NEW APROXIMATE EQUATIONS TO ESTIMATE THE DRAG COEFFICIENT OF DIFFERENT PARTICLES OF REGULAR SHAPE

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Abstract

Approximate equations for drag coefficients of different particle shapes are proposed in the form of Kaskas equation [1]. The constants of these equations have been determined by the least square method from experimental values [2, 3]. The constants of these equations are summarized in Table 1. These equations proved to be valid even at Reynolds numbers between 0,006 and 20000, the maximum relative error being 12%.

Introduction

In many cases, to discuss the particle dynamics in a fluid, the relationship between the particle Reynolds number and the drag coefficient is indispensable for the calculation of the particle motion or its trajectories.

For spherical particles extensive and adequate data have been collected and it has been found possible to present these data by empirical Equations [1, 9, 10].

However, for nonspherical particles, although invaluable data have been published, they are presented either in Tables [2, 3] or plotted in Figures [4, 6, 7, 8].

The sphericity employed by Waddel [4] is the most convenient among the several shape factors in use. This is defined as follows; $\Psi = s/S$, where s is the surface area of a sphere of the same volume as the particle; S is the actual surface area of the particle.

The maximum value obtained by this formula is 1, which is the numerical expression for the degree of true sphericity of a sphere.

Previous work

The first significant work was done by Pettyjohn and Christiansen [2], Fig. 1. They presented a correlation which is valid in Re<0.05 and allows evaluation of the effect of particle shape on free-settling velocities for isometric particles.

$$U = K\frac{gd_p^2(\rho_p - \rho_f)}{18\mu} \tag{1}$$

$$K = 0.843 \log \frac{\Psi}{0.065} \qquad (\mathrm{Re} < 0.05) \tag{2}$$

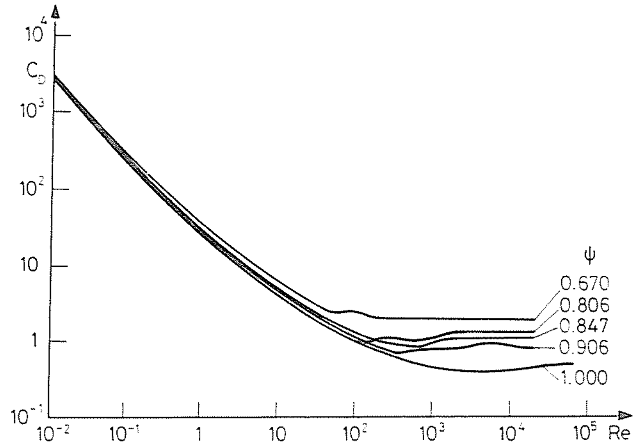

Fig. 1. Drag coefficient C_D Plotted vs Reynolds number Re from Pettyjohn [2]

where K is the Stokes law shape correction factor. Using this terminal velocity and by the Stokes law one could calculate the drag coefficient.

$$C_D = \frac{24\mu}{\rho_p U d_p} \tag{3}$$

Under highly turbulent fluid conditions (Re = 2000 to 200 000) Pettyjohn presented a direct relation between C_D and Ψ

$$C_D = 5.31 - 4.88\Psi.$$

Waddel [4] also showed that in high turbulent regions there exists a linear relation between C_D and Ψ, for sphericity range $(1-0{,}6)$.

As far as we know, Muschelknautz [5] was the first to establish approximate equations in the intermediate range for spheres, Polyhedrons, cylinders, cubes and ellipsoids, with a relative error of 2‰. These equations can be used for a limited range of Reynolds numbers (e. g. for cylinder Re=0.5 to 600).

For computer programming, using an analytical expression for calculating, the drag coefficient is more convenient, and it reduces storage space. The purpose of this article is to develop a set of equations, so as given the Re and shape factor of the particle, the drag coefficient can be predicted.

Correlation development

Our work is based on the experimental works of Pettyjohn [2] and Schmiedel [3].

