Ψ periodica polytechnica

Architecture
39/2 (2008) 73–80
doi: 10.3311/pp.ar.2008-2.05
web: http://www.pp.bme.hu/ar

RESEARCH ARTICLE

Spherical segment approximation of sexpartite vaults

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Received 2008-06-20

Abstract

As a continuation and generalization of an article dealing with quadripartite vaults using the same approach, this paper proposes spherical segment approximations of some medieval sexpartite vaults having pointed arches, with the idea of finding simple, yet representative geometric models for them. On the supposition that the bounding curves of every vault cell can be reckoned as vertical sections of spherical surfaces, it results in severe, yet expressive models of a wide range of vault shapes, revealing feasible computer representations of several vault types appearing in the books of architectural history either in textual or non-textual forms.

Keywords

sexpartite vault · spherical segment · geometry

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1 Basics

The order of the presented constructional methods, and even the conventions of the wording and the characteristics of the illustrations follow those of the previous paper [3]: for the sake of brevity and clarity architectural terms will be used parallel with the geometric ones where it seemed reasonable; and the diagrammatic figures show only the vaulting bays themselves (above the level of the imposts), disregarding the vault and rib thicknesses. Two perspective illustrations are included also, demonstrating the appearance of the vaulting forms as given by the proposed approximate construction methods.

1.1 Triangulated plan

In this paper the proportions of the vaulting bay will play an important role, and in this respect the "triangulated base plan" will be referred to quite a number of times. Triangulation has been one of the standard principles for making "geometrical construction" in medieval architecture. The quotation mark here is justified, because in geometrical terms triangulation is not an exact Euclidean construction method. Actually, it was only a way of finding certain kind of "modules", the characteristic points of which would produce equilateral triangles.

The triangulated floor plan shown in Fig. 1 [1] is particularly interesting for us, because it consists of rectangular bays the width and length of which are identical to the ratio of the height and side of equilateral triangles. The ratio of side *o* and

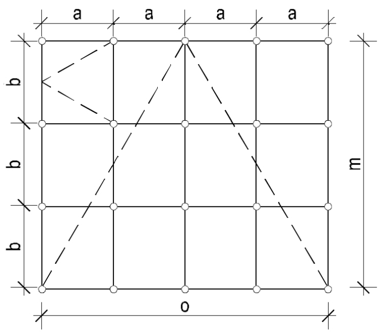

Fig. 1. Triangulated plan [1] 485.

height m of the large equilateral triangle:

$$o^2 = m^2 + \left(\frac{o}{2}\right)^2 \implies o^2 - \frac{o^2}{4} = m^2 \implies 3o^2 = 4m^2$$

As it can be seen, the width of the triangle is divided into four parts ($o = 4a$), and its length into three parts ($m = 3b$). The ratio of the sides of a given vaulting bay is as follows:

$$3(4a)^2 = 4(3b)^2 \implies 4a^2 = 3b^2 \implies \frac{a}{b} = \sqrt{\frac{3}{4}} \qquad (1)$$

In this paper the terms triangulated floor plan or triangulated ratio will be used for this ratio, thus clarifying, and at the same time, restricting the original meaning of these terms.

1.2 Principles of the geometrical construction

First let us examine the general rules of geometrical constructions of pointed sexpartite vaults.

A characteristic feature of the sexpartite vault (see Fig. 2) is that the six compartments of the bay meet at a single apex point (Z), consequently, the planes of the transverse crowns are not perpendicular to the longitudinal axis of the bay and the aisle, thus, four of the six compartments are not orthogonally symmetric.

Let the base shape of the vault be a rectangle having sides $2a$ and $2b$, and consider that the centres of the transversal, longitudinal and diagonal arches are given as X', Y', and A'. Even if these points are not known, it is easy to find them from the spans and heights of the main arches (as piercing points that the halving perpendicular lines of line sections AH, AK and AZ produce at plane ABC of the springing of the vault).

Let X, Y and V be the three centre-points of the spherical segments, and let the perpendicular offset of these sphere centres from the corresponding axes of the bay be x and y. We do not have to deal with distance v of centre-point V from transverse axis MC, because (in order to ensure symmetry) centre-points Y' and V' of the longitudinal arch must be at equal distance from point K, i.e. distances QV' and QY' are always equal ($y + \frac{b}{2}$), and this determines v ($v = y$).

Since diagonal arch AZ is a common section of adjacent spherical segments of the vault, line XY must be perpendicular to diagonal AD. Besides, since the boundary arches are also circular sections of spherical segments, the lines connecting the centres of the boundary arches and the centres of the corresponding spherical segments (i.e. X'X, Y'Y and V'V) must be perpendicular to the sides of the base shape.

In case of sexpartite vaults one has to locate the additional centre-point V. The same reasoning is applicable here as in the case of the diagonal arch: since crown KZ is the common section of two adjacent spherical segments, the line connecting the adjacent sphere centres (Y and V) must be perpendicular to horizontal projection QC of crown KZ. This is a constraint with certain consequences: for example, three straight lines with given directions must meet at point Y, including Y'Y (which is perpendicular to the side of the base shape), XY (which is perpendicular to the projection of diagonal arch AZ, i.e. to diagonal AC) and VY (which is perpendicular to the projection of crown arch KZ, i.e. to line QC).

