periodica polytechnica

Architecture
38/1 (2007) 17–20
doi: 10.3311/pp.ar.2007-1.03
web: http://www.pp.bme.hu/ar

RESEARCH ARTICLE

The elastic-plastic buckling failure load of structures made of nonlinear elastic-plastic material

Endre Dulácska

Received 2000-09-14

Abstract

The approximate determination of the elastic-plastic buckling load parameter of structures was discussed. For the critical failure load parameter a lower and an upper bound has been established, with the aid of which the results of computer calculations can be checked.

Keywords

nonlinear elastic-plastic material

Acknowledgement

This paper was produced by the Reinforced Concrete Research Group of Hungarian Academy of Sciences.

Endre Dulácska

Department of Strength of Materials and Structures, BME, H–1111 Budapest, Műegyetem rkp. 3., Hungary

e-mail: duli@silver.szt.bme.hu

1 Introduction

Nowadays we can perform the exact stability analysis of structures with the aid of computer techniques. The geometric, material and cross-sectional nonlinearities can be taken into account by the programmes (cross-sectional nonlinearity is caused for example by the cracking of the reinforced concrete).

Unfortunately, a mistake in the programme, an incorrect entry, or a change of sign may result in an entirely wrong result.

Hence the design engineer highly appreciates a method by means of which the upper and lower bounds of the critical load parameter of the structure can be determined.

In this way we can check the results of the computer calculation. Such a method will be presented in this paper.

2 The Upper and Lower Bound of the Elastic-Plastic Buckling Load Parameter of Structures

On the basis of investigations on elastic-plastic structures (Bartha, 1972) [1], (Dulácska, 1972) [2], (Horne, 1963) [6], (Horne, 1965) [7] we can state some theorem:

- the elastic-plastic buckling failure load parameter λ_F is smaller than the "elastic" critical load parameter λ_C;
- λ_F is also smaller than the rigid-plastic failure load parameter λ_P;
- and λ_F is greater than the Rankine-type straight line between points λ_C and λ_E, (see in Fig. 1).

Here λ_E is the load parameter of the elastic limit state i.e. at which yielding starts in the structure.

Thus the elastic-plastic buckling failure load parameter λ_F of structure lies on the straight line running at 45° in the hatched domain of Fig. 1. Fig. 1a shows the case of $\lambda_C > \lambda_P$ and Fig. 1b the case of $\lambda_P > \lambda_C$.

According to a more accurate analysis of structures with compact cross sections, $\lambda_F$can be approximated by the formula:

$$\lambda_{F_1} \approx \lambda_C \cdot \frac{1}{2}\left[\sqrt{\frac{1}{1+(\lambda_C/\lambda_E)^2}}+\sqrt{\frac{1}{1+(\lambda_C/\lambda_P)^2}}\right]. \quad (1)$$

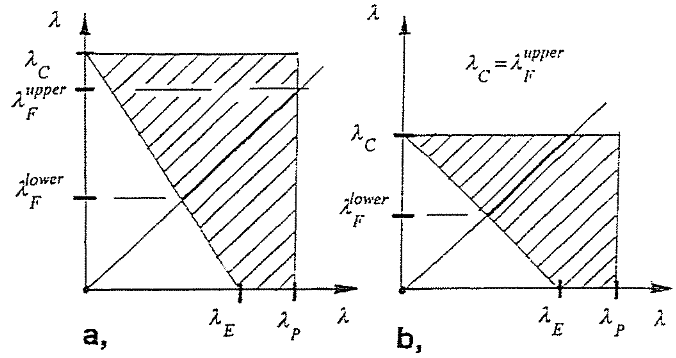

Fig. 1. The bounds of the elastic-plastic buckling failure, load parameter: *a:* $\lambda_C \geq \lambda_P$, *b:* $\lambda_C \leq \lambda_P$

For laminated and reticulated structures, and for structures with I cross sections we can estimate as:

$$\lambda_{F_2} \approx \lambda_C \cdot \frac{1}{2}\left[\sqrt{\frac{1}{1+(\lambda_C/\lambda_E)}}+\sqrt{\frac{1}{1+(\lambda_C/\lambda_P)}}\right]. \qquad (2)$$

We thus have to examine "only" how the values of the load parameters λ_E, λ_P, and λ_C can be determinated.