The data of Pettyjohn were based on experiments on spheres, cube octahedrons, octahedrons, cubes and tetrahedrons (Fig. 1). For which sphericity range were from 1 to 0.67, in the Re number range from 0,006 to 20000.

Schmiedel data [3] were based on experiments on disks of sphericities range from 0.097 to 0.253, we chose just the value of those disks having sphericity of 0.121—0.129 and 0.216 to 0.225 and approximate the first range to 0.125 and the second to 0.22.

Part 1

In the first part of this work equations between the drag coefficient and Re number in Kaskas equation form (4) for certain sphericity degrees ($\Psi=1$; 0.906; 0.847; 0.806; 0.67; 0.22; 0.125) have been developed.

$$C_D=\frac{a}{Re}+\frac{b}{\sqrt{Re}}+c \tag{4}$$

To get satisfactory accuracy, the experimental drag curves for each sphericity degree are divided into two parts (referring to the Re number). Each part of these curves is individually fitted by a Kaskas form Eq. (4) with different constants.

The constants in these equations have been determined by the least square method from experimental values in [2, 3].

Table 1 summarizes these results. For practical computation these constants can be selected from this table according to the sphericity degree and Re number range. The maximum relative error for every sphericity degree and every Re number range is also shown. Relative error is defined as (1—calculated/measured). By equation (4) and Table 1, we plotted in Fig. 2, the relation between the drag coefficient and Re number for the examined sphericity degrees.

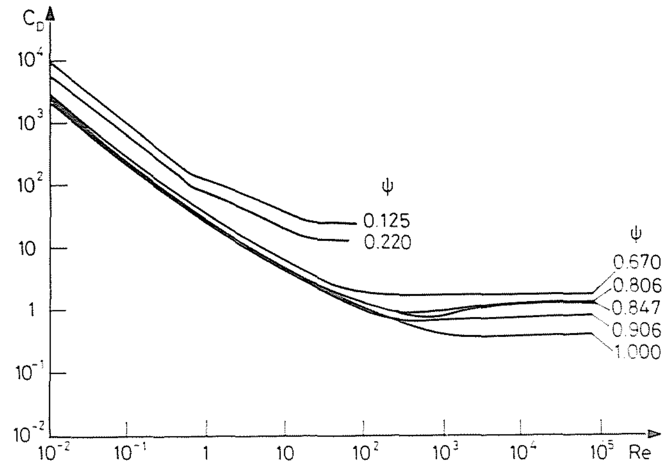

Fig. 2. Drag coefficient vs Reynolds number (calculated by constants in Table 1)

Part 2: The general equation

In order to generalize the validity of our work a direct relation between Re, and C_D developed by applying the Kaskas form equation for the whole Re number range (0.006—20000). The constants a_1, b_1, c_1, for every sphericity degree were found by the least square method (Table 2).

The dependence of these constants on the sphericity degree can be seen in Fig. 3. This Fig. 3 demonstrated the existence of an empirical relation. Therefore it have attempted to derive an equation between each one of these constants and the sphericity degree, by means of a polynomial of fourth degree.

The constant in these equations have been determined also by the least square method.

These equations are as follows.

$$c_1 = 1/(22.265\Psi^4 - 35.241\Psi^3 + 20.365\Psi^2 - 4.131\Psi + 0.304) \tag{5}$$

$$b_1 = -320.757\Psi^4 + 933.336\Psi^3 - 973.461\Psi^2 + 433.488\Psi - 67 \tag{6}$$

$$a_1 = 794.889\Psi^4 - 2294.985\Psi^3 + 2400.77\Psi^2 - 1090.0719\Psi + 211.686 \tag{7}$$

