# A MATHEMATICAL MODEL OF THE CORRUGATED PLATES PACKING OIL-WATER SEPARATOR

Lian-Kui WEN and Yuan-Yang ZHENG

University of Petroleum, China

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### Abstract

A new high-efficiency oil-water separator was developed by the authors and their coworkers [1]. The device appears like a horizontal container. Except for the parts of intake and outlet for water and the oil collecting chambers, the main body of this device is the separation chamber, in which the inclined corrugated plates are used as the separation medium.

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A new high-efficiency oil-water separator was developed by the authors and their co-workers [1]. Its structure is shown in *Fig. 1*. The device appears like a horizontal container. Except for the parts of intake and outlet for water and the oil collecting chambers, the main body of this device is the separation chamber, in which the inclined corrugated plates are used as the separation medium. Such oil-water separator has the following advantages:



Fig. 1. An illustration of the oil-water separator.

1 - intake pipe; 2 - intake chamber; 3 - oil collecting chamber; 4 - vertical plates section; 5 - horizontal plates section; 6 - corrugated plates; 7 - case body;
8 - grid; 9 - oil outlet pipe; 10 - water outlet chamber; 11 - water outlet pipe

1. As the inclined corrugated plates are placed one over the other, obversely and reversely by alternation, this oil-water separator not only can increase the oil removal efficiency with the narrowest possible plates gap, but is also easy to install and repair.

- 2. Because of the special passageway formed by the corrugated plates, more chances are thus provided for the collision and coagulation among the oil droplets and for the adhesion and coalescence between the oil droplets and the corrugated plates. This has, therefore, effectively increased the efficiency of oil removal.
- 3. Because of the corrugated plates with lower corrugation height, the equivalent diameter is small, so the state of laminar flow can still be maintained while a larger flow can be treated. Also, because of the relatively even distribution of liquid flow in the plates packing, bad effects of short circuit and stagnant zone can thus be avoided.
- 4. By installing a section of vertically placed corrugated plates at the intake end, it is not only beneficial to the distribution of liquid flow but also be of some help toward removing suspended solids.
- 5. The corrugated plates can be made of metal, plastic or glass fiber reinforced plastic, especially the kind of plastic or glass fiber plates whose surface is hydrophobic and conducive to the adhesion and coalescence of oil droplets towards the surface of corrugated plates.

Laboratory and commercial data show that the device can attain over 90% of oil-separating efficiency within a stay time of 30 minutes, so that oil droplets of 20  $\mu$ m upwards can be separated in the main.

For purpose of establishing the method of design, it is necessary to develop its mathematical model. The present study deals mainly with the work in this respect.

# Mathematical Model

In the packing, the liquid flow direction makes incessant changes in the three dimensions. The actual flow path is longer than the packing sections's total length L; the cross-section of flow is also making changes. The velocity also undergoes changes. As regards the oil droplets, moving with the water flow in the corrugated plates packing, the bigger oil droplets come up to the surface of the corrugated plates whereas a part of the smaller oil droplets can also, as a result of collision, form into bigger ones thus the oil droplets be separated after being attached to the surface of the corrugated plates. Therefore, the liquid flow and the distribution of the oil droplet sizes in the packing also make continuous changes.

It would be fairly difficult to accurately reflect the above situation in the mathematical model. For the sake of treatment, some simplified assumptions are made as follows: 1. The liquid flow in the packing is represented equivalently that the shape of the flow cross-section is a rectangle, its height being equal to the corrugation height h of the corrugated plates, and its length equals the total length L of the corrugated plates packing as shown in Fig. 2. The velocity distribution of liquid is even.



Fig. 2. The equivalent liquid flow

- 2. Use the Sauter's mean diameter  $D_{32}$  to represent the oil droplets.
- 3. As a result of adopting equivalent flow and mean grain size, a situation different from the actual conditions will necessarily arise. This difference ought to be expressed by bringing in a rectifying factor. Since the changes of the liquid and the distribution of the oil droplet sizes are related to the state of liquid flow in the corrugated plates packing, therefore, the factor so introduced should be related to the state of the liquid flow, i.e., the liquid *Re*.