1.2.1 Equal apex-heights

To construct a vault having equal rises of boundary arches, it is enough to set apex points H and K at identical heights ($h = h'$), and then to place X' and Y' at the points where the halving perpendicular lines of line sections AH and AK pierce the plane of springing. Although the locations of X'X and Y'Y are set, an infinite number of possible solutions are still available, since the location of A' can be selected arbitrarily within certain limits.

Let us examine algebraically the premise of having equal rises of boundary arches. Since the line connecting points A and H is the chord of transverse arch AH, the line erected perpendicularly to line section AH from centre-point X' of arc AH, will intersect it at its midpoint F. In this way the height of this point F is exactly the half of that of point H, i.e. ($\frac{h}{2}$). As the angles marked with two arcs (at H, and X') have perpendicular arms:

$$\frac{h}{a} = \frac{x + \frac{a}{2}}{\frac{h}{2}} \implies h^2 = 2a \cdot \left(x + \frac{a}{2}\right) \implies x = \frac{h^2 - a^2}{2a} \qquad (2)$$

A similar approach can be applied in case of the longitudinal arch, considering the angles marked with two arcs at K and Y':

$$\frac{h'}{\frac{b}{2}} = \frac{y + \frac{b}{2} + \frac{b}{4}}{\frac{h'}{2}} \implies h'^2 = b \cdot \left(y + \frac{3b}{4}\right) \implies y = \frac{h'^2 - \frac{3}{4}b^2}{b} \qquad (3)$$

Assuming equal rises of the boundary arches:

$$h^2 = h'^2 = 2a \cdot \left(x + \frac{a}{2}\right) = b \cdot \left(y + \frac{3b}{4}\right) \implies \frac{a}{b} = \frac{y + \frac{3}{4}b}{2x + a} \qquad (4)$$

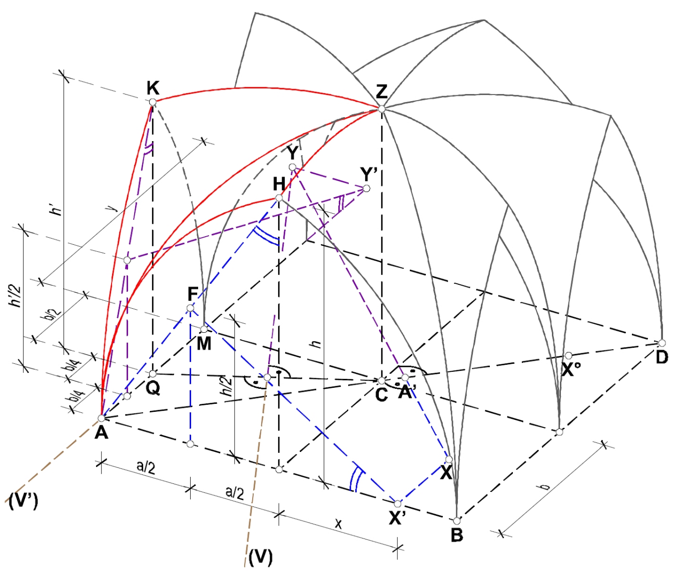

Fig. 2. Schematic layout of a sexpartite vaulting bay

It is necessary then to comply with Eq. 4 in order to have equal rises of longitudinal and transversal arches. Note that even though the reasoning is the same as in case of quadripartite vaults, the result is different. Since the geometry of the transversal arch is unchanged, so is Eq. 2. However, Eq. 3, and consequently Eq. 4 will be different, due to the dissimilar geometry of the transversal cells.

2 Variations

There are quite a number of theories trying to explain the basic construction principles of quadripartite vaults having pointed arches. As starting points, we quote statements from the textbooks used by the students of our Faculty – the same ones that were used in case of quadripartite vaults.

a. "The pointed boundary arches, which can rise to the same heights even over different spans, enabled the construction of cross-vaults in rectangular bays." [2] 131.

b. "Pointed arches became higher and more steeply pointed over time. Initially, the centres of the arches were at the tierce-point of the span, then at the quarter-point, finally at the springing, or even outside of it." [2] 134.

c. The introduction of pointed arches allowed using identical radii for the main arches of the vault. [1] 442.

d. The crowns of the pointed quadripartite vault meet each other with horizontal tangents, without breaking above the centre of the ground plan. [2] 8., [1] 444.

Let us analyse the constructional and geometrical consequences of the principles listed in points a. to d., and examine how the different character of the ground plan symmetry of sexpartite vaults in comparison to quadripartite vaults influences the geometrical construction . The general relationships described in 1.2 will stand, so we will discuss only the considerations and specifications of the constructional variations.

2.1 Vaults without diagonal groins

One of the simplest ways to form a pointed quadripartite vault is to cut out the central part of a sail vault, analogously to the transmutation of a barrel vault to a pointed barrel vault. The result of this procedure is a vault shape whose main arches are all isomorphic (corresponding to statement b.), while the centres of its four spherical segments are shifted along the diagonals of the bay to the vertices of a rectangle having the same proportions as the bay itself.