Table 1

Shape	Sphericity degree ψ	Re Range	a	b	c	Max. relative error %	Re Range	a	b	c	Max. relative error %
Sphere	1	0.01 – 152	22.275	5.47	0.312	4	152 – 22 640	145.05	–5.585	0.490	5
Cube octah.	0.906	0.06 – 297	22.861	5.903	0.336	5	297 – 17 410	104.692	–9.804	0.9715	8
Octahedron	0.847	0.008 – 340	24.008	5.883	0.466	4	340 – 17 350	298.412	–29.75	1.59	12
Cube	0.806	0.007 – 101	24.102	6.184	0.365	5	101 – 16 080	179.585	–22.084	1.638	8
Tetrahedron	0.67	0.007 – 123	27.78	4.757	1.293	5	123 – 13 310	36.920	–5.131	2.099	8
Disk	0.22	0.138 – 49.6	65.460	–9.554	14.452	4	–	–	–	–	–
Disk	0.125	0.54 – 73.03	108.662	–26.284	27.165	10	–	–	–	–	–

Table 2

The constants a, b, and c of sphericity degrees

Sphericity degree ψ	a_1	b_1	c_1
1	22.180	5.653	0.279
0.906	23.954	4.419	0.512
0.847	25.150	4.208	0.690
0.806	26.233	3.195	0.772
0.67	29.113	2.505	1.695
0.22	65.46	–9.554	14.452
0.125	108.662	–26.284	27.165

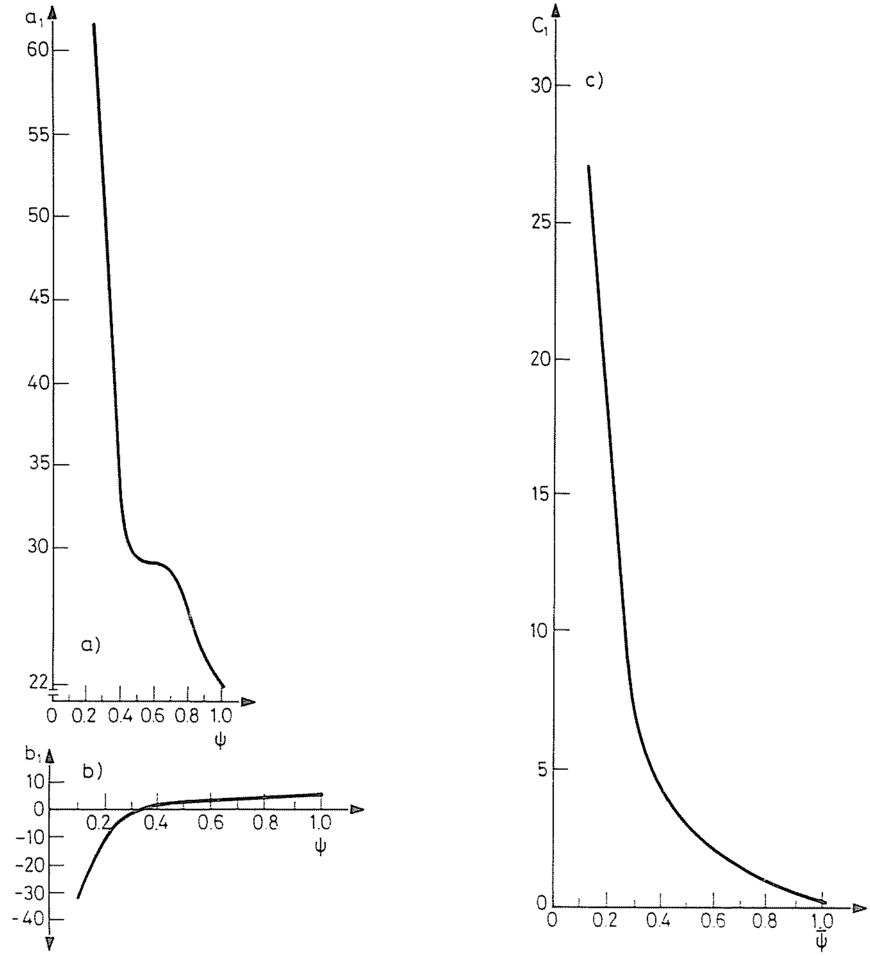

Fig. 3.a. The relation between the Ψ and the constant a_1
Fig. 3.b. The relation between the Ψ and the constant b_1
Fig. 3.c. The relation between the Ψ and the constant c_1

