

# A REVISED REDUCED COMPRESSIBILITY CHART AND FUGACITY DIAGRAM FOR FLUIDS

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## Abstract

Some tables containing reduced compressibility factors are not consistent to all thermodynamic properties of fluids. Permanent gases have an inversion point at low pressure and high temperature where the Joule-Thomson coefficient equals to 0 and the compressibility factor reaches a maximum with increasing temperature at constant pressure. A new compressibility table has been constructed using recent data given for air, ethylene and ammonia. These gases have different critical compressibility factors and represent therefore all types of fluids. Reduced fugacity charts have also been calculated from these data. Diagrams of gases with low critical compressibility factor show quantitative differences.

## Introduction

Many textbooks in Physical Chemistry and manuals for engineers depict charts of reduced compressibility factors. Books from the forties like Chemical Engineering Thermodynamics by B. F. Dodge quote data from Cope et al. [1] and from Brown et al. [2]. Characteristic feature of these diagrams is the maximum in the slope of the starting isotherms at  $T_r = 5$ .

Later, textbooks and manuals like J. H. Perry: Chemical Engineer's Handbook use diagrams published by Watson and Smith [3], Gamson and Watson [4] and by Watson [5]. On the compressibility diagram in paper [3] this maximum in the slope is not clearly observable while the two others do not publish data reaching  $T_r = 5$ . In addition, in the Compressibility Tables, published by Lydersen et al. [6], below  $p_r = 8$  the compressibility factors, passing over a minimum, monotonously increase with temperature at constant pressure until  $T_r = 15$ .

### Joule-Thomson coefficient as a tool for the consistency of thermodynamic data

Joule-Thomson coefficient can be expressed by

$$\mu_{JT} = \frac{RT_c T_r^2}{P_c P_r} \frac{1}{C_{mp}} \left( \frac{\partial Z}{\partial T_r} \right)_{p_r} \quad (1)$$

It means that in inversion points  $(\partial Z/\partial T_r)_{p_r}$  is equal to 0 otherwise its sign is identical to that of  $\mu_{JT}$ . Thus, if the compressibility factor increases until  $T_r = 15$  the Joule-Thomson coefficient is positive in the whole region. Lydersen and coworkers published in the very same work a diagram for Joule-Thomson coefficients displaying an inversion point at  $T_r \approx 3$  and  $p_r = 4$ . Unfortunately they do not follow the inversion curve over  $T_r = 7$ . Nevertheless, the former datum is in clear contradiction to their Table for compressibility factors because at  $p_r = 7$  the compressibility factor has no maximum.

Dr. Ulrich K. Deiters (Ruhr Universität Bochum) exposes that “the condition of constant pressure implies, at least for supercritical temperatures, that an increase in  $T$  has to be accompanied by a decrease in density. For high temperatures and low densities, however, the gas will approach the perfect gas behaviour ( $Z = 1$ ). Therefore  $(\partial Z/\partial T)$  is negative at high temperatures. Since this derivative is known to be positive at low temperatures, the inversion temperature must lie at a finite value between the high temperature domain and the low temperature domain.” [7].

Figure 1 depicts some inversion curves for reduced parameters. Similar curves are depicted as Brown’s “Ideal Curves” among which also the inversion curve is figured in the paper of Angus [8]. The first curve has been drawn using the thermodynamic parameters of air, after the Tables of Baehr and Schwier [9]. The inversion point have been checked by the data of their T-s chart, using the relationship

$$\mu_{JT} = - \frac{V}{T} \left( \frac{\partial T}{\partial s} \right)_h \quad (2)$$

The isoenthalpic curves of air have a maximum until  $t = 390^\circ\text{C}$  ( $T_r = 5.0$  and  $p_r = 0.5$ ). Above  $390^\circ\text{C}$  the isoenthalpic curves monotonously increase, regardless to the pressure, indicating that the Joule-Thomson coefficient is negative. Thus, the compressibility factors, above  $T_r = 5$ , regardless to the pressure, ought to decrease with increasing temperature in sharp contrast with those given in the Tables of Lydersen et al.

The second curve has been constructed from the data of ethylene published by Angus et al. [10]. The curve ends at  $T_r = 1.5$  as the data can be found up to this temperature. The third curve connects the inversion points

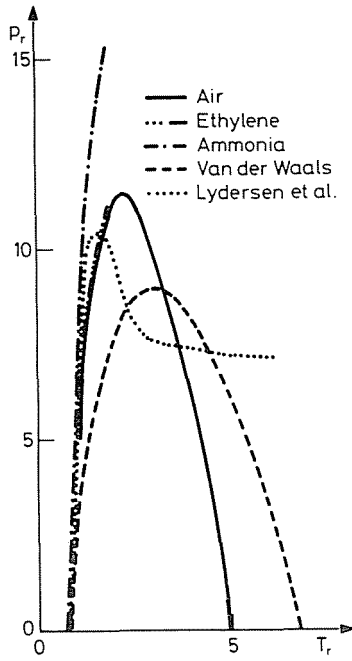


Fig. 1

of ammonia given in reduced parameters and published by Haar and Galagher [11]. The total inversion curve of a van der Waals fluid is also depicted. The corresponding equation derived from van der Waals equation, starting from the relation:

$$\mu_{JT} = \frac{V_c}{C_p} \left[ T_r \left( \frac{\partial V_r}{\partial T_r} \right)_{p_r} - V_r \right] \tag{3}$$

is

$$T_{r,i} = \frac{15 \pm 4\sqrt{9 - p_r} - \frac{p_r}{3}}{4} \tag{4}$$

The points not connected by a curve are placed on the ground of the Tables of Lydersen et al. The strange shape is due to the fact that at  $p_r = 7$  and at lower pressures no maxima are to be found in the compressibility factors.

