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RESEARCH ARTICLE

Ribbed vaults of the Nagyvázsony monastery church – Geometrical factor of safety highlights the secret

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Abstract

Using the geometrical factor of safety, the article compares three ribbed vaults which are different in their structures and complexity. Each vault could have been built for the Nagyvázsony monastery church. The spatial geometry of the three different vaults is based on the rules of control curve construction using the known geometry of the remains of the church ruins. The thrust lines for the ribs are determined for the dead load of the vault in each case. These thrust lines, as statically possible solutions, prove the suitability of the structures, and the geometrical factor of safety of the ribs helps to specify the geometry based on which the vault could be built.

The preliminary results of our research indicate that the geometrical factor of safety of the pointed groin vault proved to be the highest. From the three examined vaults, this one is the first that appeared in the history of architecture. Our conclusion is that the later built, more complex vaults raised not only geometrical or execution difficulties, but were also daring solutions from the aspect of the stability of the ribs.

Keywords

control curve construction · thrust line · geometrical factor of safety · ribbed vault

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1 Introduction

In the construction of "sky-high" gothic structures able to vault great spans boldly or even daringly, master craftsmen seemed to adopt an all-or-nothing approach, their results lasting even into the 21st century. Several very pleasing to the eye and heart-warming cathedrals, churches and monasteries demonstrate that the adoption of the appropriate proportions, mingled with tradition and daring ideas, facilitated the construction of extremely durable buildings which are now centuries old. When examining the structures of these architectural masterpieces, it seems unbelievable that the old masters never checked the stability of their buildings with static calculations. However, the combination of ideologically and aesthetically appropriate forms and proportions and structural geometry based on experience created several great surviving buildings.

The subject of our research is the once Pauline monastery church erected on the outskirts of Nagyvázsony, which unfortunately is unable to prove the perfection of its structure, as its vaults were destroyed following the Turkish invasion. The ground plan of the building and its ruined wall remains help us imagine what the structure must have looked like in its entirety. Moreover, by comparing the possible structural forms, we might find an answer to the arising question: Are the increasingly complex forms of gothic rib vaults just inventions of a creative mind, or were they intended to improve the stability of the structure?

Within the scope of the article three kinds of ribbed vaults will be examined. By creating the spatial geometry of the ribbed vaults based on the control curve construction and with the aid of the thrust lines defined by the dead load of the different structural systems, the geometrical factor of safety of these different structures will be defined. The data gained in this way provide sufficient basis to compare the different structures and give some hint to the geometry based on which the vault could be built.

2 The Pauline monastery church in Nagyvázsony

The monastery in Vázsony was founded by Pál Kinizsi and his father-in-law in 1483. The monastery of late gothic ecclesiastic architecture, however ruined it may be, is an exceptional work of art, dimensioned to seat several hundreds of believers.

It was consecrated to St. Michael, the patron saint of soldiers. The church, according to contemporary custom, was the burial place of its founder and his family. In Vázsony however, only Pál Kinizsi and the second husband of his widow, Márk Horváth were buried there. Above their tombs stone coffins decorated with carvings were laid.

Fig. 1. The current condition of the church

The monastery survived only for 70 years. In 1543 Székesfehérvár was occupied by the Turks, the monks fled, and Vázsonykő became a border fortress. In 1552 the castle of Veszprém also fell. As a precaution, the neighbouring landlords – being at the same time captains of their strongholds – blew up three monasteries, since they were transformable into fortresses. Beside the monasteries of Tálod and Városlőd the friary in Vázsony shared the same fate. After the not entirely successful explosion, stones and carvings were transported from the ruins to reinforce the fortress of Vázsonykő during the 16th and 17th centuries. Later, during the reconstruction of the village, the inhabitants also used the area of the monastery as a stone-pit. Only the firm intervention of Flóris Rómer around 1860 [1], prevented further devastation of the ruins

The excavation and preservation of the remains started only in 1959. After the excavation of the remnants of the walls, which were buried under debris, the reconstruction of the plan of the building was successful (Fig. 1).