Sexpartite vaults can also be constructed with centres of some of their spherical segments on the diagonals of their bays (see Fig. 3). Naturally, the above statement will not apply for the intermediate spherical segments (e.g. the one with centre V), which in fact on the other hand allows the construction of vaults having semi-circular diagonal arches – although this way the transverse arch also becomes semi-circular. However, this construction method fails to ensure isomorphic arches in case of sexpartite vaults.

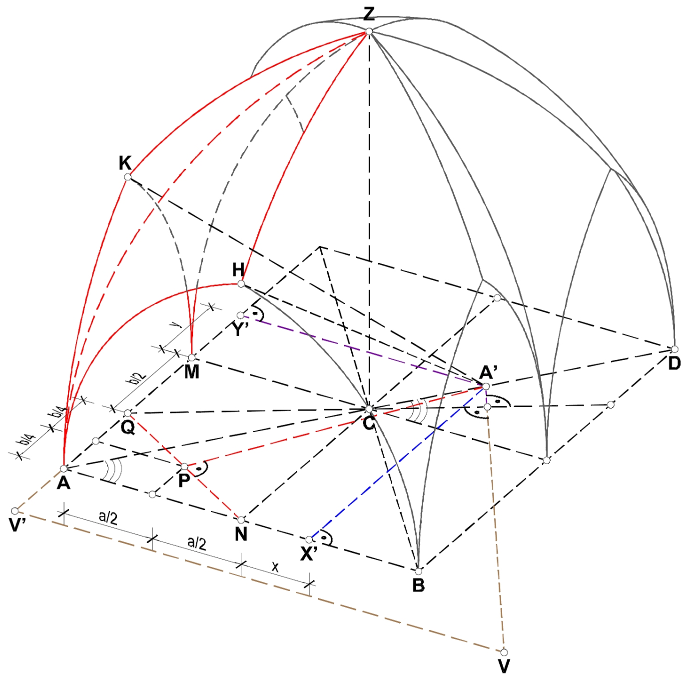

Fig. 3. Vault without diagonal groins

2.1.1 Equal apex-heights

Since A' always lies on diagonal AD, the angles marked with three arcs (at A and at C), have the same alignment, consequently:

$$\frac{a}{b} = \frac{x}{y} \Longrightarrow ay = bx \tag{5}$$

Substituting Eq. 2 and Eq. 3 into the above equation will not determine a specific proportion of the bay – but it is still possible to set up some constraints.

According to the present construction method, points H and K must be on the same spherical segment, having common centre A' – i.e. the lengths of line sections A'H and A'K are equal. Assuming that the apex heights of the boundary arches are identical, the vertical projections of these line sections will have identical length – and therefore evidently the length of their horizontal projections must also be equal. Now, as A'Q and A'N distances are equal, the halving perpendicular line of line section QV must go through A'. In a horizontal coordinate system having its centre at C, line A'P passes through point $-\frac{1}{2}a, -\frac{3}{2}b$, and its angular coefficient will be $\frac{2a}{b}$ (since it is perpendicular to line section QN having angular coefficient $\frac{-b}{2a}$):

$$y-\left(-\frac{3}{4}b\right) = \frac{2a}{b}\cdot\left(x-\left(-\frac{1}{2}a\right)\right) \Longrightarrow 4by+3b^2 = 8ax+4a^2$$

When line A'P passes through C, then x and y become 0:

$$4b\cdot 0+3b^2 = 8a\cdot 0+4a^2 \Longrightarrow \frac{a}{b} = \sqrt{\frac{3}{4}}$$

Since A' cannot be closer to A than C, A' must be located either at point C or on the part of diagonal AD beyond it, thus we can say:

$$\frac{a}{b} \le \sqrt{\frac{3}{4}}$$

Another limit can also be specified, as slope $\frac{2a}{b}$ of the halving perpendicular line PA' of section QN must be larger than slope $\frac{b}{a}$ of diagonal AD:

$$\frac{b}{a} < \frac{2a}{b}$$

This constraint is rather theoretical though, since in practice A' could not be too far beyond endpoint D of diagonal AD. Anyway, we can say, that the problem can be resolved only if the ratio of the sides of the bay remains within the following interval:

$$\sqrt{\frac{1}{2}} < \frac{a}{b} \leq \sqrt{\frac{3}{4}} \qquad (6)$$

Within this interval ($1,1547a \leq b < 1,4142a$) it is possible to assign one and only one location of A' (and naturally also of X' and Y') to any floor plan (or more accurately to any of its a/b ratio), where the apex heights of the boundary arches will become identical.