Also Beattie-Bridgeman equation [12] gives a condition for the high temperature inversion point at low pressure. The sign of the function  $y = 4cR + 2A_0T^2 - B_0RT^3$  is identical to that of the Joule-Thomson coefficient. The function equals to 0 at only one temperature i. e. the equation has only one root. These temperatures are for He: 43.6 K, for  $H_2$  227 K (calculated from the equation valid for low densities) and 231 K resp. (calculated from

the general equation), and for Ne: 254 K. For a van der Waals gas  $T_r=6.75$  while for a van der Waals liquid  $T_r=0.75$  at very low pressures. This low temperature value cannot be obtained from Beattie-Bridgeman equation as it is not valid for liquid state.

### The reduced compressibility table

A reduced compressibility table has been constructed for three gases of various critical compressibility factors. The three types are represented by ammonia [11] ( $Z_c=0.244$ ), by ethylene [10] ( $Z_c=0.278$ ) and by air [9] ( $Z_c=0.316$ ). Between  $T_1=223$  K and  $T_2=450$  K the data of air are given from two different equations the validity region of which are overlapping. In this temperature interval the calculated reduced data have been averaged weighted by  $(T-T_1)/(T_2-T_1)$  for the data given for higher temperatures while the data valid for lower temperatures have been weighted by  $(T_2-T)/(T_2-T_1)$ . Table 1 collects the compressibility factors at different parameters.

Figures 2, 3, 4 are compressibility charts in linear scale in the function of reduced parameters for air, ethylene and ammonia. Figures 5, 6, 7 are depicted in logarithmic scale where the  $45^\circ$  straight lines are also plotted in order to facilitate the determination of the pressure from known volume and temperature. Figures 2—7 have been plotted by computer Commodore PC 10 and Seconic SPC 410 plotter (IBM XT).

