DEVELOPMENT AND PROBLEMS OF STRUCTURAL OPTIMATION

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1. The inverted problem of structural engineering

1.1 Phenomena within the scope of structural engineering may be concentrated around three major concepts. In any case, the problem involves a solid body or *structure* that can be described by its geometry and material properties. This structure is affected by various *effects* (loads, thermal effects etc.), and as a *consequence*, parts of the structure undergo relative displacements. Relative displacements can occur either without or with causing discontinuity. In general, it can be stated that the fundamental problem of structural engineering is to predict the consequences of influences affecting the structure.

1.2 In practical structural engineering, the structure is not defined *a priori*, as a rule. If for instance, the designer has to construct a road bridge crossing a river, then only some parameters of the structure to be designed (e.g. the span), follow directly from the practical destination. Various other parameters (e.g. cross-sectional dimensions or material qualities) can be assumed freely, or — better said — have to be determined just in course of the design.

The fundamental problem of structural engineering, as outlined above, is essential for the knowledge of the behaviour of solid bodies. Structural engineering laws can only be studied by exposing given structures to given effects and observing their consequences. Nevertheless, answering fundamental problem of structural engineering is still insufficient to satisfy requirements inherent with its practical application. Design practice requirements are the inverse of the fundamental problem. Initially the structure is not given; on the contrary, it has to be determined. Requirements for the structure have to be reckoned with, in form of restrictions on the consequences of effects (e. g. structural discontinuities must not arise, prohibitive deformations must be avoided).

As a first approach, the inverted problem inherent with the practical application of structural engineering knowledge can be formulated as follows:

Effects on and requirements for the structure are given, these latter as restrictions on the consequences of effects. Structures for which consequences of the given effects satisfy given requirements are to be sought for.

This quite general formulation of the problem, however, is unlike to bear a practically useful outcome. It is namely impossible, even theoretically, to define every and each structure satisfying the given requirements. (Structural materials are in continuous development, the sphere of possible structural solutions is illimited.) Therefore, the set of possible structures has somehow to be circumscribed, for the sake of arriving at an exact solution. Exact methods make only possible to find the structure meeting given requirements in case of given effects, out of a well defined set of all structures possible.

In connection with the inverted problem of structural engineering as outlined above it has still to be noted that effects on and requirements for the structure cannot always be considered as to be specified independently of the structure. For instance, the dead load of the structure depends on the structure itself, or, more exactly, the sphere of possible structures contains in general structures differing by their dead load. Even requirements for the structure may not be independent of the structure itself; if for instance the sphere of possible solutions includes both steel and reinforced concrete structures, then, for the first case, the absence of any cracks is required, while for the second case, restrictions may refer to the maximum tolerated crack width.

Those said above permit a closer formulation of the inverted problem of structural engineering:

A set of possible structures is given. For each element of the set, effects and requirements are specified. By solving the fundamental problem of structural engineering for each element of the set, consequences of the given effects can be predicted, allowing to decide whether consequences meet the given requirements or not. The problem consists in delimiting that part of the set of possible structures, each element of which satisfies the given requirements, and which contains each element of the given set which meets the given requirements. This subset will be called the set of permissible structures.

1.3 Solution of the inverted problem of structural engineering yields knowledge of the sphere of structures convenient for a given practical purpose. This knowledge is of importance by providing freedom for the designer to decide between permissible structures. Namely, as long as the sphere of structures useable for a given practical purpose is unknown, the solution of the practical problem is a random one. The designer tests some elements, or just a single element out of the set of possible solutions, that is, by solving the fundamental problem of structural engineering he determines whether the structure meets requirements or not. In the case the designer can only test a single element, then no designer's freedom or free decision can be spoken of. But even if several structures can be examined by computation, the number of variants and thereby the freedom of decision is rather a restricted one. Freedom in designing is only provided for by knowledge of the set of permissible structures.

1.4 Solution of the inverted problem of structural engineering was seen to deliver a set of permissible structures. This set may be an empty one, where, in fact, the problem has no solution. It is possible too that the set of permissible structures includes a single element. In this case there is a single way to meet requirements, and there is no need of designer's decision. Practically, however, the set of permissible structures includes numerous, or an infinity of, elements. This means that the requirements set up in the inverted problem of structural engineering may be met in several ways, and the necessarily single one to be realized can only be decided through design considerations, hence by involving still other requirements. Thus, designer's freedom created by solving the inverted problem of structural engineering induces both possibility and necessity to restrict this freedom itself and to involve new requirements.

The new requirements may be introduced by two means. Either ever more restrictions are set up, eliminating ever more kinds of structures of the set, still finally the set is reduced to a single element. Or a scalar characteristic value can be given to each element of the set of permissible structures, and the structure with the least scalar value (or with the greatest one, what comes out essentially to the same) designated for execution. The first case is best illustrated by the problem of the structure of uniform strength. The second case is that of the most flexible satisfaction of practical requirements, such as defining the lightest or most economical structure.