By substituting these equations in Eq. (8) we will be able to predict the C_D for Re number range 0.006—20000 and for any sphericity degree between 1—0.125

$$C_D = \frac{a_1}{Re} + \frac{b_1}{\sqrt{Re}} + c_1 \tag{8}$$

The plot of Eq. (8) for sphericity degree 1; 0.096; 0.847; 0.806; 0.67; 0.22; 0.125) is shown in Fig. 4. The maximum relative error of 20% was found for

Re range up to 4000 and 35% for up to 20000. However, these deviations seem to be very considerable, but these maximum errors were found just at certain points.

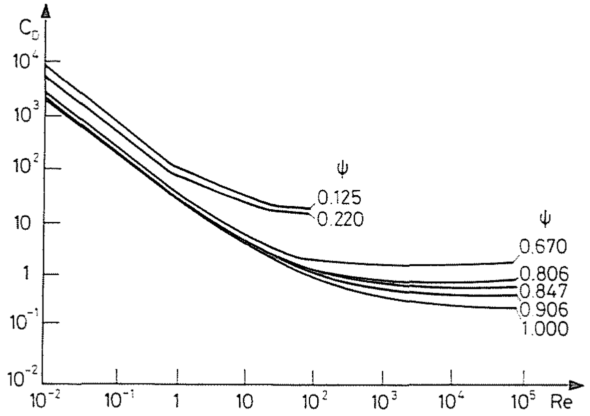

Fig. 4. Drag coefficient vs Re number (calculated by equations 5, 6, 7, 8)

It is clearly shown in both, the experimental and the calculated Figs 1, 2, 4, that regularly shaped particles have higher drag coefficients than spheres of the same volume and that they lie above each other in the order of decreasing sphericity. There is an exception to this in the range Re = 20 to 200 where the curve for octahedrons $\Psi = 0.847$ lies above that for cubes $\Psi = 0.806$ Figs 5, 1, Pettyjohn explains that this is due to orientation or to instability and occasional spinning on the part of the octahedrons which in the case of the cubes, did not occur until $Re \approx 200$.

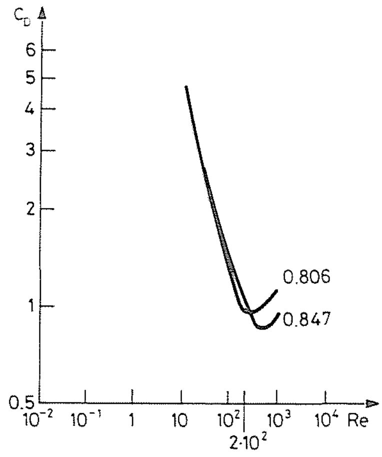

Fig. 5. In Re range of about (20—200) the C_D for sphericity degree 0.847 is larger than that of 0.806 (by data in Table 1)

It is also shown in Figs 1, 2, 4 that the effect of shape increases with increasing Reynolds numbers so that the distance between the curve of spheres and other curves increases with increasing Re values.

