EFFECTS OF IMPACT VELOCITY AND ANGLE ON COLLISION

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Abstract

In this work the effect of the particle impact velocity on the value of the restitution coefficient and the effect of the impact angle on the value of the normal coefficient of collision for four types of spherical particles are examined. It is found with increasing particle velocity its effect on the value of the restitution coefficient decreases and at impact angles exceeding 50°—60°, the impact angle has no effect on the value of the normal coefficient of collision. In a previous paper [1] the effect of particle diameter and the wall thickness on the value of the coefficient of restitution have been reported.

Introduction

The particle in the course of its movement in a pneumatic conveying system is colliding with the pipe and bend walls several times. The collision of the particle is taking place at different impact velocities and impact angles, it is seldom a normal impact. To be able to study the particle movement, the effect of the impact angle and the impact velocity on the collision should be examined.

It is necessary to mention that there are different definitions for the coefficient of restitution. BRAUER [2] and SHELDON [3] used the following:

$$\text{restitution coefficient} = CP_u / CP_e \tag{1}$$

where CP_u and CP_e are the particle velocity after and before collision, respectively.

OKUDA [4], OTTJES [5], SAWATZKI [6], TABOR [7], YAMAMOTO [8] and TSUJI [9] used the

$$\text{restitution coefficient} = VP_u / VP_e \tag{2}$$

where VP_u and VP_e are the normal components of the particle velocity after and before collision, respectively.

If the gravity force is much greater than the drag force, the restitution coefficient can be calculated from

$$\text{Restitution coefficient} = \sqrt{hi/H} \tag{3}$$

where H is the height from which the particle is dropped and hi is the rebound height after collision.

To be able to make distinction between the above ratios, the following definitions will be used in this work

coefficient of collision $e_0 = CP_u/CP_e$
normal coefficient of collision $e_n = VP_u/VP_e$
tangential coefficient of collision $e_t = UP_u/UP_e$
restitution coefficient $e = (CP_u/CP_e)_{\alpha_e} = 90°$

In the case of normal impact $e_n = e = e_0$.

Previous work

The effect of the normal component of the impact velocity on the value of the normal coefficient of collision e_n has been reported by OTTJES [5], TABOR [7], OKUDA [4] and SAWATZKI [6]. As they did not report the value of the impact angle, it is expected that $\alpha_e = 90°$, so $e = e_n$ and $CP_e = VP_e$. OTTJES [5] stated an experimental relation between the restitution coefficient and the impact velocity

$$e = \frac{0.7}{\left(1 + \dfrac{CP_e}{10}\right)} \tag{4}$$

The results of TABOR [7] and SAWATZKI [6] and OKUDA [4] are shown in Fig. 1. According to the results of [4 and 7] the coefficient of restitution decreases rapidly with increasing impact velocity, however with a further increase in the impact velocity, the value of e converged to a definite value. Sawatzki's results are different from the above ones, namely he has found with increasing impact velocity the restitution coefficient definitely decreases.

The impact angle is the angle between the direction of movement of the particle and the target wall. The effect of the impact angle on the normal coefficient of collision has been examined by BRAUER [2] and YAMAMOTO [8]. BRAUER used steel pellets of $d = 6.0$ mm and polymethylmethacrylate (PMMA) surface, (Fig. 2a) to his experiments. Due to the difficulty in measuring the e_n value in the small impact angle range, Brauer assumed that at $\alpha_e = 0°$ the normal coefficient of collision is equal to 1 and extended the nearest measured value to this point. In another series of measurements Brauer