2.2 Vaults with horizontal central tangents

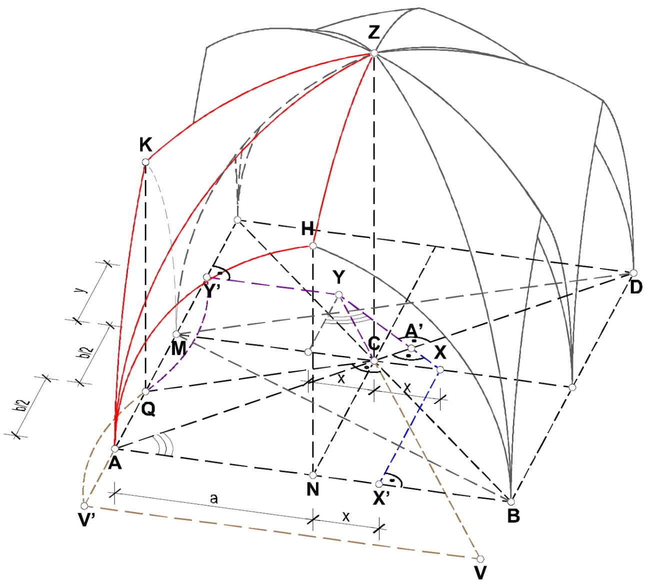

Fig. 4. Vault with horizontal central tangents

To get crowns with horizontal tangents at point Z, over the centre of the bay (according to condition d.), the centres of the spherical segments of a quadripartite vault must lie on the axes of the bay (see Fig. 4). This applies also for the longitudinal crown of a sexpartite vault; therefore every two longitudinal cells share the same centre-point. It is not true for the transverse cells, however, as they have separate centres due to the missing orthogonal plan symmetry.

In order to get horizontal tangents for crowns HZ and KZ in point Z, they must be arcs having their centres in point C, being the common sections of adjacent spherical segments of the vault. This way lines XM and YV must go through point C, the common centre-point of these arcs. In other words, centre-points X and Y must be on lines CY and CX respectively, which lines are perpendicular to horizontal projections NC and QC of crown arches KZ and HZ.

The generic method of construction changes only so much, that the centres of the longitudinal cells (e.g. X) must be on the transversal axis, while the centres of the transverse cells (e.g. Y) must be at the intersection of the line erected from point X perpendicularly to diagonal AD, with the line erected from point C perpendicularly to QC, the horizontal projection of crown KZ.

2.2.1 Equal apex-heights

As the angles marked with three arcs (at A and at Y) have perpendicular arms, assigning x and y to each other can be characterized by the following relationship:

$$\frac{a}{b} = \frac{y}{2x} \Longrightarrow 2ax = by \qquad (7)$$

Substituting Eq. 2 and Eq. 3, and assuming equal rises of boundary arches:

$$2a \cdot \frac{h^2 - a^2}{2a} = b \cdot \frac{h^2 - \frac{3}{4}b^2}{b} \Longrightarrow h^2 - a^2 = h^2 - \frac{3}{4}b^2 \Longrightarrow \frac{a}{b} = \sqrt{\frac{3}{4}} \qquad (8)$$

Consequently, this construction method can lead to equal apex heights of boundary arches only if the bay has a triangulated floor plan (see 1.1). However, if that condition is met, the rise of the boundary arches will automatically become equal, for any value of the height of point H.

2.2.2 Semicircular diagonal arch

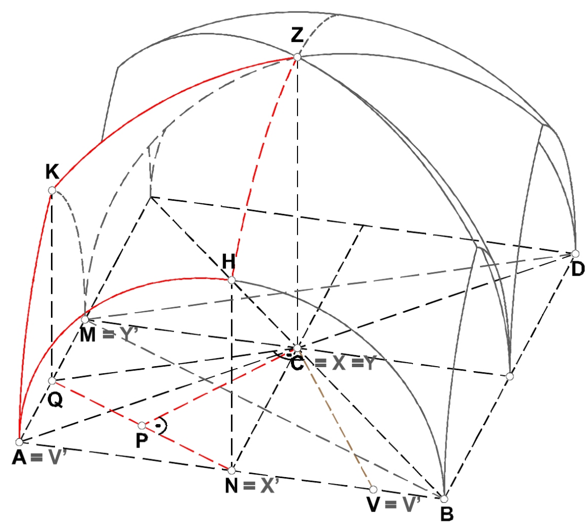

Fig. 5. Vault with semicircular diagonal arch

Centre-points X and Y can be found as the intersection of one of the axes of the vault (either X'X or Y'Y), with the line erected in centre A' of the diagonal arch AZ, perpendicularly to its plane. Consequently, if A' lies in C, X and Y will also coincide with it (see Fig. 5).

As this common centre-point lies simultaneously on the axes (MC, NC) and on diagonal (AD), both logics described above will apply: the tangent of the crowns will be horizontal in Z (as in 2.1), and there will be no ridge separating the adjacent cells (as in 2.2). Furthermore, as it lies on the longitudinal axis, centre-point X' of the transverse arch will also be there, which results in a semicircular transverse arch. This means that there will be no ridge at crown HZ either.

