PERIODICA POLYTECHNICA SER. CHEM. ENG. VOL. 38, NOS. 1-2, PP. 159-164 (1994)

OBSERVATIONS ON SOME MULTI-LIQUID LAYER PROBLEMS IN CONVECTION

Alex X. ZHAO[1] and Ranga NARAYANAN[2]

[1]Department of Chemical Engineering
University of Florida
Gainesville, Fl 32611, USA
[2] Department of Chemical Engineering
University of Florida
Gainesville, Fl 32611, USA

Received: Sept. 9, 1994

Abstract

The main result from our calculation is that the solid thickness of the lower phase does not affect the flow structure at '1 g' but does affect the flow at low gravity levels. In other words the coupling is only one-way at high gravity. We surmise that even though the solid thickness destabilizes the flow, it is insignificant compared to the overwhelming stabilization of gravity. As a result of our calculations it appears that liquid encapsulated crystal growth is better conducted under earth's gravity conditions as the configuration would be more stable.

In summary, we may state that the specific mode into which the convective state settles largely depends on the type of system that is studied but we can predict the sequential change in modes as certain key parameters such as gravity level, total depths and depth ratios are changed.

Keywords: multi-liquid layer.

Introduction

This note is concerned with some unusual problems in the study of convection in liquid bilayers. Imagine that we have two immiscible liquids superposed on each other representing a bilayer with a common interface. As a temperature gradient is applied across the interface, there are basically two mechanisms, which can generate convection viz: buoyancy and interfacial tension gradients. These two mechanisms are called Rayleigh and Marangoni effects, respectively. In a model problem we could apply a temperature gradient that is antiparallel to the gravitational field and the configuration represents an instability problem, which is associated with a bifurcation from the quiescent state to the convective state. To get the sufficient conditions for obtaining a convective state, we apply a linear stability theory, i. e., inspect the stability of the quiescent state to infinitesimal disturbances. The linear stability analyses determine the wavelength of an initially infinitesimal disturbance and the associated critical temperature gradient. The results thus obtained can then either be considered as the limiting case of a cylinder with extremely large radius or, if we fix the wave number, can represent the case of a cylinder with vertical sidewalls of vanishing vorticity.

We can understand the physical mechanisms which are involved in interfacial tension gradient convection, by considering a bilayer configuration as shown in *Fig. 1*. Let $T_1 > T_m$ and further let us pretend that gravitational effects are negligible. Suppose as a result of the perturbation the temperature at the point a is higher than at b. As most fluid bilayers have a negative interfacial tension gradient, the interfacial tension at a will be lower than at b and fluid is driven from a to b. Fluid from both phases must then rush towards a. If we have a liquid–gas system where the upper gas phase is assumed to be passive then only liquid from below will move towards a and it follows that unless the temperature gradient were reversed, the perturbations must decay.