It should be noted that the mentioned two fundamental possibilities are not different in principle, or more correctly, the second one includes the first one. Namely, structures short of a given requirement can also be excluded by giving the characteristic value 0 or $+1$ to structures meeting the requirement or not, respectively.

By completing the inverted problem of structural engineering so as to involve selection out of the set of permissible structures, then the so-called optimation problem is arrived at. The optimation problem can be formulated as follows:

The set of possible structures is given. For each element of the set the effects on and the requirements for the given structure are specified. Besides, to each element of the set, a scalar value is given. The structure belonging to the set of permissible structures, and exhibiting a characteristic value not greater than any other structure within the set of permissible structures, is sought for.

1.5 The optimation problem may have either a single solution, several solutions or no solution at all. This latter case is that of a contradiction existing between the set of possible structures and the requirements for them. Several solutions possible for an optimation problem indicate that not all practical aspects had been taken into consideration in constructing the scalar characteristic values. In such cases the problem may and has to be made unambiguous — if desired — by involving still other aspects.

There are many different possibilities to designate the characteristic value for the decision, involving aspects though absolutely pertaining to the structural design but beyond the range of structural engineering itself. Definition of the optimation characteristic values cannot be considered a structural problem, structural engineering being only concerned with the solution of the optimation problem for given characteristics.

2. Trends in the field of optimation methods

2.1 The first idea to emerge in course of the historical development of structural engineering and pertaining to the theory of optimation was the problem of structures of equivalent uniform strength. Galileo Galilei, in his book published in 1638, likely to be considered the first study on the strength of materials, has treated the problem of cantilever beams of uniform strength. From this time, outstanding scientists in mathematics and mechanics have often been interested in problems on structures of uniform strength.

The first systematic treatise on the problem of structures with uniform strength was a book published by M. Lévy in 1873. The first general theorem over the non-existence of statically redundant trusses of uniform strength is to be found in this work.

The endeavour to have a beam of uniform strength is often at the basis of the design practice. Since long, in the design of major structures, it is customary to modify assumed cross-sectional dimensions of hyperstatic beams according to the determined stresses, to adapt them for the latter, involving iterated computation of the structure and alteration of the stresses. This method — involving eventually several iterations — is preferred by designers aiming at a possibly "uniformly" loaded structure, provided there is a means to cater for the increased volume of calculations.

With the extended use of digital computers, this method gained importance anew, since it being an iteration process lends itself for computer use.

2.2 Attempts to determine the structure of the lowest weight were first successful for trusses. Based on Maxwell's ideas, in the early 1900's A. G. M. Michell studied comprehensively the problem of the structure of lowest weight to be built up of members under axial stresses, for a given load and given supporting conditions. In his studies he applied serious simplifications and attempted to arrive at closed solutions.

These studies have led to the development of a theoretical discipline dealing with the problem of the structures of minimum weight, with theoreticians mainly from English-speaking countries. Studies aimed at finding the theoretically optimum structures for some typical load cases. According to researchers of the Michell structures, although the obtained result is too abstract to be applied directly in practice, its knowledge indirectly helps us, it may act as an in fact inachievable but more or less approachable target in the design practice.

2.3 In 1933, I. M. RABINOWICH published fundamental results concerning the determination of a minimum weight bar system under non-axial (flexural and torsional) stresses. The problem has been set up as one of choosing the most favourable structure of a given family of structures of uniform strength. The idea of Rabinowich found numerous followers and developers, at first in the USSR, and from the 50's all over the world.

A typical problem of this school is for instance to determine the hyperstatic flexural bar system of lowest weight, of continuously varying cross-section, under a one-parameter load system. Studies are concerned with the case of an ideally elastic structural material. Recently, the scope has been extended to plates and shells and introduced into practice.

2.4 Mention should be made of research done in Poland on the optimum design of structures. The first work on this subject was that by Z. WASIUTYSZKI, published in 1939. Studies have been based on the minimation of the strain energy, namely, preference is given to that structure of a given volume for which to a given load the minimum of strain energy belongs. This approach is strictly related to the problem of determining the structure of minimum weight.

2.5 Alongside with the development of the theory of plasticity and with the extension of design methods reckoning with the material properties in the plastic range, research has been initiated by W. PRAGER in 1953, to determine the structure of minimum weight in the plastic range, widely extended since then.

A typical problem in this school is to determine the cross-sectional dimensions of minimum weight continuous beams and frames of given pattern, consisting of flexural bars with uniform cross-section. For the analysis of hyperstatic structures, the developed methods make use of simplifications permitted by the plastic properties, hence finding the optimum alternative requires but moderate computation work. (Otherwise, extreme computation work is typical for optimation problems.)