2.3 Vaults with main arches having equal rises

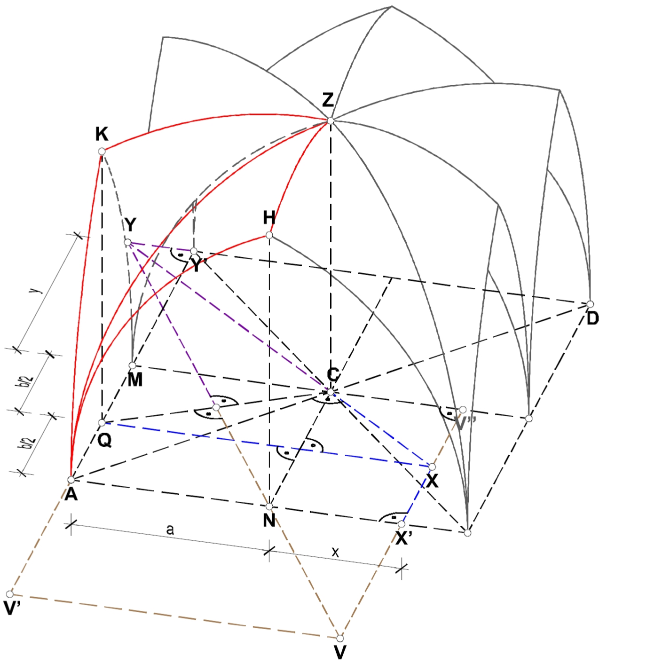

Fig. 6. Vault with main arches having equal rises

The construction method ensuring equal apex heights for all main arches in case of quadripartite vaults can be adapted quite easily for sexpartite vaults (see Fig. 6). Centre-point X of cell AHZ will be on the halving perpendicular line of horizontal projection NC of crown HZ – thus X will be on the perpendicular axis of the transverse arch (QX).

This is relevant, since V can be found as the intersection of line YV, and the line erected perpendicularly to the side in point V', which is at equal distance from Q as Y' (Y'Q = QV') – and this way X will be halfway between Y and V in longitudinal direction. As the slope of line YV is double of that of line YX, points X and V will be in equal distance from Y in transverse direction (V'V = QX). This configuration means that X', the projection of X to the plane of the transverse arch AH, and V", the projection of V to the plane of the intermediate transversal arch MZ will be on the same longitudinal line – and this way the radii of these parallel arches (AX' and MV') will be equal. The same result can be obtained from the consideration, that as A and M, and also H and Z are on the same longitudinal lines, parallel with the axis of the vault, the halving perpendicular lines of AH and MZ should pierce the base plane of construction at equal distances from A and M respectively. Centre-point Y of transversal cell AKZ, can be located according to the general construction method.

2.3.1 Equal apex-heights

This construction method obviously ensures equal apex-heights for all boundary arches no matter what proportions the bay or the boundary arches may have.

2.3.2 Four by two is six – is it?

As radii of arches AH and MZ are proved to be equal, we can designate this shape not only as a sexpartite vault, but also as two quadripartite vaults, the transverse cells of which are in asymmetrical position.

The crowns of the longitudinal cells of these hypothetical quadripartite vaults would have the same geometry as the one that has horizontal central tangents (see 2.2), which demonstrates that transversal cells having completely different geometries can be attached to the same longitudinal cell.

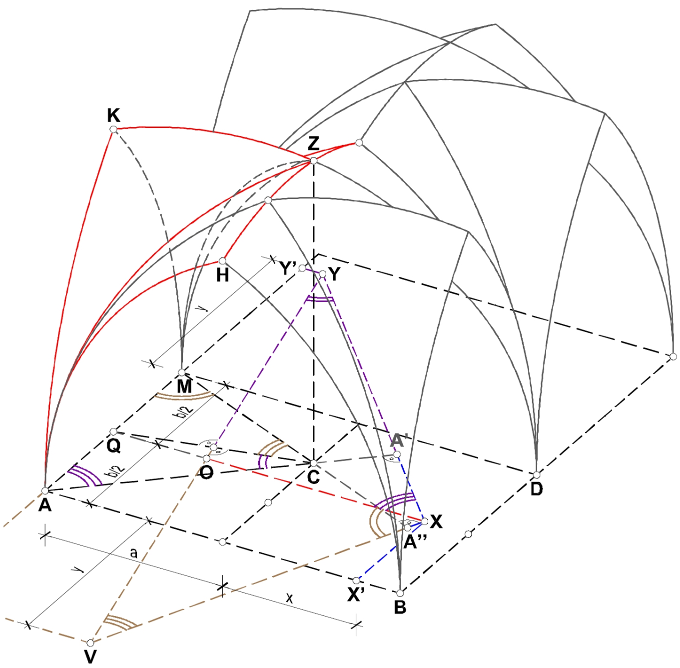

Fig. 7. Vault with main arches having equal rises

Fig. 7 illustrates another specific solution, with the crowns of the transversal cells meeting the crown of the longitudinal cell at one third of its span (resembling the vaults of St Hugh's Choir in Lincoln Cathedral).

Centre-point X remains on axis QX of the transverse arch and centre-point Y can be found as the intersection of line XY erected from X perpendicularly to AC, and of line VY, the halving perpendicular of QC, the horizontal projection of crown KZ. Centre-point V can be found as the intersection of line VY with line XV, erected from X perpendicularly to MC.

Triangles AQC and XOY or QMC and OXV are respectively isomorphic, and so, having equal (OX) base-lengths, the heights of triangles XOY and OXV are equal. As the perpendicular (longitudinal) distances of Y and of V from line QX are equal, Y' and V' will be symmetrical to Q – which means, that the resulting longitudinal arch AKM will always turn out to be symmetrical.