Nomenclature

a, b, c	constants in Eq. (4)
a_1, b_1, c_1	constants in Eq. (8)
K	Stokes' law shape correction factor
C_D	drag coefficient
Re	Reynolds number
Ψ	degree of sphericity
s	surface area of a sphere L^2
S	surface area of a solid of any shape L^2
U	particle terminal velocity (L/t)
ρ_f	fluid density M/L^3
ρ_p	particle density M/L^3
d_p	particle diameter L
ν	fluid viscosity M/Lt

References

1. KASKAS, A.: "Berechnung der stationären und instationären Bewegung von Kugeln in rehunden und strömenden Medien" Diplomarbeit dem Lehrstuhl für Thermodynamik und Verfahrenstechnik der T. U. Berlin (1964)
2. PETTYJOHN, E. S.–CHRISTIANSEN, E. B.: "Effect of Particle Shape on Free-settling Rates of Isometric Particles", Chem. Eng. Progr. *44*, 157 (1948)
3. SCHMIEDEL, J.: Physikalische Zeitschrift Bd., *29*, 605 (1928)
4. WADDEL, H.: "The Coefficient of Resistance as a Function of Reynolds number for Solids of Various Shapes", J. Franklin, Inst., *217*, 459 (1934)
5. MUSCHELKNAUTZ, B.: Partikelaerodynamik. VDI-Gesellschaft für Verfahrenstechnik und Chemieingenieurwesen GVC, Technik der Gas / Feststoffströmung-Sichten, Abscheiden, Fördern, Wirbelsichten. Düsseldorf, 7—8. December 1981.
6. LEIGHEN, E. S.: "Elements of Transport Phenomena", McGraw Hill, Japan, p. 720, (1972)
7. GOVIER, G. W.–AZIZ, K.: "The Flow of Complex Mixtures in Pipes" Van Nostrand Reinhold, Canada (1972)
8. BROWN, G. G.: "Unit Operations", John Wiley and Sons., Inc., New York (1951)
9. BOOTHROYED, R. G.: "Flowing Gas-Solids Suspension"., Chapman and hall, Ltd. p. 14 (1971)
10. MORSI, S. A.–ALEXANDER, A. J.: "An Investigation of Particle Trajectories in Two-phase Flow Systems", J. Fluid Mech., 55, 193 (1972)

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A NEW APPROACH TO THE CALCULATION OF THE VARIATION OF THE MIXING LENGTH OVER THE PIPE DIAMETER

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Abstract

Despite the approximate nature of the Prandtl concept of the mixing length, it remains one of the most easy and useful method in the prediction of the velocity distribution. It had been successfully used in the prediction of the velocity distribution in many practical problems (i. e. in pneumatic conveying). Since Prandtl's assumption on the mixing length $L/r = 0{,}14 - 0{,}08(1-y/r)^2 - 0{,}06(1-y/r)^4$ is not suitable for Re number lower than 10^5, a new approach is developed, which is also valid for Re lower than 10^5, for both smooth and rough pipes.

Introduction

Prandtl has presented an empirical Eq. (1), for calculating the mixing length as a function of position along the pipe diameter. This equation is applicable for both smooth and rough pipes, but it is only valid for $Re > 10^5$. The constants of this equation are independent of the Re number.

$$\frac{L}{r} = 0{,}14 - 0{,}08\left(1-\frac{y}{r}\right)^2 - 0{,}06\left(1-\frac{y}{r}\right)^4 \tag{1}$$

Equation (1) shows the two Prandtl's hypotheses, the first being that the mixing length at the wall is zero, and the second, that $L = \alpha y$ is confirmed for small distances from the wall. With $\alpha = 0{,}4$ it can shown that

$$\left(\frac{dL}{dy}\right)_{y=0} = 0{,}4$$

The purpose of this paper is to extend the validity of Eq. (1) for $Re < 10^5$ and to increase the accuracy.

Results and discussion

Our work is based on the experimental work of Nikuradse [1, 2]. Who made an extensive experimental work on measuring the mixing length over the pipe diameter for the smooth pipe [1] and the velocity gradient over the pipe diameter for rough pipe. The measurements were based on a very wide range of Reynolds numbers $4 \cdot 10^3 \leq \mathrm{Re} \leq 3240 \cdot 10^3$.