In fact, significance of optimation methods making use of material properties in the plastic range consisted exactly in reducing the computation work, permitting much of the practical problems to be solved manually. On the other side, however, they apply too many simplifications (e.g. assumption of ideally rigid-plastic or elasto-plastic properties, restriction of the range of stress combinations).

2.6 It should be noted that, since the 1920's, in addition to the listed tendencies and schools, several other attempts have been made in the field of optimation. Probably, several publications written in other than world languages or issued in non universally known periodicals have been concerned in merit and successfully with optimation problems. In addition to attempts in this country, I am aware of initiatives in this scope published in the twenties and thirties in Norway and in Holland, but rather unknown to the professionals.

2.7 In the preceding, tendencies arisen before the event of digital computers have been outlined. Appearance of computers was decisive for the development of this subject, both from theoretical aspects and for practical design applications.

Earlier theoretical research, e.g. that on the Michell structures, has been concerned with finding solutions in closed form, bound to extreme difficulties. This fact is responsible for the scarcity of solved problems in the world literature, for instance a single one exists on spatial structures, in spite of the rather drastic simplifications fundamental for the Michell structures. With the event of computers, research has been directed toward the numerical treatment of the Michell-type structures, a work with already interesting achievements.

At the same time, the extension of digital computers, in addition to ease the development of optimation trends based on an established system of assumptions, threw light on quite new possibilities. Thus, alongside with the existing optimation trends, new ones appeared, specially bound to computer application. A common feature of these trends is that they much reduce simplifications, accessory to earlier systems, both as to the structural requirements to be taken into account and to the economical aspects of selecting the optimum structure, and make the mathematical model to approach practical exigencies.

Optimation problems for computer use are in general formulated as follows: A set of structures is given, so that each element of this set can be described by a finite number of real parameters. Structural requirements for the problem are written as inequalities so that several different functions of the parameters must be greater than O. Satisfaction of these inequalities, or better, determination of parameter values to satisfy the inequalities represents the solution of the inverted structural problem. In addition to the inequalities, a further numerical value, the so-called target function is given as a function of the free parameters, the minimum of which designates one out of the set of structures, meeting inequalities expressing structural requirements.

Quite a wide range of optimation methods based on computer possibilities has been developed, differing by the degree of how practical structural requirements are simplified or reckoned with at full complexity, or by what of the complex economical correlations of the structure are involved, and to what degree, in formulating the target function. Also, there is a wide variety of methods for numerically solving the formulated problem. In what follows, some typical problems and solution methods will be presented, without aiming at completeness.

2.8 In certain simple cases, or when simplifications are introduced, conditional equations expressing the structural requirements and the target function may be linear in the free parameters. In this case the optimation problem is that of linear programming, feasible by several mathematical methods, especially by those pertaining to economy analysis. The linear programming problems of structural optimation are, however, mostly of a special structure and can be solved by special methods — maybe starting from the structural features of the problem. — in addition to general solution methods of linear programming problems.

2.9 Approaching structural conditional inequalities and target function to practical requirements causes the problem to inevitably lose its linearity. Mathematical methods suiting such problems, i.e. the theory of non-linear programming are in fact less developed. The problem becomes simpler if the system of requirements on inequalities can be eliminated and nothing but the minimum of a single function is to be found. This is the fundamental principle of the so-called integrated approach, developed in the USA. Conditional inequalities expressing the structural requirements are incorporated into the optimation target function by adding terms giving values tending to infinity for those sets of parameters which fail to meet conditional inequalities. Thereby the minimum condition alone — in any case that for a modified target function — is sufficient to exclude structure alternatives short of the structural requirements. By gradually reducing the effect of terms additive to the original target function and by several iterations, the integrated approach method provides for the desired accuracy limit not to be exceeded by the error due to the disturbance of the original target function.

2.10 Even with the method of integrated approach, often the additive terms replacing the conditional inequalities of the target function or of the structural requirements cannot be written as formulae but develop in course of computation, as outcomes of the fed-in algorithm. If practical structural requirements are to be reckoned with at full complexity (e.g. for reinforced concrete structures, to take into consideration various design specifications), structural conditions cannot or are not advisable to be replaced by additive terms of the target function. This is the general case when the solution involves both structural conditional inequalities and target function, but none of them in explicit form. There is, however, a computer algorithm available, yielding target function values for any value set of independent parameters, and predicting if the structure described by the given parameters satisfies the specified requirements or not.

In such a "fully numerical" approach the optimation problem is solved so that the computer makes trials with different parameter value sets, confronts outcomes from the aspects of suitability and target function value, and based on this comparison, designates other parameter value sets for trial. At last, this gradual approximation leads to that independent parameter value set which is absolutely the most favourable of all, and taking into consideration any possible cases, it yields the solution of the optimation problem at a high probability, within the desired accuracy limits. This purely numerical method permits to take into consideration almost the entire range of practical requirements, at the same time, however, the problem becomes an unduly complex one, making extremely difficult to formulate solution principles, but leading to a quite reliable practical result.