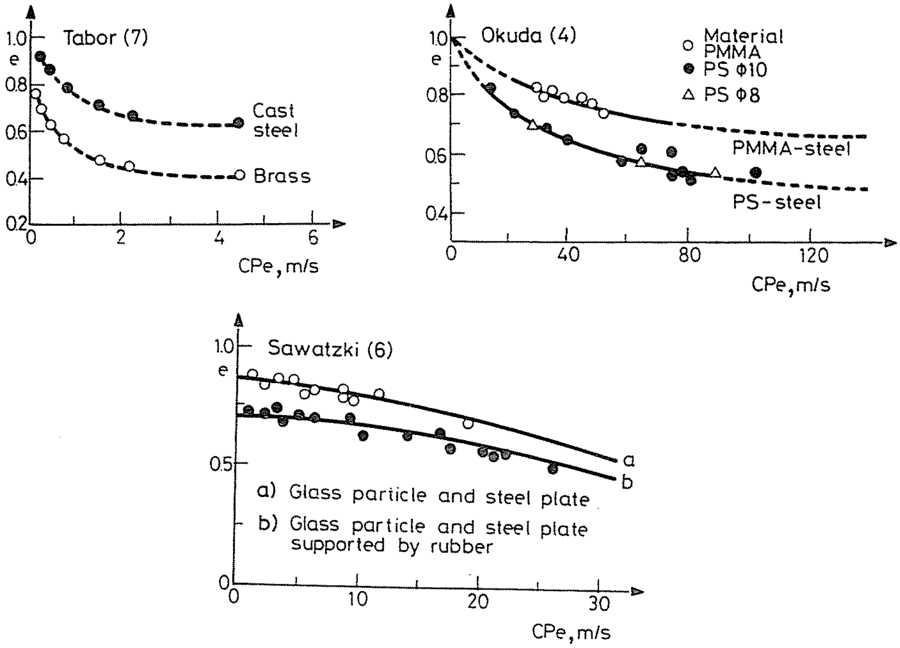

Fig. 1. Relation between the normal component of the impact velocity and *e* value

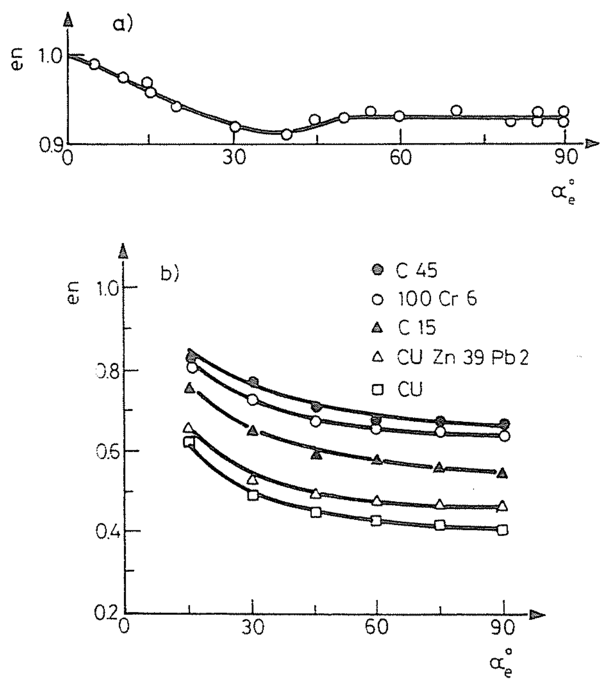

Fig. 2

measured the e_0 value as function of the impact angle α_e and $\Delta\alpha$ using steel particles of $d = 6.0$ mm impacted on 12 different types of surface material. From these measurements and Eq. (5), the relation between the impact angle and e_n, for five types of surface materials is drawn in Fig. 2b.

$$e_n = \frac{e_0}{\sin(\alpha_e)\sqrt{\frac{1}{\tan^2(\alpha_e + \Delta\alpha)} + 1}} \tag{5}$$

This figure shows that the larger the impact angles, the lower is the normal coefficient of collision e_n.

YAMAMOTO [8] reported that for polyethylene particles of 3 mm diameter and $\varphi_p = 1040$ kg/m^3, the e_n value did not change with the impact angle. SHELDON [3] impacted ball bearings of diameter 3.175 mm on aluminium surface. By processing his results Fig. 3 has been drawn, which shows that with increasing impact angle the value of e_0 abruptly decreases.

Effect of impact velocity on the value of the coefficient of restitution:

We dropped polystyrene particles of $d = 7.4$ mm diameter from different heights ($H = .2$, .4, .6, .8, 1.0 and 1.36 m) on a steel plate of 1 mm thickness ($\alpha_e = 90°$), (Fig. 4) and recorded the rebound distance (hi) for every H value. The restitution coefficients for every H value were calculated by two methods:

1— If the drag force acting on the particle is neglected, the impact velocity prior to collision can be calculated by Eq. (6)

$$CP_e = \sqrt{2Hg} \tag{6}$$

The restitution coefficient is calculated using Eq. (3).

2— If the drag force is taken into consideration, the impact velocity can be calculated by using the differential equation of motion Eq. (7), with the initial condition $Y = -H$ and $t = 0$.

$$\frac{dCP}{dt} = g - \frac{3}{4d}\frac{\varphi_a}{\varphi_p}\,\mathrm{CD}\,\mathrm{CP}^2 \tag{7}$$

CD is the drag coefficient calculated by the Kaskas equation $CD = 24/Rep +$ $+ 4/\sqrt{Rep} + .4$ where $Rep = \frac{CPd}{\vartheta}$.