From the remains of the walls it can be derived that the nave of the monastery church had an eight meter span and a polygonal apse. Tamás Guzsik claims that the church was vaulted by longitudinally arranged hexagonal stellar net compartments (Fig. 2).

The main sizes of the vault can be clearly defined from the existing characteristic features of the northern wall of the church. The height of the springers can be measured and the original height of the crowning can be estimated based on the location of the pockets of the cross-beams. Based on this data it is possible to construct a presumed vault shape with the use of the control curve construction method.

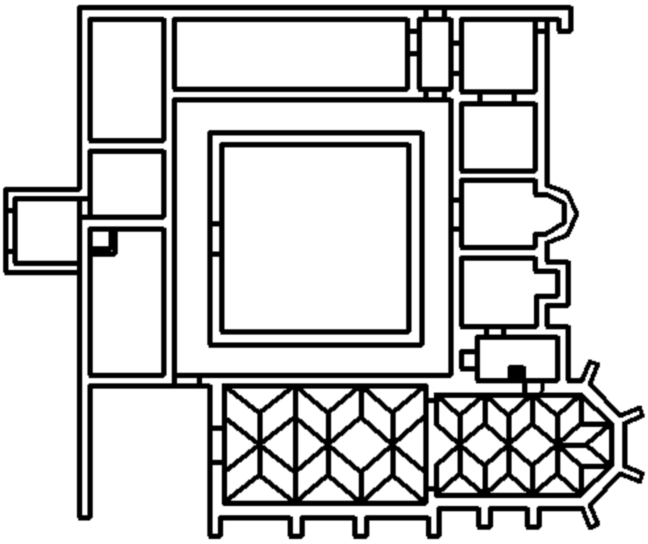


Fig. 2. The plan of the vault given by Tamás Guzsik [2]

3 The control curve construction of the ribs of the vaults

In the late Gothic period of Central Europe, control curve construction was a widespread solution for determining the geometry of the ribs of a stellar net vault [2]. This method largely facilitated the construction work, because it ensured that every rib of the vault could be drawn with the same radius, enabling the builders to carve every rib segment using the same templates.

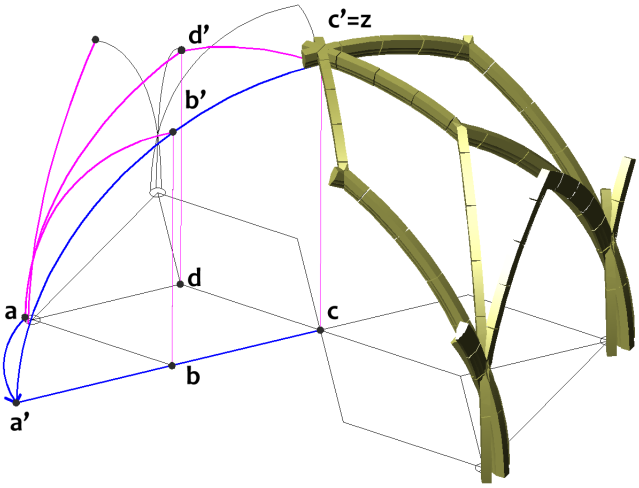


Fig. 3. The steps of drawing the directrix in case of the simple stellar vault

In case of the mentioned rib-system this means (see Fig. 3) that in the drawing of the plan abc and adc distances are equal, the a’c distance can be used as the radius of all ribs – which also means that the a’z arc can be used as the control curve for the entire vault. In this way the height of the crowning (z) and the location of the rib nodes (b’, d’) can be constructed. (This is only the minimum radius, though – obviously – a larger radius can also be used.)

4 Thrust line

“Ut Pendet Continuum Flexile, Sic Stabit Contiguum Rigidum Inversum” (Hooke).

Meaning: “As hangs the flexible line, so but inverted will stand the rigid arch”[3]. Thus the hanging chain and the masonry arch constitute essentially the same static problem.

The thrust line is the compressive forces’ pathway in the structure. So the chain – described by Hooke - loaded only by its own weight represents the ideal arch shape, the thrust line of which corresponds with the median line of the arch. In this sense if the arch is high enough and the thrust line affected by the load does fits within the arch, the load can be increased no matter how large it is, collapsing only in case of the displacement of the supports or when the strength of the cross sections is exceeded [4].