3 The Load Parameter λ_E of the Elastic Limit State of Structures

We determine the internal forces of the structure at the load parameter λ_1 and we calculate the stresses from the axial load and bending of the structure according to the elastic theory of the strength of materials.

After this we select the maximum stress $\lambda_{1,\max}$ and we compute the load parameter λ_E from Eq. (3),

$$\lambda_E = \lambda_1 \cdot \frac{\sigma_y}{\sigma_{1,\max}} \qquad (3)$$

Here σ_y is the yield stress of the material.

4 Determination of the Rigid-Plastic Failure Load Parameter λ_P of the Structure

We may exactly compute the rigid-plastic failure load (collapse load) parameter λ_P by the theory of plasticity (Kaliszky, 1984) [8].

We obtain the lower bound of $_P$ if we increase the load of the structure until the cross sectional internal force (moment) Y at any point of the structure reaches the value Y_P that causes rigid-plastic failure.

Of course, the rigid-plastic failure load Y_P may also mean coupled bending moment M and axial force N (that is the compression is usually eccentric).

The lower bound of the collapse load parameter may be computed from the equation

$$\lambda_P \leq \lambda_1 \cdot \frac{Y_P}{Y_1}. \qquad (4)$$

The collapse load Y_P of the cross section can be determined with the aid of the limit curves of the load-bearing capacity of the cross sections.

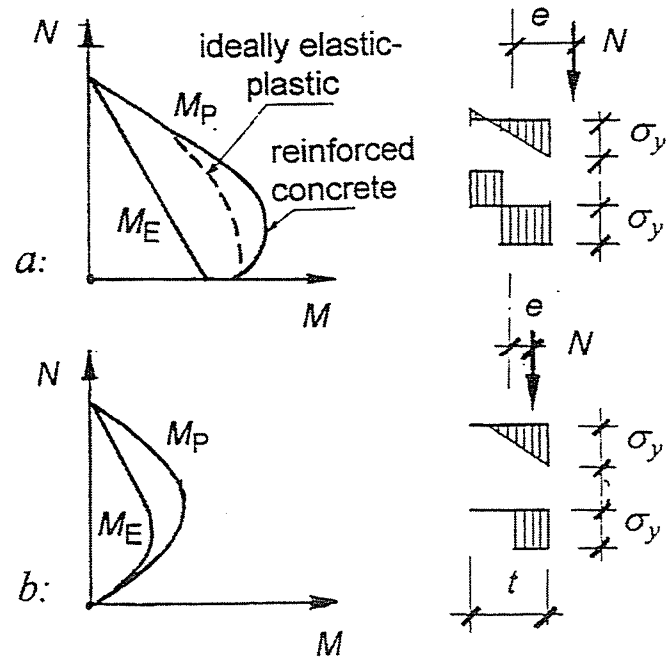

Fig. 2. Limit curves of cross sections made of various materials *a:* with tensile strength, *b:* without tensile strength

The character of these curves can be seen in Fig. 2 for various materials, or may by computed with knowledge of the strength of the materials.

5 Estimation of the Critical Load Parameter λ_C of the Structure

As the basis of the determination of the critical load parameter λ_C we use the "classical" critical load parameter $\lambda_{C,O}$ which is computed by the second-order theory, assuming small deformations (Croll-Walker, 1972) [2]. This value may be determined from the worked-out cases of the stability theory (Petersen, 1982) [10].

If we do not know the classical critical parameter of the full load $\lambda_{C,O}$ of the structure, but we know the classical critical load parameter λ_C^i of every component load separately, the critical load parameter of the complete load $\lambda_{C,O}$ can be computed by Dunkerley's approximate relationship

$$\frac{1}{\lambda_{C,O}} \approx \sum_i \frac{1}{\lambda_{C,O}^i}. \qquad (5)$$

In addition we have to analyse whether the post-buckling load-bearing capacity of our structure is decreasing, constant or increasing. Shells, shallow arches and some reticulated structures have, as a rule, decreasing post-buckling load-bearing capacity.