For smooth pipes

By plotting the $\frac{L}{r}$ value vs $\frac{y}{r}$ for every Reynolds number (Fig. 1.a, 1.b), it is shown that the value of the mixing length depends also on the Re number, especially for Re number ranges below 10^5. From these experimental results [1] and by an Eq. (2) having the same shape as Eq. (1), and by taking into

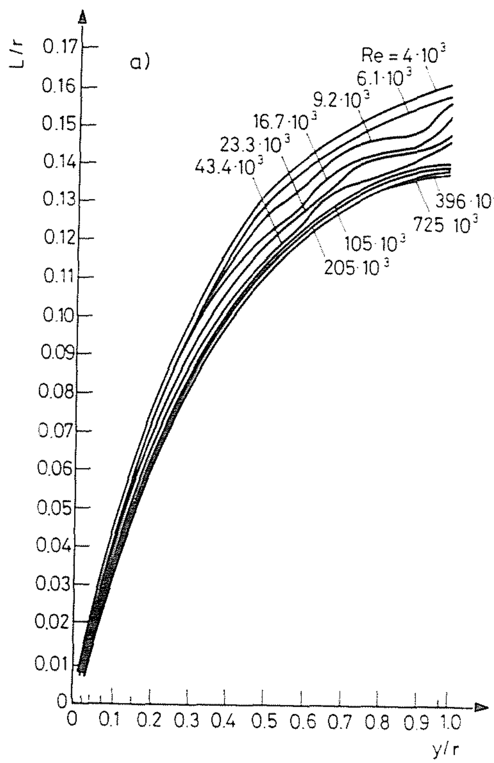

Fig. 1.a. Variation of the measured mixing length [1] over pipe diameter for smooth pipes at different Re numbers

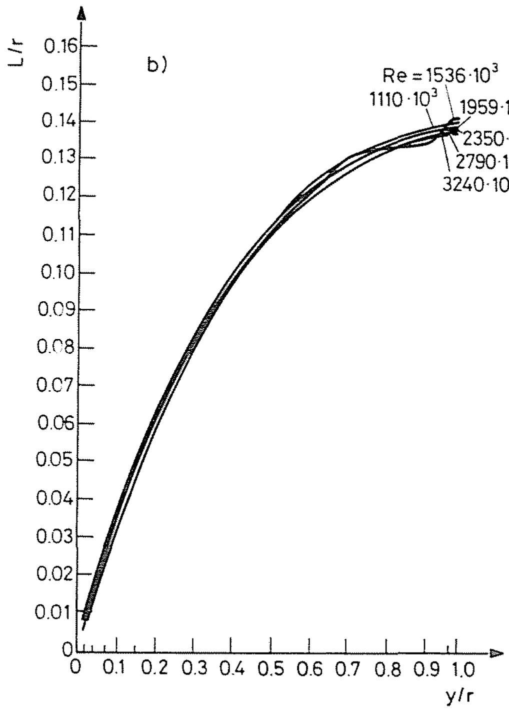

Fig. 1.b. Variation of the measured mixing length over pipe diameter for smooth pipe at different Re numbers

consideration the Prandtl's first hypothesis (3), the C value is found by the least square method for every Re number (Table 1)

$$\frac{L}{r}=C-A\left(1-\frac{y}{r}\right)^2-B\left(1-\frac{y}{r}\right)^4 \tag{2}$$

$$C=A+B \tag{3}$$

From Prandtl's second hypothesis $\left(\frac{dL}{dy}\right)_{y=0}=0,4$ and equation (2)

$$2A+4B=0.4 \tag{4}$$

From Eqs (3) and (4) it follows that

$$A=2C-0.2 \tag{5}$$

$$B=-C+0.2 \tag{6}$$

Table 1

For smooth pipe

	$Re \cdot 10^{-3}$	C
1	4	0.15817
2	6.1	0.15517
3	9.2	0.15147
4	16.7	0.14744
5	23.3	0.14496
6	43.4	0.14259
7	105	0.13997
8	205	0.13878
9	396	0.13792
10	725	0.13685
11	1110	0.13913
12	1536	0.13684
13	1959	0.13760
14	2350	0.13798
15	2790	0.13774
16	3240	0.13847