For a specific load the possible number of thrust lines is infinite, since the masonry arch is a three times indeterminate structure. Failure takes place if the thrust line touches the edges of the cross-section at four or – in a symmetrical case – at five places. Failure means that the arch becomes a mechanism due to the cracks developed at the tangent points of the thrust line, i.e. the crack is so large that the cross-section has no resistance against rotation. Three tangent points do not mean failure for the arch however, since the established three hinged arch is a statically determinate structure.

The arch is said to be stable if for the specific load at least one thrust line can be found inside the cross-section along the whole rib.

5 Geometrical factor of safety

If the cross-section of the rib is high enough, it is able to take up stress from multiple loads and the movements of supports without the thrust line exceeding the cross-section. This “surplus” height of the cross-section defines the geometrical factor of safety [4]. The geometrical factor of safety is the ratio of the actual cross section height to the minimum height of the cross-section necessitated for the thrust line. The minimum rib height could be determined by intercepting the thrust lines with lines parallel to intrados and extrados.

If the thrust line is at every point in the middle-third of a rectangular cross-section, which is inside the core of the cross-section, then the rib has a geometrical factor of safety equal to three. In this case the rib is in an elastic stress state. If the geometrical factor of safety of a rectangular cross-section is two, then the thrust line runs in the middle half section of the rib and it is in a plastic stress state. Heyman suggests that for arches or ribs the geometrical factor of safety should be greater than two, because in this case the safety factor is high enough to offset the architectural inaccuracies and thrust line changes caused by smaller support movements. With such conditions, the rib with a rectangular cross-section could be calculated considering the plastic stress state.

The geometrical factor of safety is not sufficient in itself to check the safety of the rib, since in case of an arch the shape of which is defined by the thrust line to a given load, the geometrical factor of safety is equal to infinity.

However, the stress of the ribs mentioned in this article is caused by eccentric compression, small enough to withhold the material of the rib, thus their safety can be demonstrated by examining only the geometrical factor of safety.

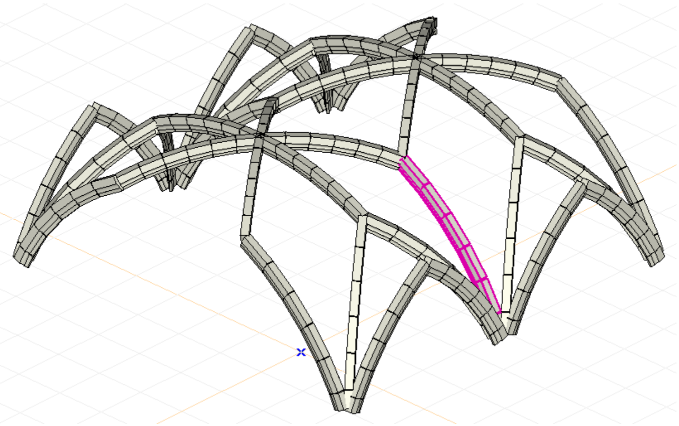


Fig. 4. The examined rib of the simple stellar vault

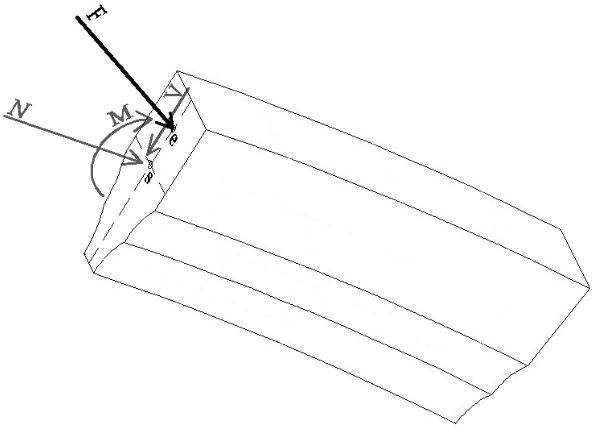


Fig. 5. Internal forces acting on a cross-section

6 Calculating masonry and masonry arches

The behaviour of the masonry structure can be examined in the light of three assumptions [5]:

i masonry has no tensile strength,

ii masonry has an infinite compressive stress, i.e. stresses are so low that there is no possibility of the masonry material failing,

iii sliding failure does not occur.

