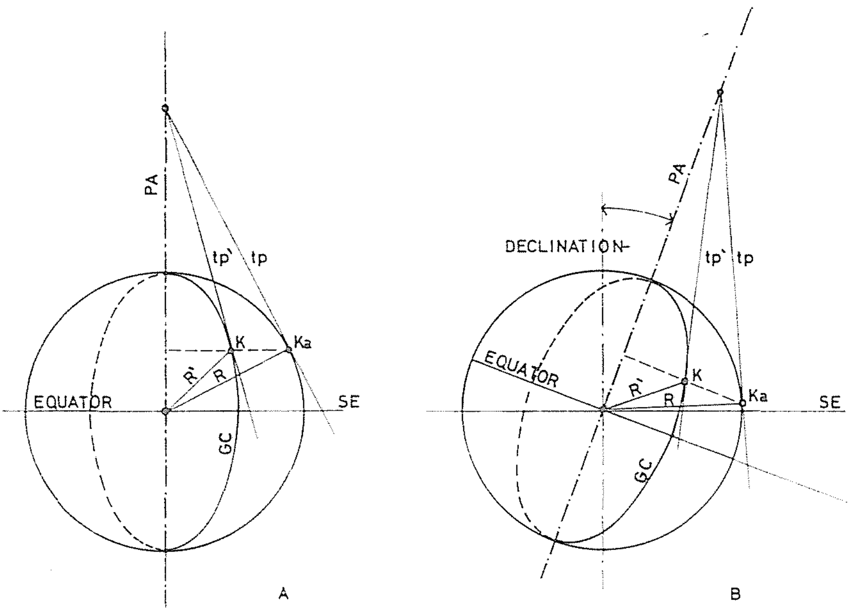

Fig. 16. Definition of polar direction

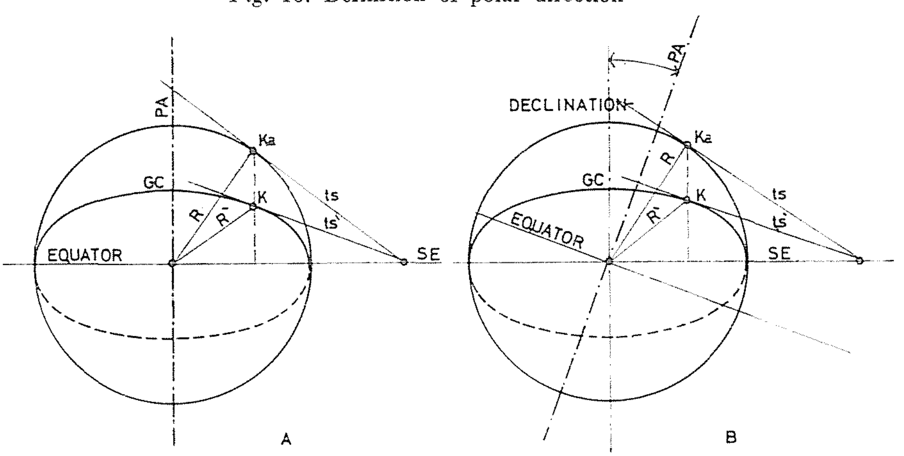

Fig. 17. Definition of the direction of sun's image

Sun direction on horizon plane is constructed by the same method (Fig. 17*a*). Point K is on a great circle passing through SE axis.

From affin point Ka, tangents *ts* and *ts'* can be defined. This is the case both at equinoxes and other days of the year (Fig. 17*b*), the SE axis being stable. Combining projections of the polar direction and the sun's image on horizon plane yields the azimuth angle at K.