Purely numerical methods mean essentially to test several alternatives of a structure so that the alternatives to be tested are designated by the computer itself, on the basis of conclusions drawn from the tests on the alternatives before. Hence, germs of the attempt to mechanize one of the most exquisite human capacities, namely to learn from experience, are involved. Various developed algorithms known from the literature differ exactly by the means how to realize this primitive „learning".

The computing work demand of entirely numerical methods is extreme. Namely, any step of the computation requires the full structural analysis of a perhaps quite complex structure for any considered load case, precisely taking into consideration all requirements for the structure and all the design specifications. Up-to-date, efficient digital computers, however, lend themselves to this immense computation work. Optimations involving great many independent parameters and rather complex requirements have been carried out in the fields of space craft construction and of airplane design (for instance, those reported of by the Boeing Aircraft Co., USA). In relation of building structures, a purely numerical optimation method has been applied e.g. in the Giprotis Institute, USSR, for designing standard large-span prestressed concrete beams for mass production.

3. Hungarian research results

Induced by the natural endeavour to design structures as advantageous as possible, development of various optimation methods began also in this country at an early date, at first without knowledge of the relevant results abroad, and independently of their encouraging effect.

As early as in the 40's G. DÉRY considered problems of favourably designing steel bridges. In the early 1950's, J. PELIKÁN carried out optimation tests on hinge arrangement of the reinforced concrete Gerber beams of the People's Stadium, Budapest, applying methods further developed since by himself and others. I. MENYHÁRD examined the problem of optimum reinforcement for concrete plates on the basis of the yield line theory. In correlation with the problematics of Michell structures, J. BARTHA set up an interesting theorem, to my knowledge being the first in the international special literature to take into consideration the phenomenon of buckling due to axial compression in optimation problems. J. PEREDY established principles of the correlation between optimations of statically determinate and indeterminate structures in elastic and plastic ranges.

In this country, a small group of workers doing research on the optimum reinforcement of concrete beams has formed. Relevant papers have been published by I. MENYHÁRD and J. PELIKÁN, and later by S. KALISZKY, Z. VISY and J. PEREDY. The initiating role of J. PELIKÁN and the interesting results of S. KALISZKY worked out on the basis of a new approximating assumption facilitating the solution of many problems and extending the field of investigations over plates and shells, should be pointed out.

Research program in this country involves optimation by means of up-to-date digital computers. J. PEREDY has been concerned with the numerical determination of Michell structure problems. T. LAKI, GY. RUSZNÁK and J. PEREDY studied "entirely numerical" methods.

4. Actual situation and future trends

A survey of the trends and home results on the field of optimation permits to draw some conclusions concerning the characteristic features of the present situation and future tasks. These conclusions express personal views, and so they are intended to raise a discussion.

4.1 Actually, optimation represents one of the structural engineering fields in the speediest development. It is strictly correlated to some most up-to-date fields of technical sciences such as either astronautics and aviation or electronics and applied cybernetics. In addition to its theoretical importance, it is also of a great practical use from direct economical aspects.

4.2 Two principal trends out of the actual complexity of optimation works are likely to crystallize. One is the endeavour to deduce general theoretical conclusions, to establish principle correlations, at the cost of omitting less important features of the extremely complex problem; the other consists in setting up and solving practical problems as complex as they are, having recourse to the latest computing techniques, in order to make possibly full use of immediate economical advantages.

4.3 This latter trend to solve complex practical problems will in all probability not get stuck in the examination of numerical problems and of particular cases disengaged of their correlations. At a farther perspective, the gradually gathered experience will probably lead to a synthese, that is, practical observations will permit to draw general theoretical conclusions representing basic features of the problem closer than do theoretical results based on the actual drastic simplifications.

4.4 As concerns the further development of research in this country, it is advisable to adher to either of the two predicted principal trends, that is, to direct optimation research either to arrive at theoretical conclusions of general validity, adding considerably to the present knowledge, or to help complex practical problems bearing immediate economic results.

Summary

There may be several different structures to meet requirements inherent with the destination of a given object. One of them should be designated for practical realization. Recently, there is a trend to base these decisions, besides the indispensable engineer's judgement, on certain exact computation methods, the so-called optimation methods.

After an exact definition of the optimation problem, a historical survey of structural optimation is given, and the principal trends described. Special consideration is given to optimation methods developed before the event of highly efficient digital computers, to the effect of these latter on the development of optimation methods, and to the evolving recent trends.

Finally, research results obtained in this country are presented, together with conclusions drawn from Hungarian and foreign observations concerning the future trends of development of optimation methods.

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