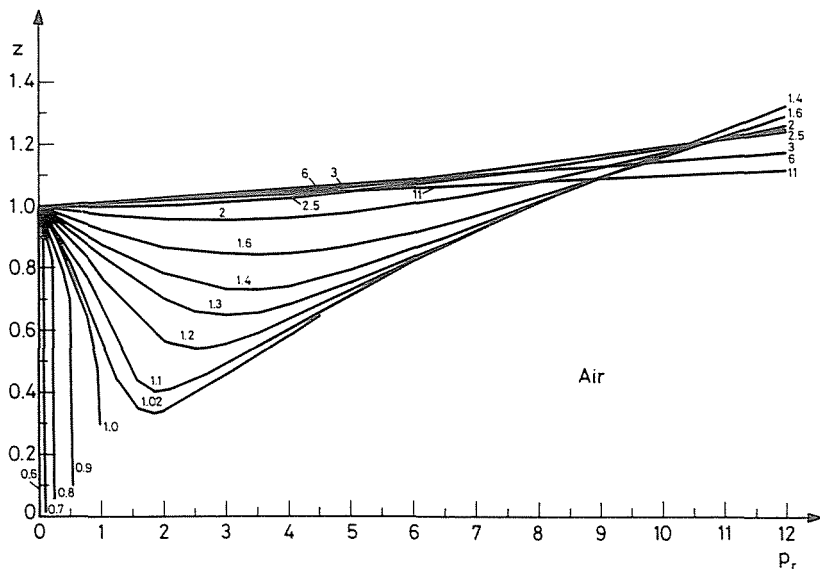


Fig. 2

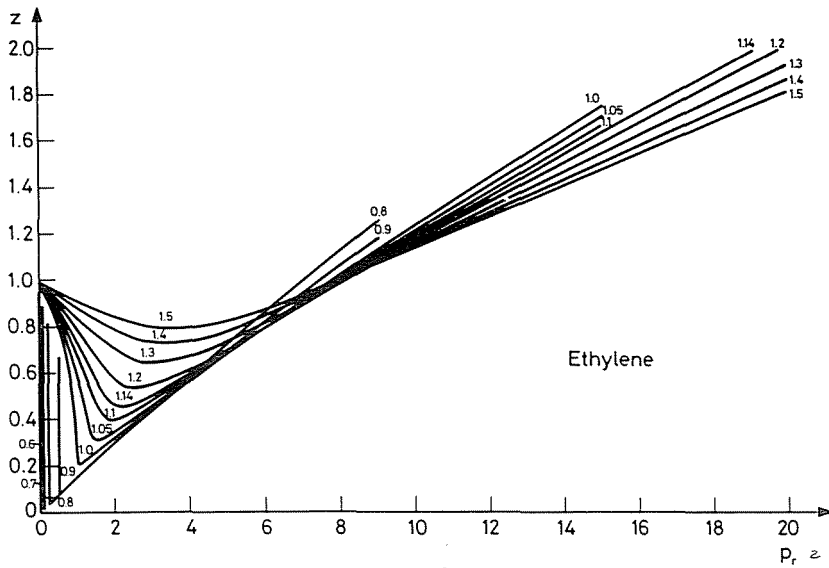


Fig. 3

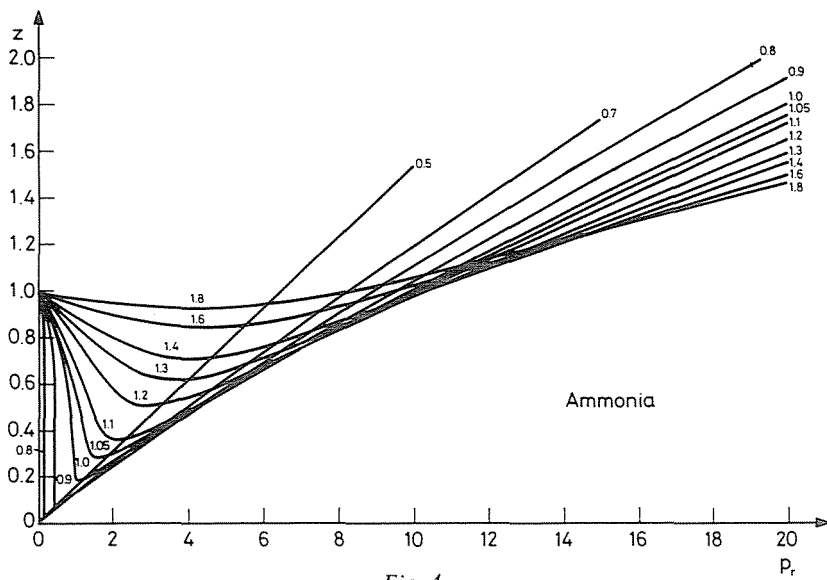


Fig. 4

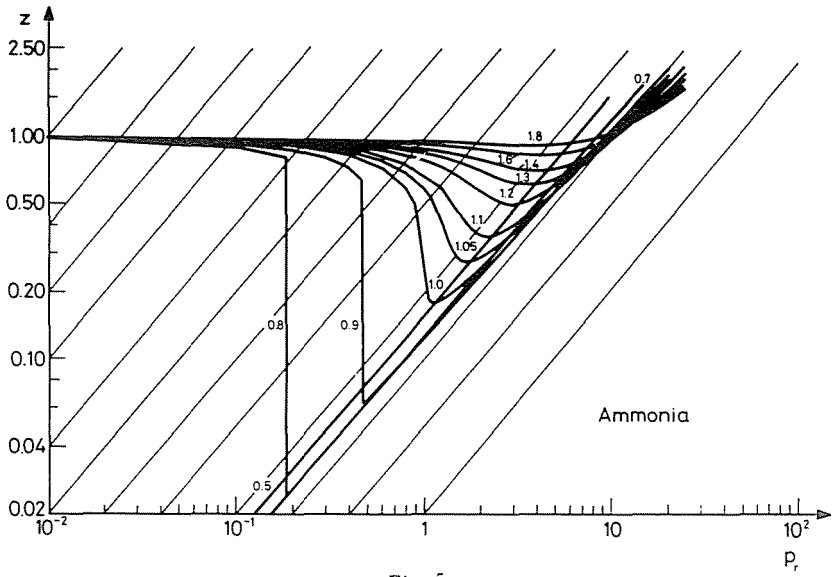


Fig. 5

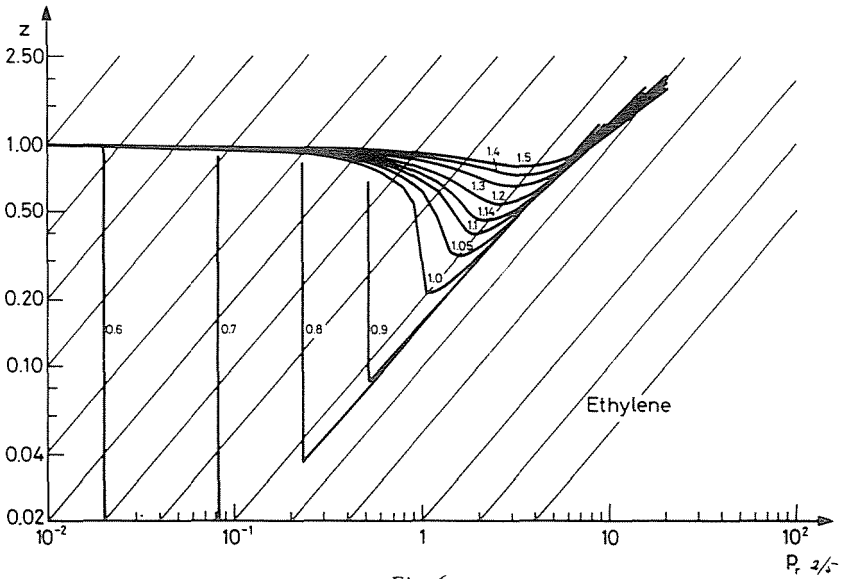


Fig. 6

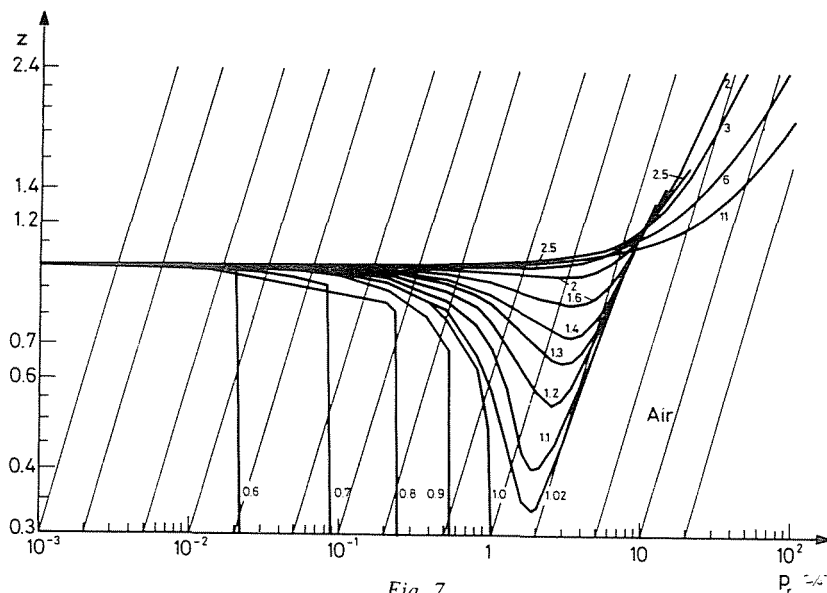


Fig. 7

### Reduced fugacity coefficient chart

Two kinds of reduced fugacity charts can be found in the literature. The first which appeared also in Perry's Chemical Engineer's Handbook has been published by Gamson and Watson [4]. It is interesting that this chart is consistent to Joule-Thomson coefficient as the isotherm belonging to  $T_r=4$  starts with a maximal slope ( $T_r=5$  is not plotted). The isotherms below the critical temperature, however, seem to be not quite reliable. The second kind of reduced fugacity chart has appeared as a member of Chemical Process Principles Charts by Hougen et al. [13]. Here, the low temperature isotherms are correct but the high temperature isotherms taken from the data of Lydersen et al. [6] are not consistent to thermodynamic properties.

Fugacity coefficients have been calculated from the data of Table I and illustrated on Figs 8 and 9. Figure 8 is related to gases of critical compressibility 0.278 and 0.316 (ethylene and air) equally. As it can be seen, below  $T_r=1.5$  the data of air (points on the Figure) are very near to the corresponding isotherms of ethylene. Below the critical temperature the saturation of air is illustrated by two points connected by a short curve being the air a mixture. Fugacity data of gases of lower critical compressibility factor (ammonia) are somewhat different so that they are illustrated on a