The tangent plane at K (Fig. 18) determined by tangents *tp'* and *ts'*, is also the horizon plane of K at equinox. The sun's azimuth appears included

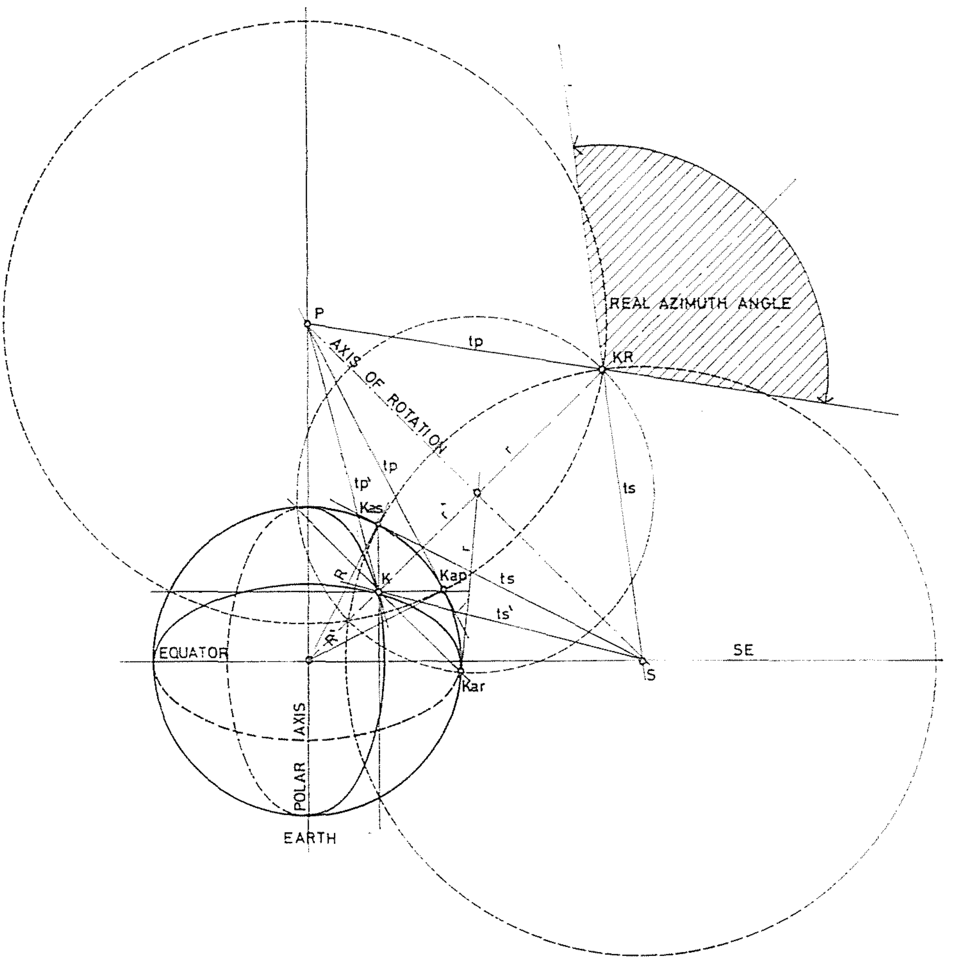

Fig. 18. Definition of azimuth angle at the time of equinoxes

between these two tangents, but it is unmeasurable on the picture. To convert it measurable, the tangent plane has to be made parallel to the plane of axes PA and SE.

In the horizon plane of K, trace points of SE and of PA are S and P, respectively. Now, the horizon plane KPS has PS, the axis of rotation as trace line. To rotate K around PS, real r' length of radius r is needed, to be constructed at point Kar, and so K is rotated to Kr. The rotated azimuth angle is included between tangents *tp* and *ts* drawn from points P and S to Kr.

Azimuth determination on any other day is done similarly, taking the declination of the polar axis into consideration (Fig. 19).

The geometric places of identical angles of azimuth provide curves on the chart.