As shown in Fig. 2, adopting a microelement length dz in the liquid flow direction z, the oil droplets of size  $D_{32}$  move forward with the velocity u of liquid flow and float upward at the terminal velocity  $v_t$ . Since the height of liquid layer is h, and the distribution of the oil droplets at the inlet of the microelement length dz is even, then, in the microelement length dz, the separation rate of oil droplets should be  $v_t dz/wh$ . The concentration of oil droplets in water is represented by C and the rectifying factor k is introduced, then, within the microelement length dz, changes in the concentration of oil droplets in water are:

$$-\mathrm{d}C = kC \frac{v_t}{uh} \mathrm{d}z \,. \tag{1}$$

Now conduct integration on the length L of the entire packing and let the concentration of oil droplets in the water at the intake and outlet be  $C_0$  and  $C_e$ , respectively, thus

$$\frac{C_0}{C_e} = \exp\left[k\frac{v_t}{uh}L\right] \,. \tag{2}$$

By the Stokes' equation, the terminal velocity can be expressed, and the oil separation efficiency  $\eta$  is

$$\eta = 1 - \exp\left[-\frac{k(\rho - \rho_0)gD_{32}^2L}{18\mu uh}\right].$$
 (3)

This is the basic mathematical expression of the said oil-water separator.

# Experiment

The experimental device is a horizontal container, inside which there is the corrugated plates packing of the given length. The oil-water mixture containing dispersed oil droplets is under preparation in the agitating tank with crude oil and water tap. Spectrophotometry with a Shimadzu 120-02 UV/VIS spectrometer is used for analysis of oil concentration in water [2]. In order to determine the distribution of the oil droplets, the method of floatation is adopted and then checking by the way of microphotographic counting.

Table 1 shows the structural parameters of the relevant experimental device. The oil concentration of oily water prepared for the experiment is in the range of 50 - 2600 mg/L, the particle size of oil droplets is within  $150 \,\mu\text{m}$ , oil density under  $20^{\circ}\text{C}$  is  $810 - 860 \,\text{kg/m}^3$ , and water temperature for the experiment stays at  $10 - 20^{\circ}\text{C}$ .

Serial No.	Inner diameter mm	Length of corrugated plates packing L, mm	Corrugation height h, mm	Inclined angle of corrugated plate
1	95	450	5	45°
$\overline{2}$	95	300	5	45°
3	95	150	5	45°
4	400	3000	12	45°
5*	400	3000	12	45°

Table 1The structural parameters of experimental devices

\* Treating oily waste water from production at Da Gang Oilfield.

#### **Results of Experiment**

### Distribution of Oil Droplet Diameters

By omitting the truncation factor, the distribution of oil droplet size can be expressed as

$$f(x_i) = \frac{1}{\sqrt{2\pi}x_i \ln \sigma_g} \exp\left[-\frac{\ln \frac{x_i}{x_g}}{\sqrt{2}\ln \sigma_g}\right].$$
 (4)

Using the data of the distribution of oil droplets, the corresponding geometrical mean value  $x_g$  and geometrical standard deviation  $\sigma_g$  can be obtained, and by the following equation, the corresponding Sauter diameter  $D_{32}$  is obtained [3]:

$$\ln D_{32} = \ln x_g - 0.5(\ln \sigma_g).$$
(5)

# Correlation of Rectifying Factor

The correlation between the rectifying factor k and the Re is

$$k = ARe^{b}, \qquad Re = \frac{d_e up}{\mu}.$$
 (6)

Calculating on the basis of non-linear least-square estimation by Newton's iterative method, within the range of 2.96 - 81.68 of Re, A = 0.0453 and b = 0.701, their correlation coefficient will be R = 0.994.

Comparing the model prediction with the experimental data, the relative errors are within the range of 5%. It shows that the mathematical model and the corresponding equations are in good agreement with the actual situation.

#### References

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# Symbols

A	constant ()
<u>ь</u>	constant(-)
C	oil concentration $(mg/L)$
$D_{32}$	Sauter diameter (m)
$d_e^{}$	equivalent diameter of packing (m)
h	height of corrugated plate (m)
k	rectifying factor ()
L	length of packing (m)
u	velocity water flow (m/s)
$v_t$	terminal velocity of oil droplet (m/s)
$x_i$	diameter of oil droplet (m)
$x_a$	geometrical mean diameter of oil droplets (m)
Ž	coordinate of liquid flow direction (m)
$\sigma_a$	geometrical standard deviation (—)
μ	viscosity of water (Pa·s)
ρ	density of water $(kg/m^3)$
$\rho_0$	density of oil (kg/m <sup>3</sup> )