When examining such structures we must take the reduction of the critical load parameter, caused by imperfections and eccentricities, into account. This can be done by the following formula:

$$\lambda_C = \rho \cdot \lambda_{C,O}. \qquad (6)$$

For shells, the reduction factor , based on (Kollár-Dulácska, 1984) [9] is shown in Fig. 3.

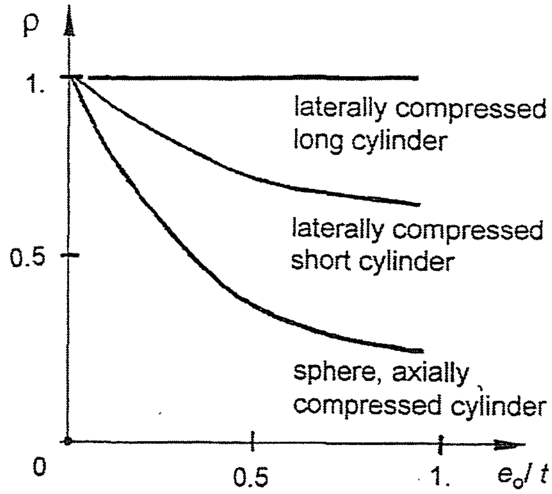

Fig. 3. Decrease of the critical load of shells with increasing eccentricity e_0

Plates and frames have a constant (or increasing) post-buckling load bearing capacity, hence with these structures the imperfections do not influence the critical load.

Reinforced concrete structures form, however an exception. With these, the cracks reduce the stifness, and, thus also the critical load.

The critical load of structures made of reinforced concrete, timber and plastic is reduced by the creep as well. The influence of creep may be taken into account by reducing modulus of elasticity according to the formula:

$$E = E_0/(1+\varphi) \tag{7}$$

where E_0 is the initial value of the modulus of elasticity and is the creep factor (Dulácska, 1981) [5]. The effect of the variation of the modulus of elasticity with the stress in the case $e_0 = 0$ is taken into account by the Eq (1) (Dulácska, 1972) [3].

We may estimate the decrease of critical load parameter of a reinforced concrete structure with the aid of Fig. 4, based on (Dulácska, 1978) [4]. Here e_0 is the eccentricity of the compressive force of the most onerous cross section of the structure, and $\lambda_C^{\mathrm{lower}}$ is the lower bound of the critical load parameter of the reinforced concrete structure, which is determined by the second order theory of elastic stability theory taking the effect of the decrease of stiffening caused by cracks and creeps. That is, we compute the value of the lower critical load parameter with the bending stiffnesses of the cracked reinforced concrete cross section on the basis of stadium II. (cracked elastic state), with the creep reduced modulus of elasticity.

The value $\lambda_C^{\mathrm{lower}}$ is valid in the range $e_0 \geq e_{0,\mathrm{lim}} = \frac{t}{2}(1 - \frac{\lambda_C^{\mathrm{lower}}}{\lambda_{C,O}})$ where t is the height of the cross section. In the range $0 \leq e_0 \leq e_{0,\mathrm{lim}}$ the value $_C$may be computed from the formula:

$$\lambda_C^{\mathrm{rc}} = \lambda_{C,O}\left[1 - \left(1 - \frac{\lambda_C^{\mathrm{lower}}}{\lambda_{C,O}}\right)\left(2 - \frac{e_0}{e_{0,\mathrm{lim}}}\right)\frac{e_0}{e_{0,\mathrm{lim}}}\right]. \tag{8}$$

We may use the value $\lambda_C^{\mathrm{lower}}$ in the entire range of e_0 as a lower bound.

