DESIGN OF DROP-SHAPED TANKS WITH SPECIFIED CAPACITY

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1. Drop-shaped tanks

The generatrix of a rotational symmetric tank subject to permanent internal pressure may and perhaps has to be selected so that stresses acting in the tank walls are equal at any point and in any direction. Tanks so developed are termed drop-shaped tanks, and are structures free of either flexion or shear, fluid pressure being offset by stresses acting in the tank wall plane.

Shape of and force distribution in a drop-shaped tank are the same as for a liquid drop. Intermolecular forces produce a drop-surface layer subject to equal tensile stresses in any point and direction, these constituting the surface tension. Relationship between surface layer and drop is the same as the inter-action between the drop-shaped tank wall and the compressed liquid completely filling out the tank.

Fig. 1 — Tank welded of steel plates, Port-Arthur (Texas), for petroleum storage. Height: 12·15 m, diameter: 43·0 m.

In what follows, it is intended to examine only the filled state as that of principal loading, rather than to test force effects upon filling, in empty state, and due to wind loads. Neither will tanks of either totally or partially other than pure drop shape, i.e. of a different force distribution be investigated.

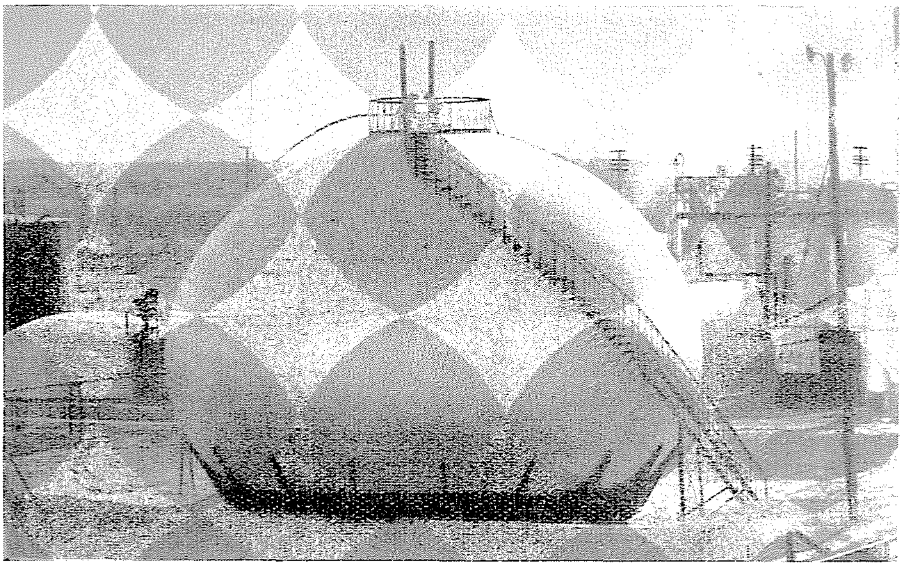

Fig. 2 — Tank welded of steel plates, Colorado (Texas). Height: 12·0 m, diameter: 16·15 m, capacity: 1,500 m³.

There exist several examples of "pure" drop-shaped tank, such as those shown in the pictures 1, 2 and 3.

If the specific weight of the stored liquid, the liquid pressure at the tank top — the overload — and the specific stress acting in the tank wall are known, the problem can be solved by methods described in the literature [1], [2], leading to the generatrix of the drop-shaped tank, hence design of this latter pertains to the form determination problems in structural design.