In calculating the C value by the least square method, Prandtl's second hypothesis (4) has been taken into consideration. This leads to Eq. (2) with only one constant, and by comparing the mixing length calculated by this C value with the experimental result for different Re numbers [1], it did not seem to be a good result. That is the reason why we took into consideration Prandtl's second hypothesis after the least square method. The values of C vs Re numbers (Table 1) are plotted in Fig. 2. It is clearly shown that an

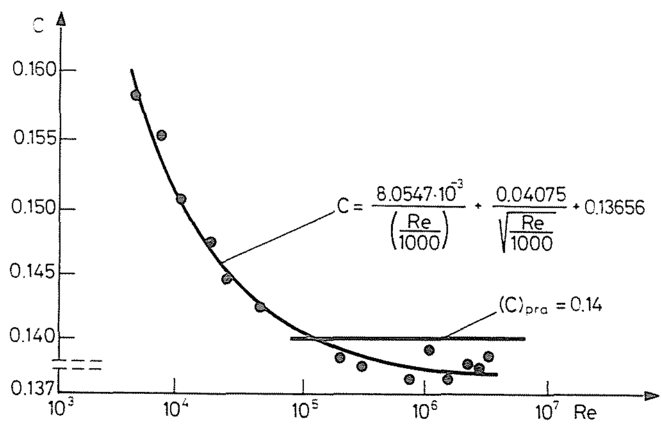

Fig. 2. The relation between C and Re for smooth pipes

empirical relation exists between C and Re values. Thus it was attempted to derive an equation for calculating the C value for any Re number:

$$C = \frac{K_1}{\left(\frac{Re}{1000}\right)} + \frac{K_2}{\sqrt{\left(\frac{Re}{1000}\right)}} + K_3 \tag{7}$$

Constants K_1, K_2 and K_3 have been determined by the least square method. So Eq. (7) becomes

$$C = \frac{8.0547 \cdot 10^{-3}}{\left(\frac{Re}{1000}\right)} + \frac{0.0407}{\sqrt{\left(\frac{Re}{1000}\right)}} + 0.1365 \tag{8}$$

Comparing Prandtl's equation (1), where $C = 0.14$, and the developed approximation method Eqs (2, 5, 6, 8) with the experimental results [1] shows that this approach is not only valid for Re numbers below 10^5, but it also gives a better overall agreement with the experimental data than Eq. (1) (see Table 2 and Fig. 2).

Table 2

$Re \cdot 10^{-3}$	$\sigma \cdot 10^3$ developed approach (2, 5, 6, 8)	$\sigma \cdot 10^3$ equation (1)
4	6.565	–
6.1	6.106	–
9.2	5.916	–
16.1	5.422	–
23.3	4.138	–
43.4	3.259	–
105	2.545	2.555
205	2.350	2.442
396	1.733	2.161
725	1.961	2.792
1110	1.388	1.548
1536	2.225	3.233
1959	1.619	2.541
2350	1.603	2.451
2790	1.955	2.847
3240	2.113	2.744

Rough Pipes

Using the velocity gradient in rough pipes measured by Nikuradse [2], for relative roughnesses of 507; 252; 126; 30.6 and 15, the mixing length along the diameter has been calculated for different Re numbers by Eq. (9)

$$L=\frac{\overset{*}{V}\cdot\sqrt{1-\frac{y}{R}}}{\left(\frac{dv}{dy}\right)} \tag{9}$$

The C values for different relative roughnesses and different Re numbers are calculated by Eq. (2) and by the least square method, and are presented in Table 3.

Comparing Eq. (8) with the above calculated results for different relative roughnesses and Re numbers, it can be seen that Eq. (8) gives in this case a better agreement with experimental results than Eq. (1). (See Table 4 and Fig. 3.)