Individual stone blocks of the rib may be strong in tension, but mortar between stones is indeed weak. An attempt to impose tensile forces would pull the work apart.

The assumption of unlimited compressive strength of the material will be approximately correct if average stresses are in question. Basically it means that the failure of the masonry material caused by pressure is highly unlikely.

Finally, there is sometimes evidence of slip between individual stones or bricks. However the masonry structure generally retains its shape remarkably well; evidently a very small compressive stress is all that is required to avoid the danger of slip and general loss of cohesion. One should take into account the slip between individual stones as a possible failure mode. The structure can be checked against that knowing the compressive force along the thrust line and the friction factor between the elements.

The three assumptions given above are in fact those required to apply the lower bound theorem of plasticity [6] for masonry structures. The lower bound theorem of plasticity claims that any statically admissible load intensity is lower than or equal to the load-intensity causing failure. Considering this, it may be said that if at least one thrust line can be found inside the arch height for a specific load, then the arch is safe. The thrust line is not necessarily the actual thrust line, but if it is a statically possible one, then there is complete evidence that the structure is safe. Thus it may be claimed, that the arch can carry the specific load, since its load-bearing capacity is equal to or larger than the specific load.

Due to the movement of the supports and the drying of the mortar, the arch will crack and the shape of the thrust line will also be altered, while reaching equilibrium. However, the new thrust line cannot get out of the arch and the arch cannot become a mechanism.

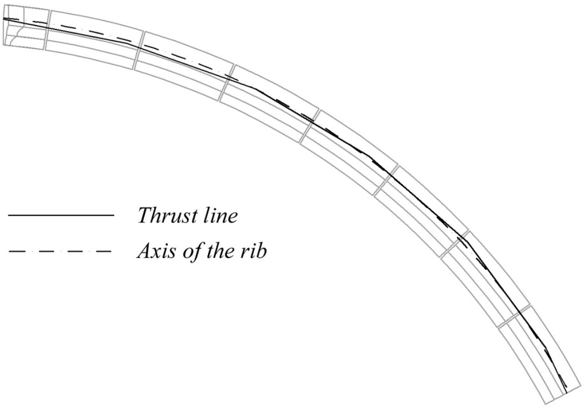


Fig. 6. The thrust line of a given rib

7 Calculating the rib systems

The spatial calculation model makes it possible to define exactly the load acting on the ribs as a consequence of the dead load of the vault. The aim of the calculation is to find a thrust line in the arches of the ribs, which corresponds to the loads.

The vault surfaces were considered as thin shells and defined as high density but low stiffness material [7]. Thus the model could determine the loads transferred from the vault to the ribs correctly.

Knowing the loads on the rib, the compressive force, the bending moment and the shearing force acting in the ribs can be determined (Fig. 5) e.g. by simple elastic analysis. From these internal forces the value and the direction of the resultant, eccentric force acting on the cross-section can be found. The vector of the resultant force defines the location of the thrust line and its tangent for a given cross-section. The eccentricity of the resultant force provides the point of action of the force, which itself defines the coordinate of the thrust line for a specific