2. Problem of the specified capacity

Of course, the required tank capacity is function of service needs and is to be considered a design datum. And though, it is not listed above among initial data. In fact, the developed design processes refer to the determination of tank generatrix, in the knowledge of which the capacity can be computed. The calculated and the specified capacities may often differ. This can only be

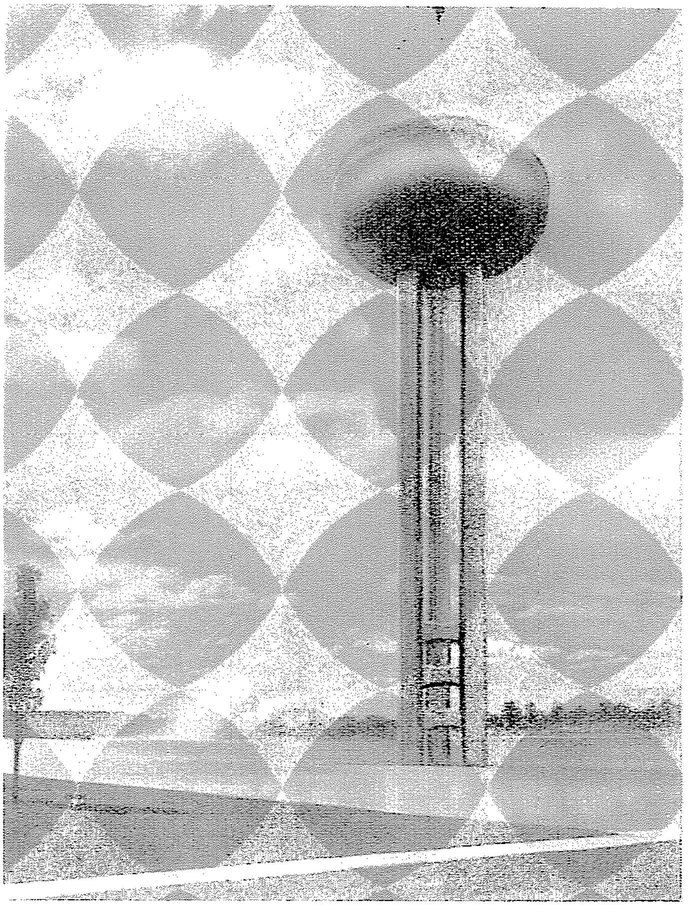

Fig. 3 — Water tank made of steel plates in the General Motors Technical Center, Detroit.

overcome by altering the stress value and applying it to calculate again the drop-shaped tank generatrix and then its capacity. This iteration can be continued to the desired accuracy.

According to a method developed by W. Flügge in his "Stresses in Shells" [2] an initial stress value may be assumed so that iteration may end within a few steps, leading to highly accurate results. This method is based on the recognition that liquids of given specific weight, volume and overload can only be balanced by a single, homogeneous stress system. Of course, the desired drop shape pertains just to this stress system. Relationship between factors defining this drop shape has been treated by Flügge as follows: (Fig. 4).

He plotted in ordinata the log. scale quotient from the drop-shaped tank capacity (V) by the third power of overload (h), and in abscissa the fraction h/a, where

$$a = \sqrt{\frac{N}{\gamma}} \tag{1}$$

In this expression N is the specific stress and γ the liquid density. Knowing correlated curve points, one can proceed as follows. From given V and h values the value of fraction $h/a = p$ can be read off the curve. Expressing N from Eq. (1):

$$p = \frac{h}{a}\ ;\quad p^2 = \frac{h^2}{a^2} = h^2\frac{\gamma}{N}\ ;\quad N = \frac{h^2\gamma}{p^2} \tag{2}$$

Calculation with this N value leads to the generatrix of the drop-shaped tank, which in turn permits to compute the tank capacity, that has to equal the specified value, apart from the reading errors.

All that has been described above is well known and needs no support.