Table 1

## Chart of generalized reduced compressibility function

| $Z_c$ | $p_r$ | 0.01  |       |       | 0.1   |       |       | 0.2   |       |       | 0.3   |       |       |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|       |       | $T_r$ | 0.244 | 0.278 | 0.316 | 0.244 | 0.278 | 0.316 | 0.244 | 0.278 | 0.316 | 0.244 | 0.278 |
|       | 0.5   | 0.002 |       |       | 0.016 |       |       | 0.032 |       |       | 0.047 |       |       |
|       | 0.6   | 0.971 |       |       | 0.014 |       |       | 0.028 |       |       | 0.042 |       |       |
|       | 0.7   | 0.986 |       | 0.989 | 0.013 |       |       | 0.026 |       |       | 0.039 |       |       |
|       | 0.8   | 0.992 |       | 0.993 | 0.908 |       | 0.926 | 0.026 |       | 0.844 | 0.038 | 0.048 |       |
|       | 0.90  | 0.995 |       | 0.995 | 0.943 |       | 0.950 | 0.881 |       | 0.897 | 0.810 |       | 0.840 |
|       | 0.92  | 0.995 |       | 0.995 | 0.948 |       | 0.953 | 0.891 |       | 0.904 | 0.828 |       | 0.852 |
|       | 0.94  | 0.995 |       | 0.996 | 0.952 |       | 0.956 | 0.901 |       | 0.911 | 0.844 |       | 0.863 |
|       | 0.96  | 0.995 |       | 0.996 | 0.956 |       | 0.959 | 0.909 |       | 0.917 | 0.858 |       | 0.873 |
|       | 0.98  | 0.996 | 0.996 | 0.996 | 0.959 | 0.963 | 0.962 | 0.916 | 0.924 | 0.923 | 0.869 | 0.882 | 0.882 |
|       | 1.00  | 0.996 | 0.997 | 0.996 | 0.962 | 0.965 | 0.964 | 0.922 | 0.929 | 0.928 | 0.880 | 0.890 | 0.889 |
|       | 1.01  | 0.996 | 0.997 | 0.997 | 0.963 | 0.966 | 0.966 | 0.925 | 0.931 | 0.930 | 0.885 | 0.894 | 0.893 |
|       | 1.02  | 0.997 | 0.997 | 0.997 | 0.965 | 0.967 | 0.967 | 0.928 | 0.933 | 0.932 | 0.889 | 0.897 | 0.897 |
|       | 1.03  | 0.997 | 0.997 | 0.997 | 0.966 | 0.968 | 0.968 | 0.931 | 0.935 | 0.934 | 0.894 | 0.901 | 0.900 |
|       | 1.04  | 0.997 | 0.997 | 0.997 | 0.967 | 0.969 | 0.969 | 0.933 | 0.937 | 0.936 | 0.898 | 0.904 | 0.903 |
|       | 1.05  | 0.997 | 0.997 | 0.997 | 0.968 | 0.970 | 0.970 | 0.936 | 0.939 | 0.938 | 0.902 | 0.907 | 0.906 |
|       | 1.06  | 0.997 | 0.997 | 0.997 | 0.970 | 0.971 | 0.971 | 0.938 | 0.941 | 0.940 | 0.905 | 0.910 | 0.909 |
|       | 1.07  | 0.997 | 0.997 | 0.997 | 0.971 | 0.972 | 0.971 | 0.940 | 0.943 | 0.942 | 0.909 | 0.913 | 0.912 |
|       | 1.08  | 0.997 | 0.997 | 0.997 | 0.972 | 0.973 | 0.972 | 0.942 | 0.945 | 0.944 | 0.912 | 0.916 | 0.915 |
|       | 1.09  | 0.997 | 0.997 | 0.997 | 0.973 | 0.974 | 0.973 | 0.944 | 0.947 | 0.946 | 0.915 | 0.919 | 0.917 |
|       | 1.10  | 0.997 | 0.997 | 0.997 | 0.973 | 0.975 | 0.974 | 0.946 | 0.948 | 0.947 | 0.918 | 0.921 | 0.920 |
|       | 1.12  | 0.998 | 0.998 | 0.998 | 0.975 | 0.976 | 0.975 | 0.950 | 0.951 | 0.950 | 0.924 | 0.926 | 0.925 |
|       | 1.14  | 0.998 | 0.998 | 0.998 | 0.977 | 0.977 | 0.977 | 0.953 | 0.954 | 0.953 | 0.929 | 0.930 | 0.929 |
|       | 1.16  | 0.998 | 0.998 | 0.998 | 0.978 | 0.979 | 0.978 | 0.956 | 0.957 | 0.956 | 0.934 | 0.934 | 0.933 |
|       | 1.18  | 0.998 | 0.998 | 0.998 | 0.980 | 0.980 | 0.979 | 0.959 | 0.959 | 0.958 | 0.938 | 0.938 | 0.937 |
|       | 1.20  | 0.998 | 0.998 | 0.998 | 0.981 | 0.981 | 0.980 | 0.962 | 0.961 | 0.961 | 0.942 | 0.942 | 0.941 |
|       | 1.3   | 0.999 | 0.999 | 0.999 | 0.986 | 0.985 | 0.985 | 0.972 | 0.971 | 0.970 | 0.958 | 0.956 | 0.955 |
|       | 1.4   | 0.999 | 0.999 | 0.999 | 0.989 | 0.989 | 0.989 | 0.979 | 0.977 | 0.977 | 0.968 | 0.966 | 0.965 |
|       | 1.5   | 0.999 | 0.999 | 0.999 | 0.991 | 0.991 | 0.991 | 0.983 | 0.983 | 0.982 | 0.975 | 0.974 | 0.973 |
|       | 1.6   | 0.999 |       | 0.999 | 0.993 |       | 0.993 | 0.986 |       | 0.986 | 0.980 |       | 0.979 |
|       | 1.7   | 0.999 |       | 0.999 | 0.994 |       | 0.995 | 0.988 |       | 0.989 | 0.982 |       | 0.984 |
|       | 1.8   | 0.999 |       | 1.000 | 0.994 |       | 0.996 | 0.989 |       | 0.991 | 0.984 |       | 0.987 |
|       | 1.9   |       |       | 1.000 |       |       | 0.997 |       |       | 0.993 |       |       | 0.990 |



Table 1 (cont.)