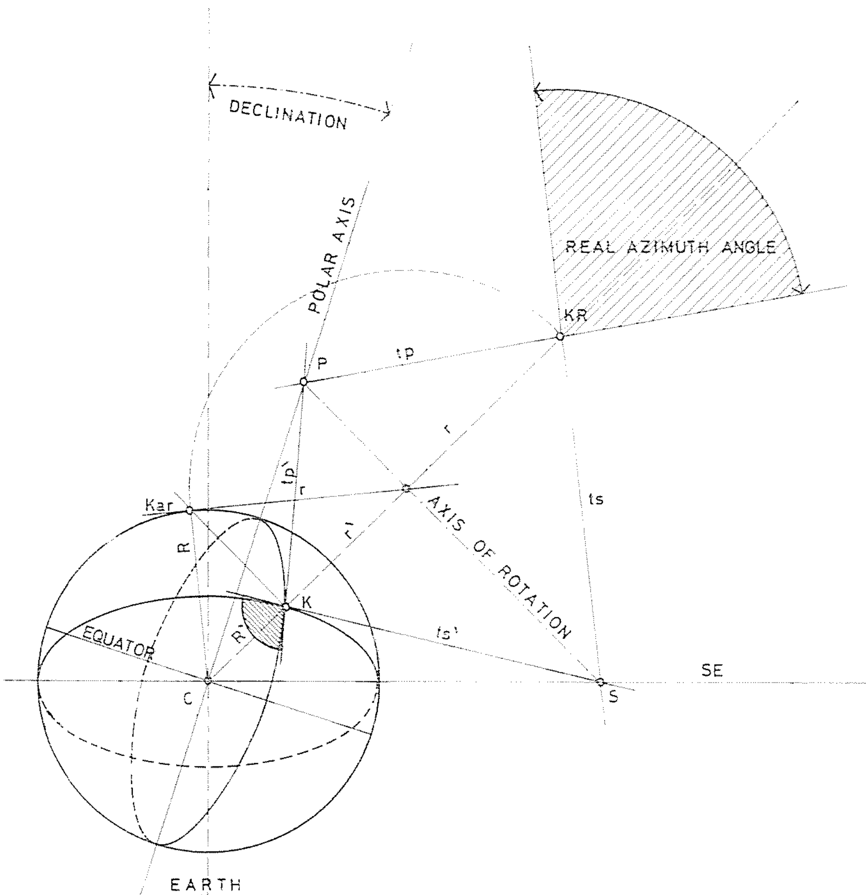

Fig. 19. Definition of azimuth at any time of the year

Construction of identical azimuth curves

The integrate of geographic spots of identical azimuth readings at a particular day is obtained by the reverse of the above method (Fig. 20), since evidently, on a given day a certain azimuth angle belongs to one geographic point.

Hence, choosing one azimuth angle, the integrate of identical points can be determined.

On the lengthened radius R' of the earth the point pertaining to the initially assumed azimuth should be found.

Axis of rotation PS being perpendicular to R', a triangle over base $p'' - s''$ should be constructed, with external angle at kr'' on the extension of R' equal to the given azimuth. This triangle $p'' - s'' - kr''$ carrying the given