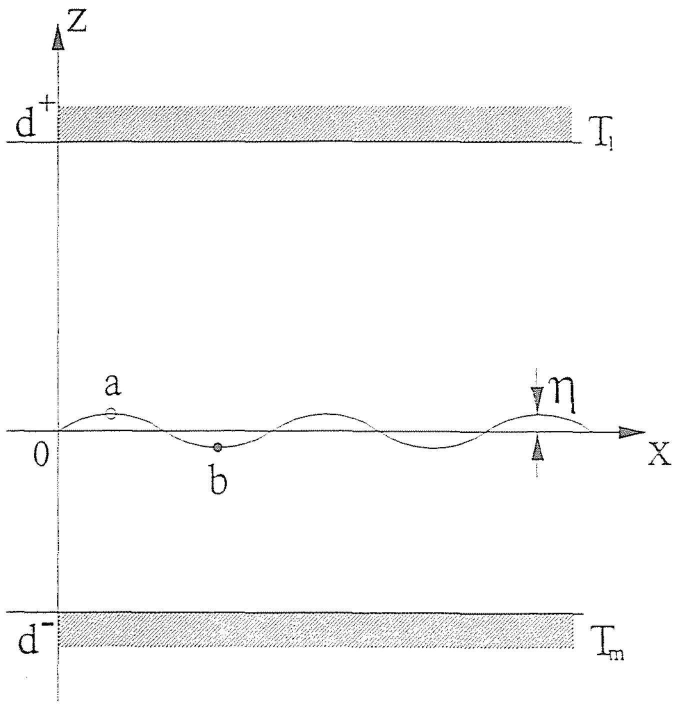

Fig. 1. Schematic of the bilayer system

Theoretical Development

The model that we analyze is schematically shown in *Fig. 1*. The governing equations are derived in a manner identical to that of FERM and WOLLKIND (1982). Several important dimensionless groups will arise. These are the Rayleigh number (R), Marangoni number (M), Cripsation number (C), the Bond number (G) and Prandtl number (P). The first two groups represent the ratio of forcing effect of convection, i. e. buoyancy and interfacial tension gradients respectively to the damping effects of convection viz: thermal and viscous diffusivities. The Cripsation number relates the viscous and thermal damping effects to interfacial tension while the Bond number relates gravity waves to capillary waves. The Prandtl number gives us the relative importance of momentum to thermal effects.

The governing equations are the usual continuity, momentum and energy equations in both phases with conservation laws at a deflecting interface. The Rayleigh and Marangoni numbers must therefore occur in the domain and interfacial condition, respectively.

A trivial solution to the above problem exists. It is basically the quiescent, conductive state in both bulk phases with the hydrostatic pressure gradient balancing the buoyancy. Linear instability theory is applied whereby infinitesimal disturbances on the trivial state result in an eigenvalue problem and the obtained temperature difference or critical R, M pair is ascertained for the onset of steady convection. The search for oscillatory solutions is precluded in this limited study.

It is clear that there are 4 possible 'flow modes' or 'scenarios' and these words will be used interchangeably. These are depicted in *Fig. 2* and assigned Roman numerals. These flow models indicate hot or cold fluid rising into troughs or crests. The numerical results will center on these flow scenarios and the sequence of flow mode changes as we change operating parameters.

The Numerical Results

a) Liquid-Liquid Systems

In this preliminary study, we consider only the case of a liquid bilayer that is '*heat from above*' because this is of application to liquid encapsulated crystal growth. We predict a sequence from mode II to mode I as we increase gravity. The reasoning is as follows. '*Heating from above*' can only cause convection that is started by a Marangoni influence. Now, this flow must necessarily be of either mode I or mode II in nature because fluid has to flow from hot regions towards the cold spots at the interface in a Marangoni dominant regime. However, gravity serves the purpose of stabilization and therefore delays the instability. If there is a mode change on a lowering of gravity then the flow must be driven into mode II because the hot region at the interface prefers to move towards the hot plate which is now above and this arrangement is more unstable than mode I. However, as gravity is increased we ought to see mode I because it is easier for the fluid to push a shorter column of fluid against gravity in both phases in mode I than in mode II.