This time the restitution coefficient is calculated using Eq. (2).

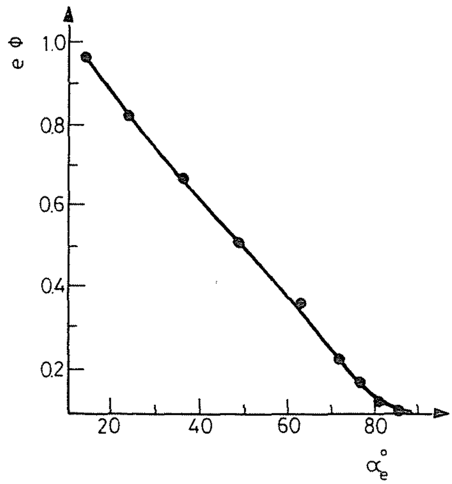

Fig. 3

It was found that the relative error between the two methods R_V increase with increasing H value, in the case of $H=1.36$ m the relative error $R_V=2.16\%$. Since $\alpha_e=90°$, $VP_e=CP_e$.

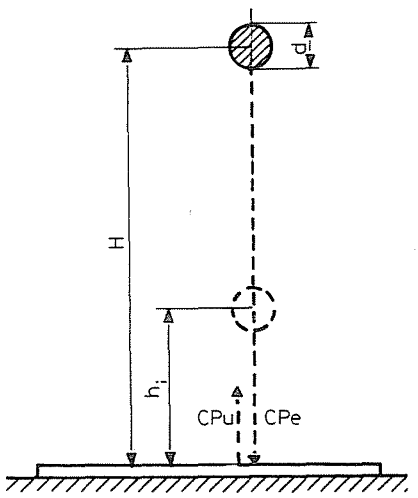

Fig. 4. Determination of the restitution coefficient

Figure 5 shows the relation between the coefficient of restitution e and CP_e for a POL particle of $d=7.4$ mm. From this figure it can be seen that by increasing the particle velocity the value of e decreases, however on increasing the velocity further, the value e converges to a constant value.

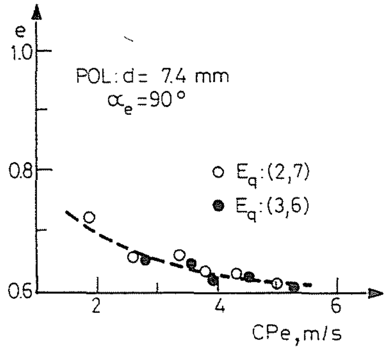

Fig. 5. Effect of the impact velocity on the value of the restitution coefficient

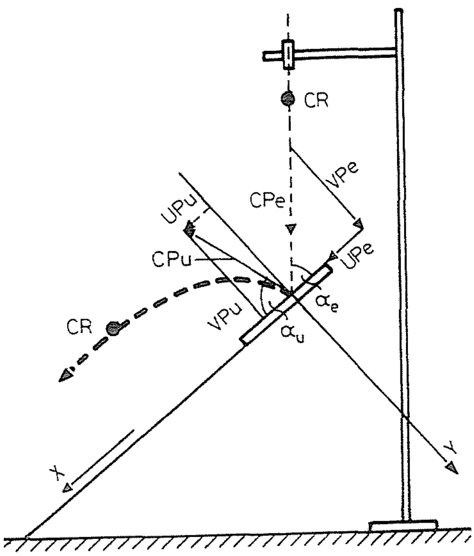

Fig. 6

Effect of impact angle on the value of the normal coefficient of collision:

The experimental work consisted of dropping particles of various diameters and materials (ALO, GLS, FER and POL) from the height $H = .4$ m on an inclined steel plate of 1 mm thickness (Fig. 6). The component VP_e of the particle velocity normal to the plate before collision was calculated by Eq. (8).

$$VP_e = \sin(\alpha_e)\sqrt{2Hg} \tag{8}$$

The normal component of the particle velocity after collision VP_u is calculated from the trajectory of the particle after collision [10]. It was calculated by neglecting the drag force, so the horizontal component of