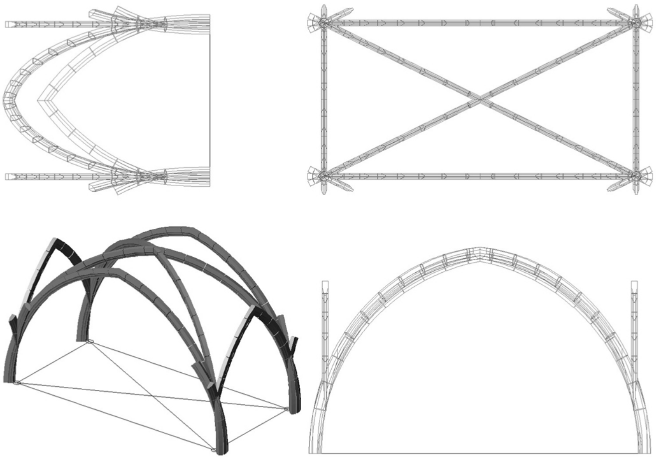


Fig. 7. Quadripartite vault

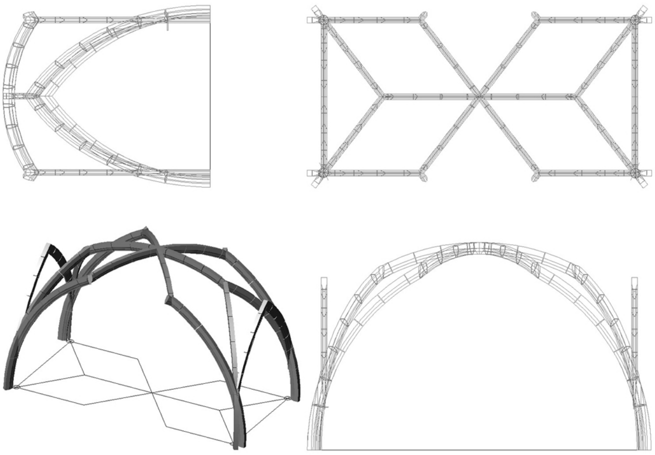


Fig. 8. Simple stellar vault

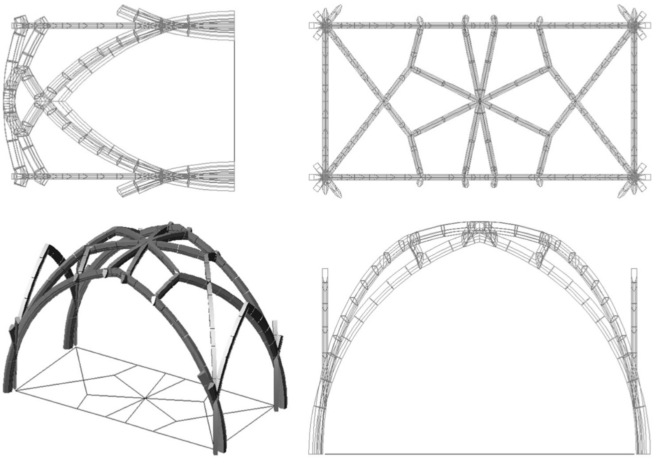
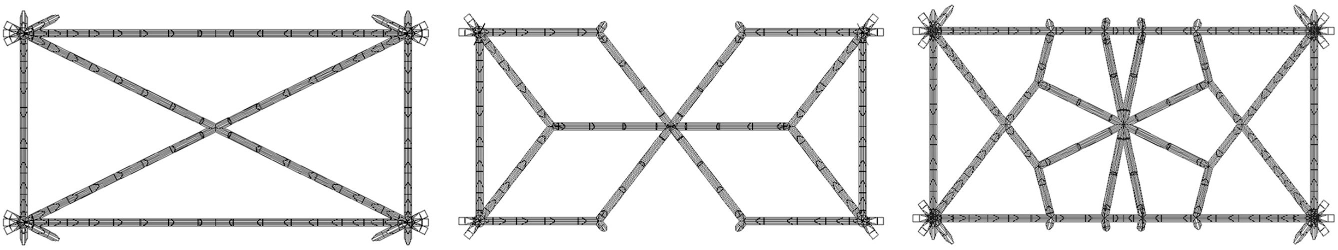


Fig. 9. More complex stellar vault



Quadripartite vault γ_{geom}=4,6

Simple stellar vault γ_{geom}=1,9

More complex stellar vault γ_{geom}=2,5

Fig. 10. Geometrical factor of safety for the examined vaults

cross-section. If one connects the points of action for several cross-sections of the rib being distributed quite densely along the rib axis, it would approximate to the thrust line sufficiently well (Fig. 6).