| $P_r$ |       | 0.4   |       |       | 0.5   |       |       | 0.6   |       |       | 0.7   |       |       |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $Z_c$ | $T_r$ | 0.244 | 0.278 | 0.316 | 0.244 | 0.278 | 0.316 | 0.244 | 0.278 | 0.316 | 0.244 | 0.278 | 0.316 |
| 0.5   | 0.063 |       |       |       | 0.079 |       |       | 0.094 |       |       | 0.110 |       |       |
| 0.6   | 0.056 |       |       |       | 0.070 |       |       | 0.084 |       |       | 0.098 |       |       |
| 0.7   | 0.052 |       |       |       | 0.065 |       |       | 0.078 |       |       | 0.091 |       |       |
| 0.8   | 0.051 | 0.063 |       |       | 0.063 | 0.078 |       | 0.076 | 0.095 |       | 0.088 | 0.110 |       |
| 0.90  | 0.726 |       |       |       | 0.066 |       |       | 0.079 | 0.098 |       | 0.091 | 0.113 |       |
| 0.92  | 0.756 |       |       | 0.794 | 0.667 |       | 0.726 | 0.081 |       |       | 0.094 | 0.116 |       |
| 0.94  | 0.781 |       |       | 0.810 | 0.707 |       | 0.751 | 0.615 |       |       | 0.098 |       |       |
| 0.96  | 0.802 |       |       | 0.825 | 0.739 |       | 0.771 | 0.665 |       | 0.709 | 0.570 |       |       |
| 0.98  | 0.819 | 0.837 |       | 0.838 | 0.764 | 0.787 | 0.790 | 0.702 | 0.731 | 0.736 | 0.628 |       |       |
| 1.00  | 0.835 | 0.848 |       | 0.849 | 0.786 | 0.804 | 0.805 | 0.732 | 0.754 | 0.757 | 0.671 | 0.698 | 0.702 |
| 1.01  | 0.842 | 0.854 |       | 0.854 | 0.796 | 0.811 | 0.813 | 0.746 | 0.764 | 0.767 | 0.690 | 0.712 | 0.716 |
| 1.02  | 0.848 | 0.859 |       | 0.859 | 0.805 | 0.818 | 0.820 | 0.757 | 0.774 | 0.777 | 0.706 | 0.726 | 0.727 |
| 1.03  | 0.855 | 0.864 |       | 0.864 | 0.819 | 0.825 | 0.826 | 0.769 | 0.783 | 0.785 | 0.720 | 0.737 | 0.740 |
| 1.04  | 0.860 | 0.869 |       | 0.868 | 0.821 | 0.832 | 0.832 | 0.779 | 0.792 | 0.793 | 0.734 | 0.748 | 0.750 |
| 1.05  | 0.866 | 0.873 |       | 0.873 | 0.828 | 0.838 | 0.838 | 0.788 | 0.799 | 0.801 | 0.746 | 0.759 | 0.760 |
| 1.06  | 0.871 | 0.878 |       | 0.877 | 0.835 | 0.843 | 0.844 | 0.798 | 0.807 | 0.808 | 0.758 | 0.768 | 0.770 |
| 1.07  | 0.876 | 0.882 |       | 0.881 | 0.842 | 0.849 | 0.849 | 0.806 | 0.814 | 0.814 | 0.768 | 0.777 | 0.778 |
| 1.08  | 0.881 | 0.886 |       | 0.885 | 0.848 | 0.854 | 0.854 | 0.814 | 0.821 | 0.821 | 0.778 | 0.786 | 0.786 |
| 1.09  | 0.885 | 0.889 |       | 0.889 | 0.854 | 0.859 | 0.859 | 0.821 | 0.827 | 0.827 | 0.787 | 0.794 | 0.794 |
| 1.10  | 0.889 | 0.893 |       | 0.892 | 0.859 | 0.864 | 0.863 | 0.828 | 0.833 | 0.833 | 0.796 | 0.801 | 0.801 |
| 1.12  | 0.897 | 0.900 |       | 0.898 | 0.869 | 0.872 | 0.872 | 0.841 | 0.844 | 0.844 | 0.811 | 0.815 | 0.814 |
| 1.14  | 0.904 | 0.906 |       | 0.905 | 0.879 | 0.880 | 0.879 | 0.852 | 0.854 | 0.853 | 0.825 | 0.827 | 0.827 |
| 1.16  | 0.911 | 0.911 |       | 0.910 | 0.887 | 0.888 | 0.887 | 0.863 | 0.864 | 0.862 | 0.838 | 0.839 | 0.838 |
| 1.18  | 0.917 | 0.917 |       | 0.915 | 0.895 | 0.895 | 0.893 | 0.872 | 0.872 | 0.871 | 0.850 | 0.849 | 0.848 |
| 1.20  | 0.922 | 0.921 |       | 0.920 | 0.902 | 0.901 | 0.899 | 0.881 | 0.880 | 0.878 | 0.860 | 0.858 | 0.856 |
| 1.3   | 0.943 | 0.941 |       | 0.940 | 0.929 | 0.925 | 0.924 | 0.915 | 0.910 | 0.909 | 0.900 | 0.895 | 0.893 |
| 1.4   | 0.958 | 0.954 |       | 0.954 | 0.947 | 0.943 | 0.942 | 0.937 | 0.932 | 0.931 | 0.926 | 0.920 | 0.919 |
| 1.5   | 0.967 | 0.965 |       | 0.964 | 0.960 | 0.956 | 0.955 | 0.952 | 0.948 | 0.947 | 0.945 | 0.939 | 0.938 |
| 1.6   | 0.974 |       |       | 0.972 | 0.968 |       | 0.965 | 0.962 |       | 0.958 | 0.957 |       | 0.952 |
| 1.7   | 0.978 |       |       | 0.978 | 0.973 |       | 0.973 | 0.969 |       | 0.968 | 0.965 |       | 0.963 |
| 1.8   | 0.980 |       |       | 0.983 | 0.976 |       | 0.979 | 0.973 |       | 0.975 | 0.970 |       | 0.971 |
| 1.9   |       |       |       | 0.987 |       |       | 0.984 |       |       | 0.981 |       |       | 0.978 |

Table 1 (cont.)