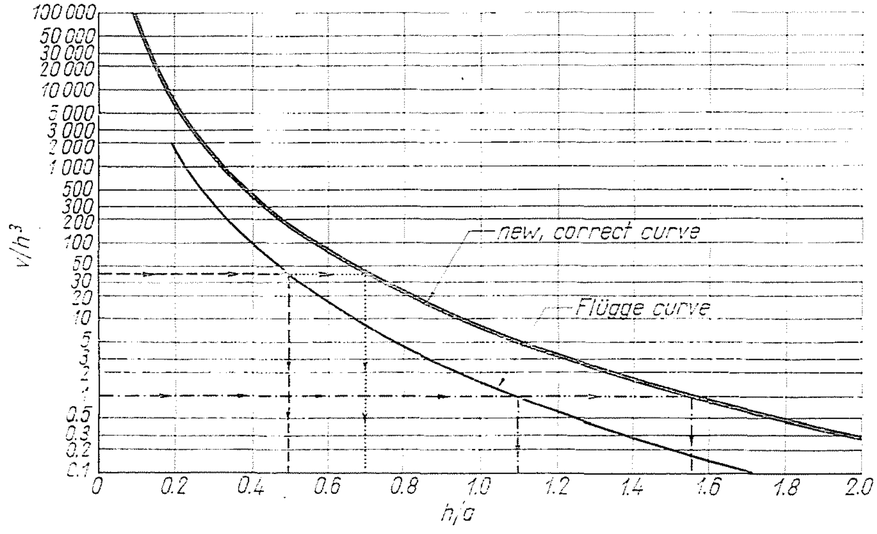

Fig. 4 — Capacity of drop-shaped tanks vs. h/a.

3. Suggested method to virtually provide for the specified capacity

In studying form determination problems, the described principal solution has been applied to computer programming the determination of drop-shaped tank generatrices. As soon as solving the first problem the outcomes appeared to be irrealistic. Checking demonstrated the deviation to be due to an error in the mentioned Flügge curve.

Thereupon it became imperative to correctly determine the curve shape. Digital computation has led to the thick-line diagram in Fig. 4. Co-ordinates of some points of the new curve are given below. Two numerical examples (2 – 3 and 4 – 5) will be presented for the sake of confronting the curves proposed by FLÜGGE and elaborated by us, respectively.

Co-ordinates of some points of the new curve:

h	h/a	V/h^3
1·0	0·1	$8.253469 \cdot 10^4$
2·0	0·2	$6.309401 \cdot 10^3$
3·0	0·3	$1.321272 \cdot 10^3$
4·0	0·4	$4.203044 \cdot 10^2$
5·0	0·5	$1.685705 \cdot 10^2$
6·0	0·6	$7.840814 \cdot 10$
7·0	0·7	$4.044264 \cdot 10$
8·0	0·8	$2.251806 \cdot 10$
9·0	0·9	$1.330155 \cdot 10$
10·0	1·0	8.236358
11·0	1·0	5.300591
12·0	1·2	3.523015
13·0	1·3	2.406599
14·0	1·4	1.683257
15·0	1·5	1.201840
16·0	1·6	$8.738406 \cdot 10^{-1}$
17·0	1·7	$6.457051 \cdot 10^{-1}$
18·0	1·8	$4.840895 \cdot 10^{-1}$
19·0	1·9	$3.676994 \cdot 10^{-1}$
20·0	2·0	$2.826270 \cdot 10^{-1}$

Notice that the modification of the curve does not affect the principial solution of the problem, but only the numerical output, this latter, however, rather significantly.

The program written in ALGOL-60 language applied to determine the proposed curve is given in Annex. It has been developed according to an algorithm by DR. SZMODITS [1], pp. 188 – 189. Applying as parameter the angle included between the curve tangent and the horizontal, divisions have been taken each degree, between which the curvature is considered constant, and the radius of curvature for the next interval has been determined in each division. Division co-ordinates are the output. After shaping the generatrix, the program effects the computation of the tank surface and capacity, approximating curve sections between divisions by straights, and drop-shaped tank components by truncated cones.

There are three varieties of the program. The first variety is for the co-ordinates of the proposed curve. The program is only correct if fraction $(H3-H1)/H2$ gives an integer number!

Here $H1$ first value of overload,
$H2$ difference between overload values,
$H3$ final value of overload.