2.4 Vaults with the main arches having equal radii

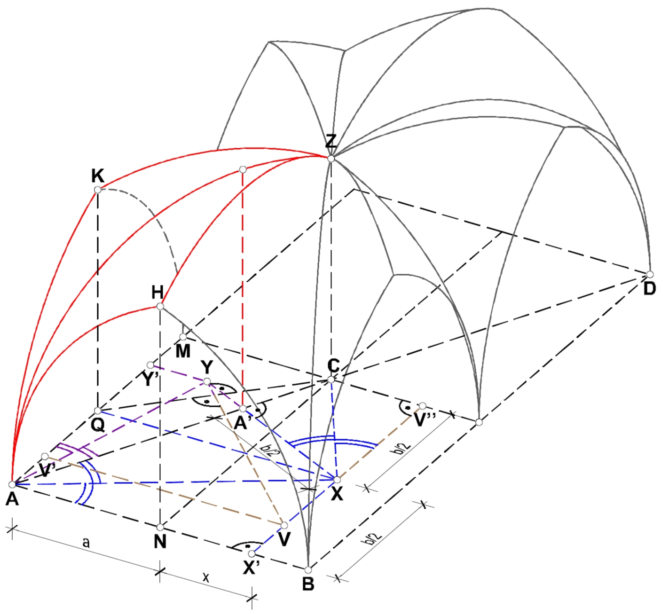

Fig. 8. Vault with the main arches having equal radii

In order to ensure equal radii for all main arches of a quadripartite vault (see statement c.), the centres of the spherical segments (e.g. X) must be at equal angles from the boundary and diagonal arches, therefore X and Y must be on angle bisectors AX and AY respectively (see Fig. 8).

By adopting the quadripartite method presented before to sexpartite vaults, the radii of arches having their centre-points in X', A', and Y', will be equal, but the radius of the intermediate transversal arch having its centre in V" will normally be different.

The location of this V" point is determined by V, the location of which is determined by Y, the location of which in turn is determined by X. Consequently, to set the correct location for V" requires the correct placement of X.

As we have seen in 2.3, the radii of the two transversal arches (having their centres in X' and V") will be equal, if centre-point X has an offset of $\frac{b}{2}$ from the frontal side of the bay. This method however fails to produce equal radii for the other arches.

To meet the two conditions at the same time, we have to unite the logics of the two above construction methods: X should be on the intersection of the bisector of angle BAC, with the halving perpendicular QX of horizontal projection NC of crown-curve HZ. However, the line erected from this point perpendicularly to diagonal AD meets the diagonal on its AC section – which means, that Z will not be the highest point of diagonal arc AZD.

Anyway, this location is more realistic in case of sexpartite vaults than it would be in case of quadripartite vaults, as intermediate transverse arch MZW still supports the inner portion of the infilling.

2.4.1 Equal apex-heights

The three above possibilities offer different solutions. The solution described in 2.3, which ensures equal radii for the transverse arches only, would always produce the correct result without any restrictions.

The method described above in 2.4 obviously requires a specific $a/b = 1/2$ ratio for the bay plan, as it ensures equal radii for all main arches with the sole exception of the intermediate transversal arch. Since the boundary arches having their centres at X' and Y' have equal radii, their rise can only be equal if their span is equal ($2a = b$). The rise of the boundary arches (and thus their radii) still can be varied though, so this construction method can still produce an infinite number of solutions.

The third method, ensuring equal radii for all main arches, inherits the above $a/b = 1/2$ proportion of the bay plan. Furthermore, as this method restricts the position of the centre-points of spherical segments, in this case there is only one possible solution – the one illustrated in Fig. 8.

2.5 Vaults with the highest possible boundary arches

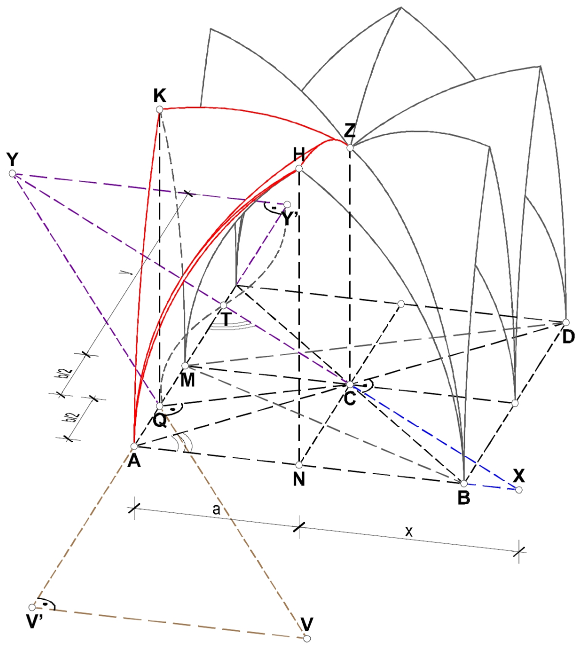

Fig. 9. Vault with the highest possible boundary arches

To set up a sexpartite vault with the highest points of its crowns being at its boundary planes, sphere centre X has to be located on the side of the bay, but points Y and V do not, because the plane of the boundary arches of the side cells is not perpendicular to the plane of their crown (see Fig. 9). In order to have the highest point of crown KZ located over point Q, sphere centre Y should be on the line erected from point Q perpendicularly to line segment QC. This way point Y can be obtained as the intersection of this line, and the line erected from point X perpendicularly to diagonal AD.