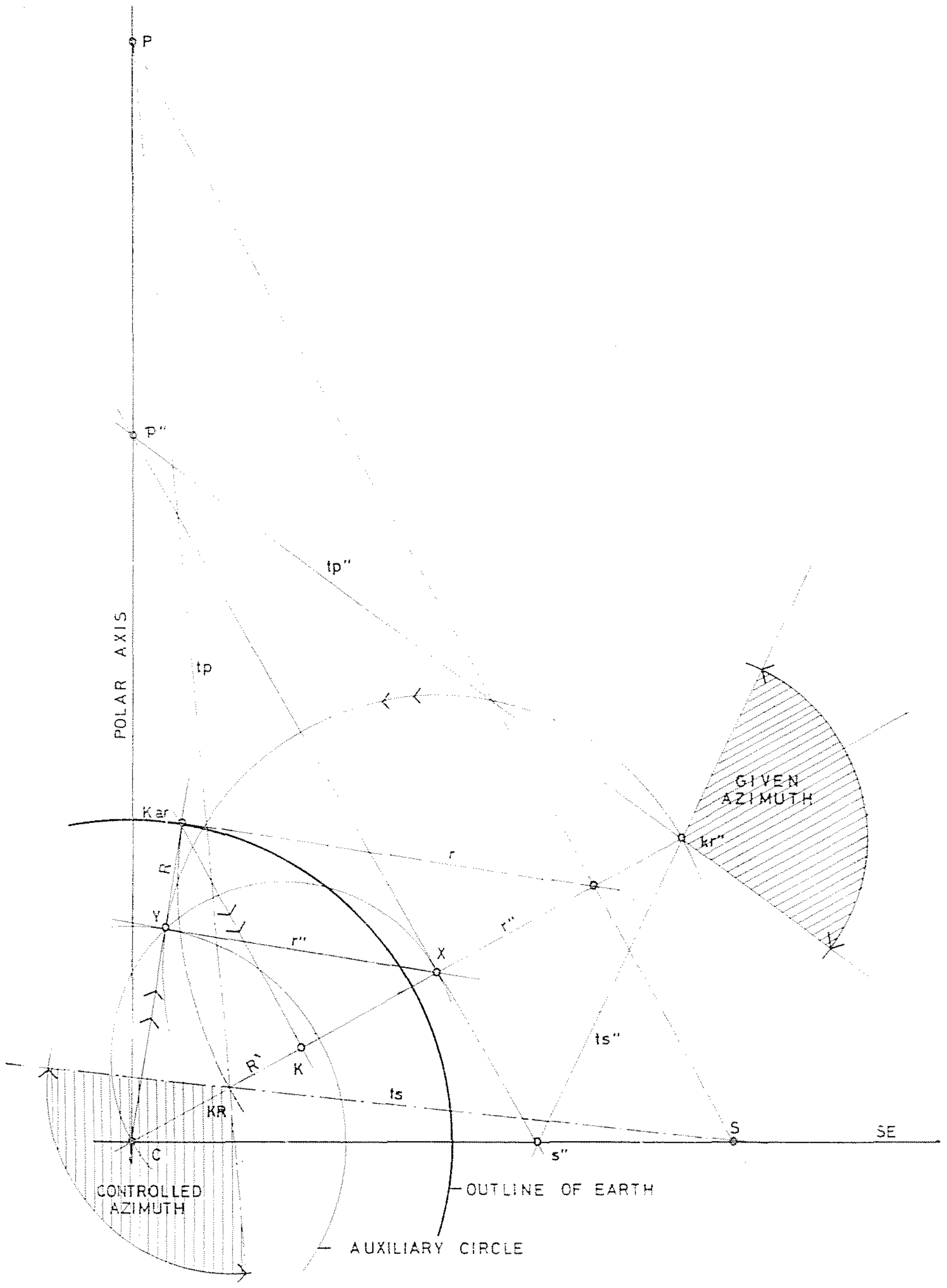

Fig. 20. Geometric definition of one geographical spot of given azimuth at the time of equinoxes

azimuth angle can be rotated around $p'' - s''$ axis. It is a plane tangential to an auxiliary and definite sphere, concentrical with the earth. The rotated tangent plane $p'' - s'' - kr''$ being normal to the radius CY of the auxiliary sphere its perimeter can be determined from the Thales law. The radius of rotation r'' is tangential to the auxiliary sphere at Y. Projecting Y normally to R' would give the geometric spot on the auxiliary sphere of the given azimuth. The same point R is, however, sought on the earth (between C and Kar), so point Y is to be projected radially to the outline of the earth to give point Kar, and therefrom, in accordance with the previous considerations, point K is determined.

For proof a control projection is shown in dotted lines.

Repeat construction in Fig. 20 until a number of points with identical azimuth angles is obtained, sufficient to construct the azimuth curve at a due accuracy.

Summary

For the widening professional activity comprehensively applicable data are required. The present study offers a new method of solar angle calculation. Seven diagrams replace the 180 single latitude charts necessary up to now. One shadow angle protractor is also attached for the transformation of sun angles into shadow angles. The method of space geometrical construction of the diagrams is also being presented.

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