The second variety lends itself to determine capacity, surface area and generatrix of drop-shaped tanks, provided parameters such as overload, liquid density and stress are known. This variety has been applied to solve problems 2 to 5. In the second and third varieties the values $H1$, $H2$ and $H3$ are always for overload and the three values are equal in each problem.

The third variety functions similarly as the second one, excepted, however, that it determines the generatrix of the drop-shaped tank of specified capacity by iteration, varying the stress. In the program the first datum DB stands for the number of problems to be solved, S is their serial number, and T the number of the variety (first, second or third).

Problem 1

Find some points of the new, correct curve in Fig. 4.
$S = 1$, $T = 1$
Other input data:
$N = 100$ Mp/m, $\gamma = 1{\cdot}0$ Mp/m^3.
The h value is ranging from 1,0 to 20,0 m, in 1,0 m steps.
Thus $H1 = 1{\cdot}0$, $H2 = 1{\cdot}0$, $H3 = 20{\cdot}0$.

$$\text{Constant } a = \sqrt{\frac{N}{\gamma}} = \sqrt{\frac{100{\cdot}0}{1{\cdot}0}} = 10{\cdot}0,$$

hence the p value varies from 0·1 to 2·0 due to the variation of h, with steps of 0·1.
Output is as presented in Chapter 3.

Problems 2 and 3

Determine the generatrix of a water-filled drop-shaped tank of 5,000 m^3 capacity, subject to an overload of 5·0 m, by applying the values of the Flügge curve and of the new curve, in problems 2 and 3, respectively.
Input data in problem 2 are:
$S = 2$, $T = 2$,
$H1 = H2 = H3 = 5{\cdot}0$ m.
$\gamma = 1{\cdot}0$ Mp/m^3
Thus $V/h^3 = 40$ hence
$p = h/a = 0{\cdot}5$ (See continuous line in Fig. 4)
$N = 100{\cdot}0$ Mp/m.

Input data in problem 3 are:
$S = 3$, $T = 2$,
$H1 = H2 = H3 = 5{\cdot}0$ m
$\gamma = 1{\cdot}0$ Mp/m^3
Thus $V/h^3 = 40$ hence
$p = h/a = 0{\cdot}7017$ (See dotted line in Fig. 4)
$N = 50{\cdot}7735$ Mp/m.
Outputs of the problem are compiled in the table below, and the tank generatrix is shown in Fig. 5. Deviation between both capacity values is rather perspicuous. Correlating both values with the desired capacity of 5,000 m^3, correct values seem to be obtained by the new curve.

	Problem 2		Problem 3	
Specified capacity (m^3)	5000.0			
Calculated capacity (m^3)	21670.0		5002.0	
Surface area (m^2)	394900.0		135000.0	
Stress (Mp/m)	100.0		50.7735	
α (degrees)	Y(m)	Z(m)	Y(m)	Z(m)
0	0·0	0·0	0·0	0·0
15	9·35	1·17	4·97	0·63
30	15·18	3·52	8·53	2·08
45	18·59	6·09	10·78	3·79
60	20·53	8·60	12·11	5·51
75	21·51	10·93	12·79	7·13
90	21·79	13·03	12·99	8·60
105	21·56	14·84	12·83	9·88
120	20·95	16·33	12·40	10·93
135	20·07	17·48	11·78	11·74
150	19·02	18·28	11·04	12·31
165	17·90	18·75	10·26	12·63
180	16·77	18·90	9·49	12·73

Problems 4 and 5

Determine the generatrix of a water-filled drop-shaped tank of 1,000 m^3 capacity, subject to an overload of 10·0 m, applying the values of the Flügge curve and of the new curve in problems 4 and 5, respectively.