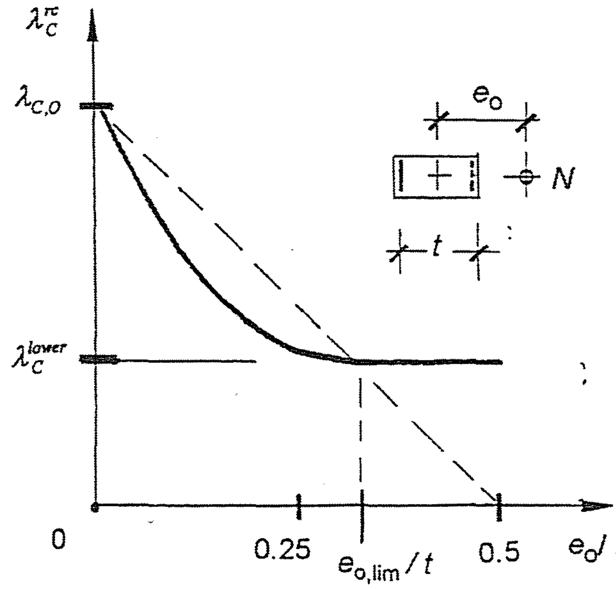

Fig. 4. Decrease of the critical load parameter λ_C^{rc} of reinforced concrete structures with increasing eccentricity e_0

If the reinforced structure has a geometrically decreasing load -bearing character (e.g.shells), we must take also this effect, characterised by , into account.

Notation

λ_F	The elastic-plastic buckling failure load parameter.
λ_C	The elastic-classical critical load parameter.
$\lambda_{C,O}$	The initial value of λ_C.
λ_P	The rigid-plastic failure load parameter.
λ_E	The load parameter of the elastic-limit state.
λ_1	The load parameter of the elastic state.
λ_C^i	The load component parameter.
λ_C^{rc}	The λ_C value for reinforced concrete.
$\lambda_C^{\mathrm{lower}}$	The lower value of λ_C.
$\lambda_C^{\mathrm{upper}}$	The upper value of λ_C.
λ_{F1}	The λ_F for compact cross section.
λ_{F2}	The λ_F for sandwich and I cross section.
E	The modulus of elasticity.
E_0	The initial value of E.
M	The bending moment.
M_E	The bending moment at the elastic limit state.
M_P	The bending moment at the plastic limit state.
N	The axial force.
φ	The creep factor.
e	The eccentricity of the external force.
e_0	The initial eccentricity of the external force.
t	The height of the cross section.
ρ	Factor for shells.
σ_y	The yield stress of the material.
$\sigma_{1,\max}$	The cross sectional stress at the elastic state.
Y	The cross sectional internal action effect.
Y_1	The Y value at the elastic state.
Y_P	The Y value at the rigid-plastic failure state.

References

1 **Bartha A Th**, *Some Simple Thoughts on Column Buckling*, Proceedings of International Colloquium on Column Strength, (1972).

2 **Croll JA, Walker AC**, *Elements of Structural Stability*, Macmillan, London, 1972.

3 **Dulácska E**, *Praktische Stabilitätsuntersuchung von zentrisch gedrückten Tragwerken aus Material mit värenderlichem Elastizitätsmodul*, Bautechnik **10** (1972), 340–345.

4 ———, *A rugalmas vasbetonrúd kihajlása: Buckling of Elastic Reinforced Concrete Columns*, Építés-Építészettudomány (1978), no. 1–2, 46–65. in Hungarian.

5 ———, *Der Einfluss des Kriechens auf die Stabilität der Tragwerke*.

6 **Horne MR**, *Elastic-plastic Failure Loads of Plane Frames*, Proc.of Royal Society A, 1963, pp. 346.

7 **Horne MR, Merchant W**, *The Stability of Frames*, Pergamon Press, New-York, 1965.

8 **Kaliszky S**, *Plastizitätslehre. Theorie und Technische Anwendungen*, VDI Verlag, Düsseldorf, 1984.

9 **Dulácska E, Kollár L**, *Buckling of Shells for Engineers*, Wiley and Sons, Chichester, New-York, 1984.

10 **Petersen Ch**, *Statik und Stabilität der Baukonstruktionen*, Vieweg and Sohn, Braunschweig, 1982.

11 **Tarnai T**, *A szerkezeti stabilitáselmélet Dunkerley típusu tételei: Dunkerley Type Theorem of the Stability Theory of Structures*, Építés-Építészettudomány (1984), no. 3–4, 427–467. (in Hungarian).