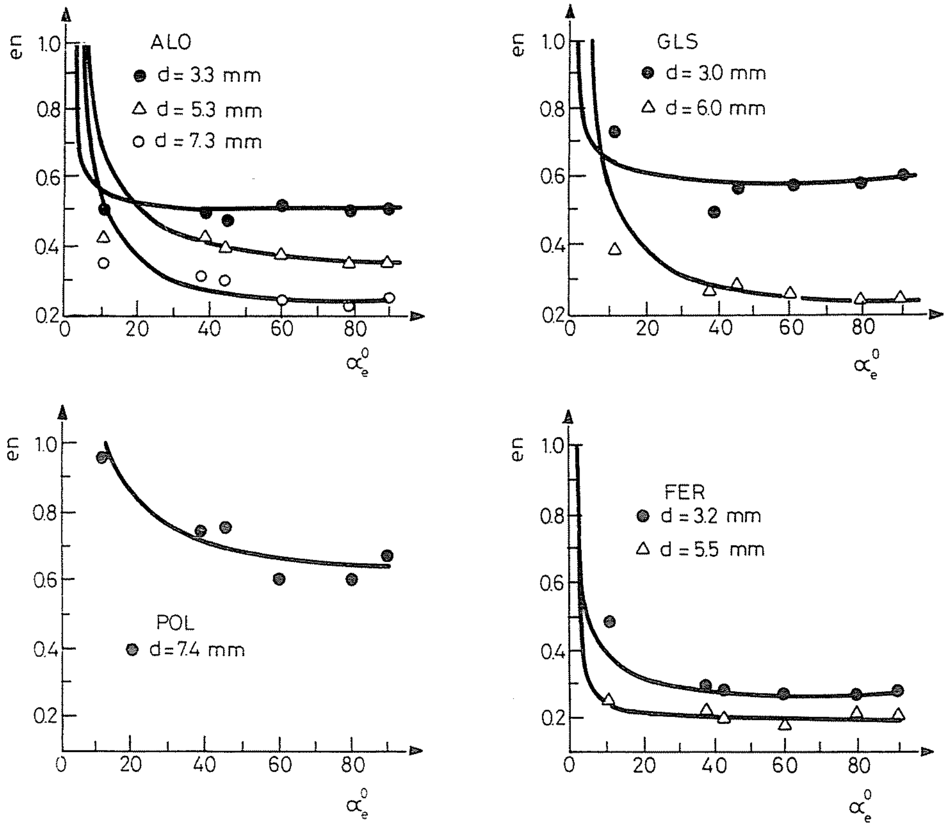

Fig. 7. Effect of the impact angle on the value of the normal coefficient of collision

acceleration is zero, and the acceleration in the normal direction is due only to the weight of the particle. By using 12 points of the trajectory of the particle after collision, the velocity after collision (VP_u) was calculated by the least squares method. In the knowledge of the normal components of the particle velocity before and after collision e_n can be determined. The points shown in Fig. 7 represent the calculated e_n values for different impact angles. The curves plotted in the figure have been calculated by an approximate equation developed in a previous paper [10].

This figure shows that with increasing impact angle e_n decreases, up to angles of 50°—60°. Above that angle value the normal coefficient of collision does not change. The same phenomenon was found for other particle diameters.

In the horizontal conveying system, the impact angles between the particles and the wall were always small. As the effect of the impact angle on the e_n value increases with decreasing angle, it is expected that the effect of the impact angle on e_n is very important in horizontal conveying systems. So it is not allowed to attribute a constant value to e_n in horizontal pneumatic conveying systems.

Nomenclature

CP	: particle velocity (m/sec)
CD	: particle drag coefficient
d	: particle diameter (m)
e	: restitution coefficient $(CP_u/CP_e)_{\alpha_e} = 90°$
e_0	: coefficient of collision CP_u/CP_e
en	: normal coefficient of collision VP_u/VP_e
e_t	: tangential coefficient of collision UP_u/UP_e
g	: acceleration of gravity (m/sec^2)
H	: the height from which the particle is dropped (m)
hi	: rebound height of the particle after collision (m)
Rep	: particle Reynolds number
R_V	: relative error
UP	: component of the particle velocity parallel to the wall (m/sec)
VP	: component of the particle velocity normal to the wall (m/sec)
ϑ	: kinematic viscosity of air (m^2/sec)
φ_a	: density of air (kg/m^3)
φ_p	: density of particle (kg/m^3)
α_e	: impact angle (degree)
α_u	: reflection angle (degree)
$\Delta\alpha$	: $\alpha_e - \alpha_u$ (degree)

Suffix

e	: value before collision
u	: value after collision

Subscripts

ALO	: Aluminium oxide
GLS	: Glass
POL	: Polystyrene
FER	: Fertilizer

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