8 The examined rib-systems

Besides the historically most probable structure (Fig. 8), two other rib systems – one with a simpler and one with a more complex rib system – were also examined (Fig. 7, Fig. 9).

The geometry of each of the vaults was determined using the rules of control curve construction. However having different ground plans the spatial models also have different features.

i The first model was a quadripartite vault whose pointed arches were constructed with the radius derived from the rise and span of its (imaginary) diagonal arch [8] (Fig. 7).

ii The second model was a stellar net vault whose geometry was obtained by adapting the control curve construction method to the vault plan suggested by Tamás Guzsik (Fig. 8).

iii The third model was a more complex stellar net vault. Since in this case the route following the horizontal projections of the ribs from springer to springer was longer, the radius of the ribs became longer, eventuating in a higher crown (Fig. 9).

9 Geometrical factor of safety of the rib

Using the above described method, one may calculate the highest eccentricity of the thrust lines in the rib, and the geometrical factor of safety can be determined.

From the calculated values a surprising result is obtained, since that vault has the lowest geometrical factor of safety, which was supposedly applied at the church of Vázsony (Fig. 10). It is also surprising, that the stellar vault with the more complex rib system – which is thus relatively heavier, but has more primary load bearing elements – has a lower geometrical factor of safety than the rib of the pointed groin vault.

The smallest geometrical factor of safety of the rib considering elastic stress state and using the core of the cross-section is 30/8.85= 3.3 (Fig. 11). Thus it can be said, that in case of the pointed groin vault the calculated result is not only possible, but also gives a compatible result, i.e. the thrust line is inside the core of the section, so the elastic analysis is realistic. That is, cracks do not occur and in case of using the right constitutive law, the result is correct even concerning the deflection. Moreover the applied load could be increased up to the failure of the material of the cross-section.

In a historically expressive way: this structure could be spanning the nave of the church for ever.

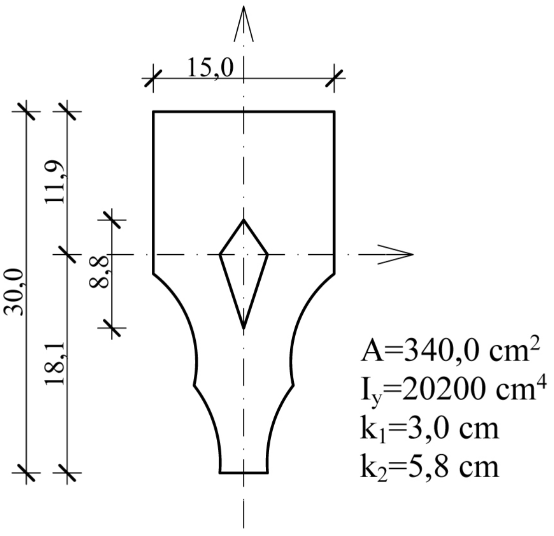


Fig. 11. The geometry of the applied cross-section of the rib

10 Conclusions

The method used for the research results in a possible solution calculated by an elastic finite element model of the arches. The thrust line determined from the calculated internal forces defines a statically possible solution, which also proves the suitability of the rib.

With the aid of the presented method, the geometrical factor of safety of formerly and currently built arches can be estimated, which often says more about the building than the safety condition of the elements. This way of thinking could provide a serious basis to classify current structures according to their safety, and during the theoretical reconstruction of a non existing structure; the method could be taken into account as another control to the decision.

While calculating the vaults, we got the surprising result that the most likely built rib system proved to be the least safe structure, whereas the more complex stellar vault has almost a one and a half times higher factor of safety. At the same time it is another surprising result that the geometrical factor of safety of the groin vault, built with the smallest number of ribs and therefore being affected by the largest internal forces, is well above the satisfactory level claimed by Heyman [5].

The conclusion of this specific examination is that the more complex vaults may have created difficulties, both from geometrical and constructional aspects, although at the same time seem to be a daring solution concerning stability. In addition, with small changes to the geometry of the vaults (e.g.: crowning height and rib radius), the safest rib system of a specific vault can be found. This could lead us to a deeper understanding of the Gothic style.

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