2.5.1 Equal apex-heights

As the highest point of crown KZ is located over point Q, the fall of line YV over the length of segment YQ will be $y + \frac{b}{2}$. The slope of line YX (the tangent of angle XYY') is half of the steepness of line YV (the tangent of angle VYY'), therefore it is sure that the distance between points T and Q (intersections of the side of the base rectangle with lines YX and YV) will be $\frac{2y+b}{4}$, half of the Y'Q distance.

As the angles marked with three arcs (at A and at T) have perpendicular arms, assigning x and y to each other can be characterized by the following relationship:

$$\frac{a}{b} = \frac{\frac{b}{2} + \frac{2y+b}{4}}{a + x} \Longrightarrow 4a^2 + 4ax = 3b^2 + 2by \tag{9}$$

Substituting Eq. 2 and Eq. 3, and assuming equal rises of boundary arches:

$$\begin{aligned} 4a^2 + 4a \cdot \frac{h^2 - a^2}{2a} &= 3b^2 + 2b \cdot \frac{h^2 - \frac{3}{4}b^2}{b} \\ \Longrightarrow 8a^2 + 4h^2 - 4a^2 &= 6b^2 + 4h^2 - 3b^2 \\ \Longrightarrow \frac{a}{b} &= \sqrt{\frac{3}{4}} \end{aligned} \tag{10}$$

Consequently, this construction method can lead to identical apex heights of boundary arches only if it has a triangulated floor plan (see 1.1). However, if that condition is met, the rise of the boundary arches will automatically become equal, irrespective of the height of point H.

3 Combinations

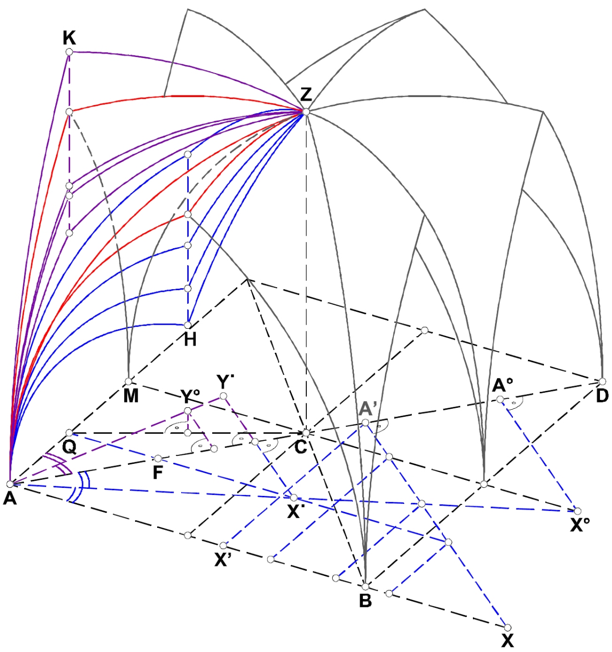

Fig. 10. Different sexpartite vault shapes using the same diagonal arch.

Since only the diagonal arch connects the two adjacent cells, evidently even cells constructed with entirely different methods can be used in a single vaulting bay.

As it can be seen in Fig. 10, the centres of the cells derived from a given diagonal arch having its centre in point A' will lie on line A'X drawn from point A' perpendicularly to diagonal AD.

The special characteristics ensured by the above construction methods can exist simultaneously only if the cell centres are located in the intersection points of the lines matching the given constructions. Obviously, there is no way to combine the attributes attainable by locating the cell centres on lines parallel with boundary arch AHB. The same way, we can exclude the intersection point of diagonal AD with bisector AX° of angle DAB, as this point (A) would be in inappropriate place. This leaves the six intersection points of the above two groups of lines: points A, F and C on the diagonal, A, X˙ and X° on the bisector. As the pointed arches definitely require radii greater then half of the span, we can dismiss points A and F. Normally, we would have excluded point C as well, but as we have seen in 2.2, it can be dealt with as a special case.

Intersection point X˙ of bisector AX˙ and line FX˙ again gives an unlikely result, as the line drawn from this point perpendicularly to diagonal AD meets it on its wrong side, resulting in a bent over diagonal arch. However – as we have discussed it in 2.4 – this solution cannot be ruled out completely.

By placing the cell centre to point X°, the intersection of angle bisector AX° and transversal axis MC, it is possible to construct a vault having horizontal tangents in point Z, and some of its main arches still having equal radii. Only some of them, because in the four transversal cells the same construction method would give a different result (point Y°), and in case of sexpartite vaults we cannot find the kind of symmetry which in case of quadripartite vaults enabled Y° to be on line X°A°. Furthermore, the centre of the diagonal arch obtained from Y° would appear on an inappropriate location (on AC segment).

The construction method described in 2.3 ensures equal apex-heights for the boundary arches. However (unlike the square bay in case of quadripartite vaults) we cannot find a single bay shape that would always guarantee the expected result. In some cases (see 2.4), we were not able to find a completely satisfying solution. In other cases (see 2.1) we did not find a certain shape, but we could specify limits for the proportions of the bay. Finally, in some cases (discussed in 2.2 and 2.5) we learned that a rectangular bay having the triangulated proportions described in 1.1 can guarantee the correct result.