Input data in problem 4 are:
$S = 4$, $T = 2$,
$H1 = H2 = H3 = 10{\cdot}0$ m.
$\gamma = 1{\cdot}0$ Mp/m^3.
Thus $V/h^3 = 1{\cdot}0$ hence

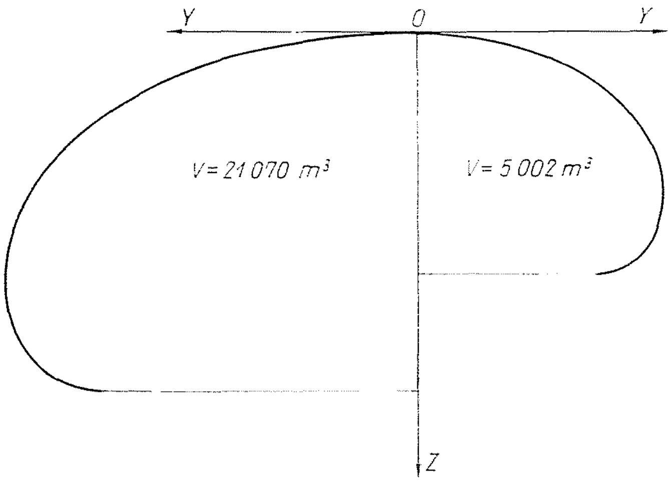

Fig. 5 — Outputs of problems 2 and 3: drop-shaped tank generatrices.

$p = h/a = 1{\cdot}1$ (See dash-and-dot line in Fig. 4)
$N = 82{\cdot}644628$ Mp/m.

Input data in problem 5 are:
$S = 5,\ T = 2,$
$H1 = H2 = H3 = 10{\cdot}0$ m.
$\gamma = 1{\cdot}0$ Mp/m³.
Thus $V/h^3 = 1{\cdot}0$ hence
$p = h/a = 1{\cdot}557$ (See dash-and-two-dots line in Fig. 4)
$N = 41{\cdot}257652$ Mp/m.

Outputs of the problem are compiled in the table below, and the tank generatrix is shown in Fig. 6.

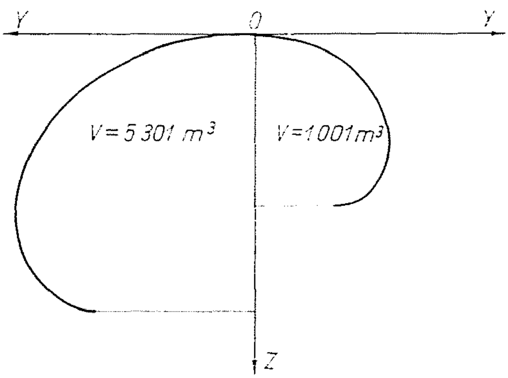

Fig. 6 — Outputs of problems 4 and 5: drop-shaped tank generatrices.

	Problem 4		Problem 5	
Specified capacity (m^3)	1000.0			
Calculated capacity (m^3)	5301.0		1001.0	
Surface area (m^2)	119800.0		34680.0	
Stress (Mp/m)	82.6446		41.2577	
α (degrees)	Y(m)	Z(m)	Y(m)	Z(m)
0	0·0	0·0	0·0	0·0
15	4·17	0·54	2·11	0·28
30	7·58	1·94	3·94	1·03
45	9·99	3·77	5·33	2·09
60	11·52	5·75	6·26	3·30
75	12·33	7·68	6·78	4·53
90	12·57	9·47	6·93	5·70
105	12·37	11·03	6·80	6·74
120	11·84	12·33	6·44	7·61
135	11·07	13·34	5·92	8·29
150	10·17	14·03	5·31	8·76
165	9·21	14·43	4·68	9·02
180	8·29	14·55	4·08	9·10