| Pr             |                | 0.8   |       |       | 0.9   |       |       | 1.00  |       |       | 1.05  |       |       |
|----------------|----------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Z <sub>c</sub> | T <sub>r</sub> | 0.244 | 0.278 | 0.316 | 0.244 | 0.278 | 0.316 | 0.244 | 0.278 | 0.316 | 0.244 | 0.278 | 0.316 |
| 0.5            | 0.126          |       |       |       | 0.141 |       |       | 0.157 |       |       | 0.165 |       |       |
| 0.6            | 0.112          |       |       |       | 0.126 |       |       | 0.140 |       |       | 0.146 |       |       |
| 0.7            | 0.104          |       |       |       | 0.117 |       |       | 0.129 |       |       | 0.136 |       |       |
| 0.8            | 0.100          | 0.125 |       |       | 0.113 | 0.141 |       | 0.125 | 0.156 |       | 0.131 | 0.164 |       |
| 0.90           | 0.104          | 0.128 |       |       | 0.116 | 0.143 |       | 0.128 | 0.158 |       | 0.134 | 0.166 |       |
| 0.92           | 0.106          | 0.131 |       |       | 0.118 | 0.146 |       | 0.130 | 0.161 |       | 0.136 | 0.168 |       |
| 0.94           | 0.110          | 0.135 |       |       | 0.122 | 0.150 |       | 0.134 | 0.164 |       | 0.140 | 0.172 |       |
| 0.96           | 0.117          | 0.370 |       |       | 0.128 | 0.157 |       | 0.139 | 0.171 |       | 0.145 | 0.178 |       |
| 0.98           | 0.529          |       |       |       | 0.141 |       |       | 0.152 |       |       | 0.156 | 0.188 |       |
| 1.00           | 0.598          | 0.631 | 0.634 | 0.494 | 0.538 |       |       | 0.244 | 0.278 | 0.316 | 0.185 | 0.213 |       |
| 1.01           | 0.626          | 0.652 | 0.655 | 0.546 | 0.574 |       |       | 0.430 | 0.454 |       | 0.353 | 0.365 |       |
| 1.02           | 0.647          | 0.670 | 0.674 | 0.582 | 0.605 | 0.608 | 0.482 | 0.516 | 0.515 | 0.419 | 0.452 | 0.453 |       |
| 1.03           | 0.667          | 0.686 | 0.688 | 0.605 | 0.627 | 0.628 | 0.528 | 0.552 | 0.548 | 0.482 | 0.502 | 0.492 |       |
| 1.04           | 0.686          | 0.701 | 0.702 | 0.627 | 0.647 | 0.647 | 0.566 | 0.584 | 0.577 | 0.531 | 0.546 | 0.531 |       |
| 1.05           | 0.700          | 0.714 | 0.716 | 0.649 | 0.665 | 0.665 | 0.592 | 0.609 | 0.605 | 0.562 | 0.576 | 0.569 |       |
| 1.06           | 0.715          | 0.726 | 0.729 | 0.669 | 0.681 | 0.682 | 0.618 | 0.619 | 0.630 | 0.591 | 0.600 | 0.601 |       |
| 1.07           | 0.728          | 0.738 | 0.739 | 0.684 | 0.695 | 0.695 | 0.638 | 0.646 | 0.647 | 0.613 | 0.619 | 0.620 |       |
| 1.08           | 0.740          | 0.748 | 0.749 | 0.700 | 0.708 | 0.708 | 0.657 | 0.662 | 0.663 | 0.634 | 0.638 | 0.639 |       |
| 1.09           | 0.751          | 0.758 | 0.759 | 0.713 | 0.720 | 0.721 | 0.674 | 0.679 | 0.680 | 0.653 | 0.656 | 0.658 |       |
| 1.10           | 0.762          | 0.767 | 0.768 | 0.726 | 0.732 | 0.732 | 0.689 | 0.693 | 0.694 | 0.669 | 0.672 | 0.673 |       |
| 1.12           | 0.781          | 0.784 | 0.784 | 0.749 | 0.752 | 0.752 | 0.716 | 0.719 | 0.718 | 0.699 | 0.701 | 0.700 |       |
| 1.14           | 0.798          | 0.799 | 0.799 | 0.769 | 0.771 | 0.770 | 0.740 | 0.741 | 0.739 | 0.725 | 0.725 | 0.724 |       |
| 1.16           | 0.813          | 0.813 | 0.812 | 0.787 | 0.787 | 0.785 | 0.760 | 0.760 | 0.758 | 0.747 | 0.746 | 0.744 |       |
| 1.18           | 0.826          | 0.825 | 0.824 | 0.803 | 0.801 | 0.800 | 0.778 | 0.777 | 0.775 | 0.766 | 0.764 | 0.762 |       |
| 1.20           | 0.839          | 0.836 | 0.835 | 0.817 | 0.814 | 0.812 | 0.795 | 0.792 | 0.790 | 0.784 | 0.781 | 0.778 |       |
| 1.3            | 0.885          | 0.879 | 0.878 | 0.871 | 0.864 | 0.862 | 0.856 | 0.849 | 0.846 | 0.849 | 0.841 | 0.839 |       |
| 1.4            | 0.917          | 0.909 | 0.907 | 0.906 | 0.898 | 0.896 | 0.896 | 0.886 | 0.885 | 0.891 | 0.881 | 0.879 |       |
| 1.5            | 0.937          | 0.931 | 0.929 | 0.930 | 0.923 | 0.920 | 0.923 | 0.914 | 0.912 | 0.920 | 0.910 | 0.908 |       |
| 1.6            | 0.952          |       | 0.945 | 0.947 |       | 0.939 | 0.942 |       | 0.932 | 0.939 |       | 0.929 |       |
| 1.7            | 0.961          |       | 0.958 | 0.957 |       | 0.953 | 0.954 |       | 0.948 | 0.952 |       | 0.946 |       |
| 1.8            | 0.967          |       | 0.967 | 0.964 |       | 0.964 | 0.962 |       | 0.960 | 0.961 |       | 0.958 |       |
| 1.9            |                |       | 0.975 |       |       | 0.972 |       |       | 0.970 |       |       | 0.969 |       |