4 Conclusions

Sexpartite Gothic vaults are surely amongst the most unique shapes ever used in architecture. Up to the present time they were described and typified almost exclusively by the shapes and proportions of their bays and boundary arches, but the development of informatics and computer graphics enabled us to

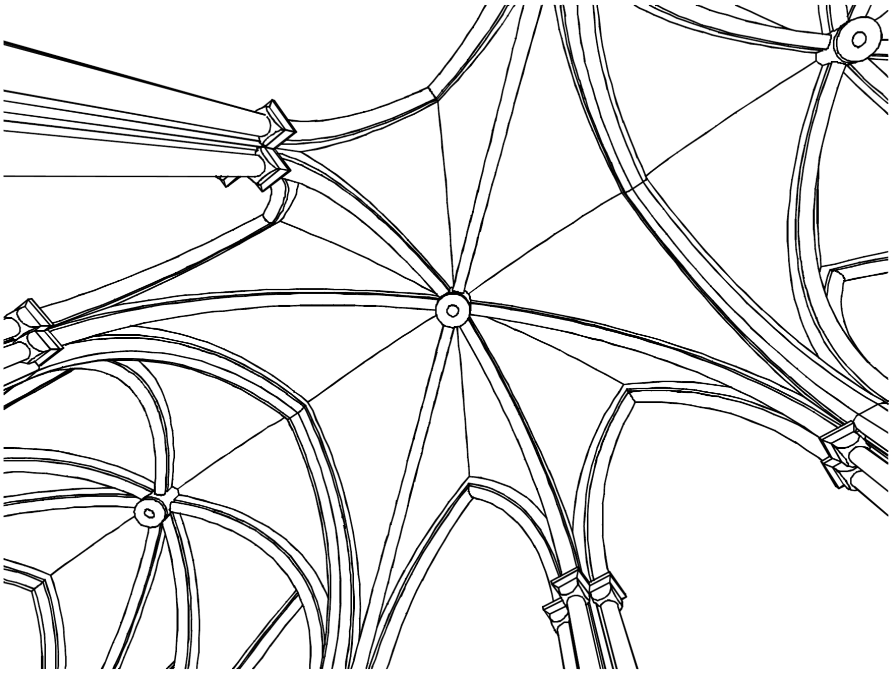

Fig. 11. Perspective view of a vault described in 2.3, whose main arches have equal rises.

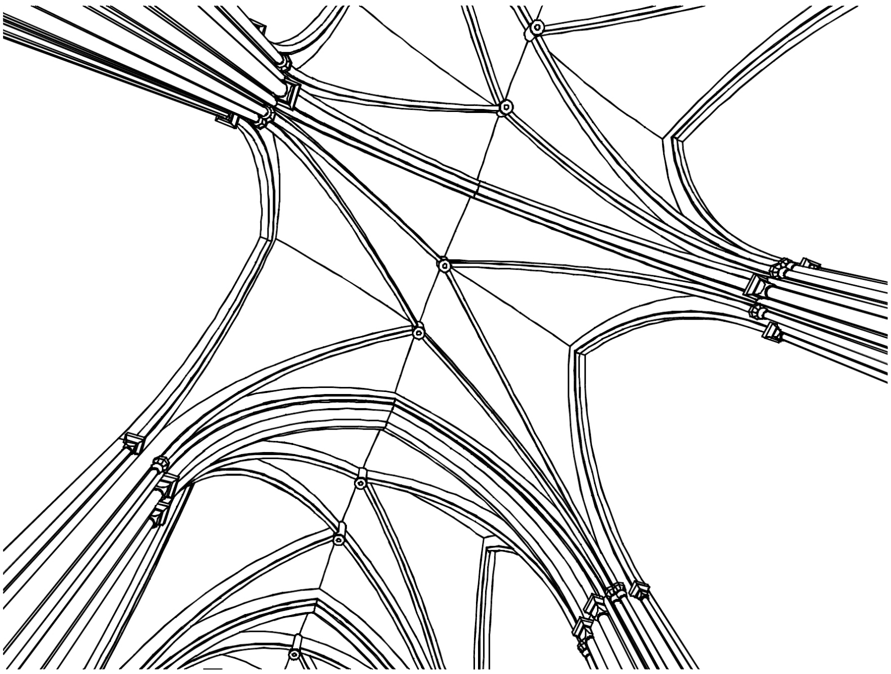

Fig. 12. Perspective view of a vault set up with asymmetric layout, described also in 2.3.

deal with the actual three-dimensional shape of the vault proper, instead of only trying to imagine it based on its certain sections. This paper proposes potential geometrical approximations of the shapes of sexpartite vaults having pointed arches. The generalization of the construction methods applicable to quadripartite vaults reveals how the modified form of the ground plan symmetry influences these methods, separating the unique results from the general solutions. On the supposition that the bounding curves of every vault cell can be reckoned as vertical sections of a single sphere, it results in severe, yet expressive models of a wide range of vault shapes, revealing feasible computer representations of several vault types.

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