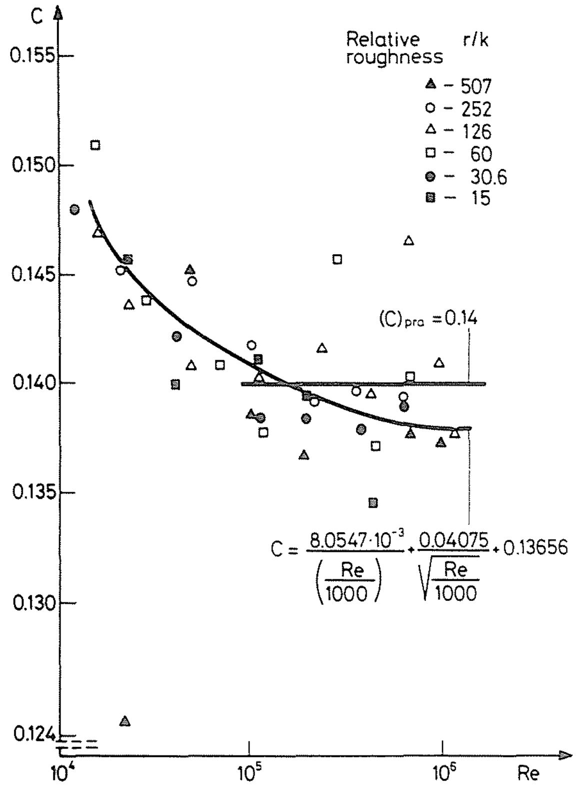

Fig. 3. The relation between C and Re for rough pipes

Table 3
For rough pipe

$Re \cdot 10^{-3}$	C $r/K = 507$	$Re \cdot 10^{-3}$	C $r/K = 252$	$Re \cdot 10^{-3}$	C $r/K = 126$	$Re \cdot 10^{-3}$	C $r/K = 60$	$Re \cdot 10^{-3}$	C $r/K = 30.6$	$Re \cdot 10^{-3}$	C $r/K = 15$
22.7	0.1246	21.4	0.1452	16.7	0.1469	15.3	0.1509	12.1	0.148	11.3	0.1238
49	0.1450	51	0.1446	23.7	0.1435	29.5	0.1438	23	0.1706	22.2	0.1456
106	0.1385	103.5	0.1416	50.5	0.1406	70	0.1409	43	0.1420	43	0.1399
186	0.1367	202	0.1390	112	0.1402	116	0.1378	104	0.1384	108	0.1410
427	0.1343	344	0.1394	231	0.1415	271	0.1455	195	0.1383	197	0.1394
680	0.1377	624	0.1392	417	0.1394	438	0.1369	372	0.1379	430	0.1345
970	0.1373			640	0.1463	677	0.1401	638	0.1387		
				960	0.1407						

Table 4

R/K	$\sigma_{abs(max)}$
507	$5.0068 \cdot 10^{-3}$
252	$4.505 \cdot 10^{-3}$
126	$5.094 \cdot 10^{-3}$
60	$6.413 \cdot 10^{-3}$
30.6	$5.544 \cdot 10^{-3}$
15	0.07468

Nomenclature

L = mixing length
Re = Reynolds number
V = velocity in axial direction
y = distance normal to pipe
r = radius of the pipe axis
$\overset{*}{V}$ = shear-stress velocity
C, A, B = constants in equation (2)
K_1, K_2, K_3 = constant in equation (7)
r/K = relative roughness
σ = standard deviation

References

1. NIKURADSE, J.: Gesetzmassigkeiten der turbulenten Strömung in glatten Rohren. Forsch. Arb. Ing-Wes. No. 356 (1932).
2. NIKURADSE, J.: Strömungsgesetze in rauhen Rohren. Forsch., Arb. Ing-Wes. No. 361 (1933).

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