ANNEX

Algorithm in ALGOL–60 language for determining tank drop shapes.

```
begin real GAMMA, H1, H2, H3, N, P, V;
      integer DB, S, T;
      comment here DB is to be valued;
   B: comment here the following variables have to be valued: S, T, H1, H2, H3, N,
              GAMMA, if T = 2 then also P, if T = 3 then also V;
      begin real A, V1, F;
            integer I, K, KE, L, LS, M;
            array SIZE [0:180, 1:2],
                  OUTPUT [1:(H3–H1)/ H2+1, 1:2];
            SIZE [0,1]:=0; M:=K:=L:=LS:=0;
         C:A:=sqrt(N/GAMMA);
            for SIZE [0,2]:=H1 step H2 until H3 do
            begin real KR, NR;
                  KR:=NR:=2×A↑2/SIZE[0,2]; V1:=F:=0;
                  P:=SIZE[0,2]/A;
                  for I:=1 step 1 until 180 do
                  begin real K1, K2, YA, YF, ZA, ZF, ALFA, BETA, DELTA;
                  K1:=(I–1)×0·0174532925;
                  K2:=I×0·0174532925;
                  ZF:=KR×cos (K1); YF:=KR×sin (K1);
```

```
        ZA:=KR×cos (K2); YA:=KR×sin (K2);
        SIZE [I,1]:=SIZE [I-1,1]-YF+YA;
        SIZE [I,2]:=SIZE [I-1,2]+ZF-ZA;
        if I=180 then go to D;
        NR:=SIZE [I,1]/sin (K2);
        KR:=NR/(NR×SIZE [I,2]/A ↑ 2-1);
      D:ALFA:=SIZE [I,1]+SIZE [I-1,1];
        BETA:=SIZE [I,1]-SIZE [I-1,1];
        DELTA:=SIZE [I,2]-SIZE [I-1,2];
        V1:=0·26179938×DELTA×(3×ALFA ↑ 2+BETA ↑ 2)+V1;
        if T=1 then go to E;
        F:=3·1415927×(SIZE [I-1,1] ↑ 2-SIZE [I,1] ↑ 2+ALFA×sqrt
        (BETA ↑ 2+DELTA ↑ 2))+F;
      E:end;
        if T=1 then
        begin M:=M+1; OUTPUT [M,1]:=P;
              OUTPUT [M,2]:=V1/SIZE [0,2] ↑ 3;
        end;
    end if T = 1 then print out block OUTPUT here;
        go to R;
        if T=3 then
        begin KE:=K; if abs ((V-V1) /V) < 0·001 ∨ V=V1
              then go to G else if V>V1 then
              begin P:=P-0·002; K:=-1; LS:=LS+1; end;
         else begin P:=P+0·002; K:=1; LS:=LS+1; end;
              if K ≠ KE then L:=L+1;
              if L=2 then go to G;
              N:=GAMMA×H1 ↑ 2/P ↑ 2; go to C;
        end;
      G: begin comment print out outputs here
        end;
      R: if DB≠S then go toB;
    end;
end;
```


Summary

When determining drop shapes for tanks, it is rather problematic to ensure the capacity required for practical purposes; in general, this is only possible by iterated calculations. There exists a nomogram, well-known from the literature [2], aimed at accelerating the iteration by furnishing initial informative data. (Such informative data may also be of use in e.g. preliminary design.) Digital computer analysis, however, shows this nomogram to be erroneous. A nomogram, correct in our feeling, has been elaborated and applied to solve checking problems.

References

[1] SZMODITS, K.: Statik der Schalenkonstruktionen. Leipzig, Taubner. 1966.
[2] FLÜGGE, W.: Stresses in Shells. Berlin, Göttingen, Heidelberg. Springer-Verlag. 1960 and 1962.
[3] SZÁNTAY, B.: Design of Chemical Apparatuses. Budapest, Tankönyvkiadó 1963.
[4] Réservoirs métalliques à grande capacité, de forme sphéroïdale. *L'Ossature Métallique*, 1938. 305. p.
[5] New Spheroidal Design for Large Oil Tank. *Engineering News-Record*. Vol. II. 1929. 416 p.