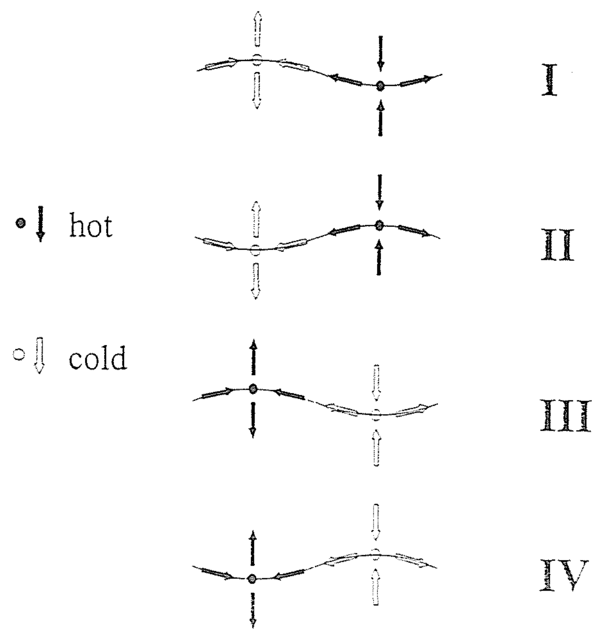

Fig. 2. Four flow modes of bilayer convection

Likewise for the '*heated from above*' case the mode switching goes from mode I to mode II as we decrease total depth. If we again consider gravity and total depth to remain constant but vary l (the ratio of the upper fluid depth to the lower fluid depth) for the '*heated from above*' case, we see that we go from mode II to mode I as l is increased. Remember here that the **onset** is necessarily Marangoni driven and so it is not possible to get mode III. Moreover, buoyancy stabilizes the flow in both phases but an increase in l has the effect of making the upper phase more stabilizing than the bottom one. That is, the upper phase offers more 'buoyancy' resistance as l is increased and it is easier for the fluid to get into mode I because a shorter column of fluid is pushed against gravity in mode I compared to mode II. If l is decreased, the bottom phase exerts more 'buoyancy' resistance, i.e. the Rayleigh effect causes more stabilization in the lower phase but the interface is also now closer to the upper hot plate; the Marangoni effect comes into the fore by bumping the hot part of the interface towards the upper plate. This interfacial instability meanwhile helps the fluid settle into mode II and helps it push a taller column of liquid against gravity. We did calculations for a number of bilayer liquid systems and verified our conjectures.

b) Solidifying Phase Below a Liquid-Liquid Bilayer

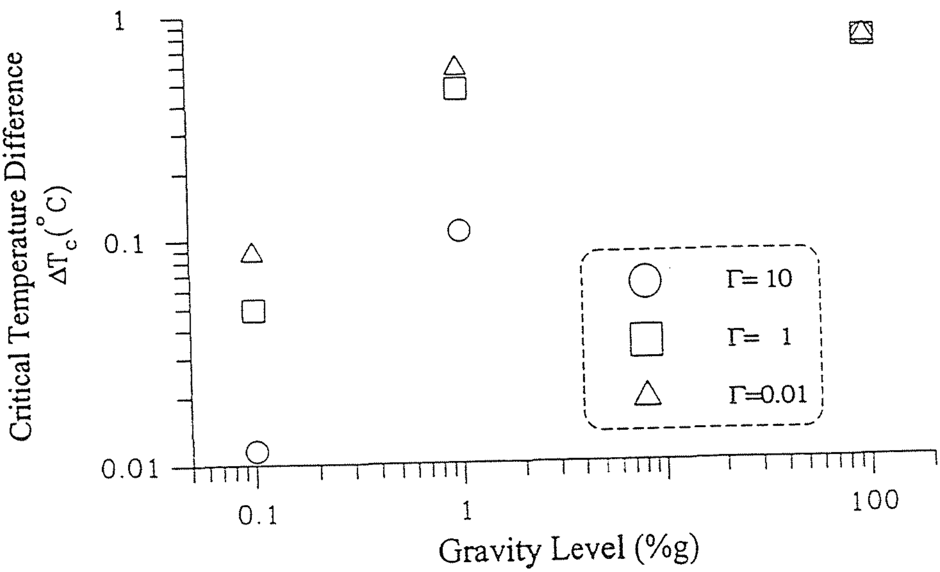

Fig. 3. Onset temperature difference versus Γ for different gravity levels, where Γ is the dimensionless thickness of the solidifying phase scaled with respect to the lower liquid thickness.

This case involves a solidifying phase below the lower layer of liquid. The condition at the boundary of the lower liquid and the adjacent solidifying phase is consequently replaced by a boundary condition that incorporates the thermal distribution in the solidifying phase. It is also implicitly assumed that the rate of solidification is slow enough to be negligible and so that there is no net flow in the base state.

The main result from our calculation is that the solid thickness of the lower phase does not affect the flow structure at '1 g' but does affect the flow at low gravity levels. In other words the coupling is only one-way at high gravity. We surmise that even though the solid thickness destabilizes the flow, it is insignificant compared to the overwhelming stabilization of gravity. As a result of our calculations it appears that liquid encapsulated crystal growth is better conducted under earth's gravity conditions as the configuration would be more stable. *Fig. 3* represents a summary of these results.

In summary, we may state that the specific mode into which the convective state settles largely depends on the type of system that is studied but we can predict the sequential change in modes as certain key parameters such as gravity level, total depths and depth ratios are changed.

Reference

FERM, E. N. – WOLLKIND, D. J.: Onset of Rayleigh-Bénard-Marangoni Instability: Comparison between Theory and Experiment, *J. Non-Equilib. Thermodyn.* Vol 7(3), pp. 169-189, 1983.