| $Z_c$ | $P_r$ | 1.10  |       |       | 1.15  |       |       | 1.20  |       |       | 1.25  |       |       |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|       |       | $T_r$ | 0.244 | 0.278 | 0.316 | 0.244 | 0.278 | 0.316 | 0.244 | 0.278 | 0.316 | 0.244 | 0.278 |
|       | 0.5   | 0.173 |       |       | 0.180 |       |       | 0.188 |       |       | 0.196 |       |       |
|       | 0.6   | 0.153 |       |       | 0.160 |       |       | 0.167 |       |       | 0.174 |       |       |
|       | 0.7   | 0.142 |       |       | 0.149 |       |       | 0.155 |       |       | 0.161 |       |       |
|       | 0.8   | 0.137 | 0.171 |       | 0.143 | 0.179 |       | 0.149 | 0.186 |       | 0.155 | 0.194 |       |
|       | 0.90  | 0.140 | 0.173 |       | 0.146 | 0.180 |       | 0.152 | 0.188 |       | 0.158 | 0.195 |       |
|       | 0.92  | 0.142 | 0.175 |       | 0.148 | 0.183 |       | 0.154 | 0.190 |       | 0.160 | 0.197 |       |
|       | 0.94  | 0.146 | 0.179 |       | 0.152 | 0.186 |       | 0.157 | 0.193 |       | 0.163 | 0.200 |       |
|       | 0.96  | 0.151 | 0.184 |       | 0.157 | 0.191 |       | 0.162 | 0.198 |       | 0.168 | 0.205 |       |
|       | 0.98  | 0.161 | 0.194 |       | 0.166 | 0.200 |       | 0.171 | 0.206 |       | 0.176 | 0.213 |       |
|       | 1.00  | 0.180 | 0.215 |       | 0.188 | 0.218 |       | 0.189 | 0.221 |       | 0.191 | 0.226 |       |
|       | 1.01  | 0.275 | 0.294 |       | 0.202 | 0.261 |       | 0.201 | 0.240 |       | 0.200 | 0.240 |       |
|       | 1.02  | 0.351 | 0.384 | 0.363 | 0.284 | 0.311 | 0.303 | 0.263 | 0.280 | 0.279 | 0.245 | 0.257 | 0.272 |
|       | 1.03  | 0.429 | 0.449 | 0.419 | 0.376 | 0.387 | 0.366 | 0.333 | 0.336 | 0.338 | 0.294 | 0.304 | 0.322 |
|       | 1.04  | 0.490 | 0.504 | 0.474 | 0.447 | 0.453 | 0.427 | 0.395 | 0.403 | 0.396 | 0.347 | 0.355 | 0.370 |
|       | 1.05  | 0.526 | 0.540 | 0.527 | 0.490 | 0.500 | 0.488 | 0.447 | 0.458 | 0.452 | 0.406 | 0.414 | 0.418 |
|       | 1.06  | 0.562 | 0.569 | 0.569 | 0.532 | 0.536 | 0.535 | 0.499 | 0.501 | 0.499 | 0.465 | 0.463 | 0.460 |
|       | 1.07  | 0.587 | 0.592 | 0.591 | 0.560 | 0.563 | 0.561 | 0.531 | 0.533 | 0.528 | 0.501 | 0.501 | 0.494 |
|       | 1.08  | 0.610 | 0.613 | 0.613 | 0.586 | 0.587 | 0.586 | 0.560 | 0.561 | 0.557 | 0.534 | 0.533 | 0.527 |
|       | 1.09  | 0.631 | 0.634 | 0.635 | 0.609 | 0.610 | 0.611 | 0.586 | 0.587 | 0.586 | 0.563 | 0.562 | 0.559 |
|       | 1.10  | 0.649 | 0.651 | 0.652 | 0.629 | 0.630 | 0.630 | 0.608 | 0.608 | 0.608 | 0.587 | 0.585 | 0.585 |
|       | 1.12  | 0.682 | 0.683 | 0.681 | 0.664 | 0.664 | 0.663 | 0.646 | 0.645 | 0.643 | 0.628 | 0.626 | 0.624 |
|       | 1.14  | 0.709 | 0.709 | 0.708 | 0.694 | 0.693 | 0.691 | 0.678 | 0.676 | 0.674 | 0.662 | 0.660 | 0.658 |
|       | 1.16  | 0.733 | 0.732 | 0.729 | 0.719 | 0.717 | 0.714 | 0.706 | 0.703 | 0.699 | 0.692 | 0.688 | 0.685 |
|       | 1.18  | 0.754 | 0.752 | 0.749 | 0.741 | 0.739 | 0.736 | 0.729 | 0.726 | 0.723 | 0.716 | 0.713 | 0.709 |
|       | 1.20  | 0.772 | 0.769 | 0.766 | 0.761 | 0.758 | 0.754 | 0.750 | 0.746 | 0.743 | 0.738 | 0.734 | 0.731 |
|       | 1.3   | 0.841 | 0.834 | 0.831 | 0.834 | 0.826 | 0.823 | 0.827 | 0.819 | 0.815 | 0.819 | 0.811 | 0.808 |
|       | 1.4   | 0.886 | 0.876 | 0.874 | 0.881 | 0.870 | 0.868 | 0.876 | 0.865 | 0.863 | 0.871 | 0.860 | 0.857 |
|       | 1.5   | 0.916 | 0.906 | 0.904 | 0.913 | 0.902 | 0.900 | 0.909 | 0.898 | 0.896 | 0.906 | 0.895 | 0.892 |
|       | 1.6   | 0.937 |       | 0.926 | 0.934 |       | 0.923 | 0.932 |       | 0.920 | 0.930 |       | 0.917 |
|       | 1.7   | 0.951 |       | 0.943 | 0.949 |       | 0.941 | 0.948 |       | 0.939 | 0.946 |       | 0.937 |
|       | 1.8   | 0.960 |       | 0.957 | 0.959 |       | 0.955 | 0.958 |       | 0.953 | 0.957 |       | 0.952 |
|       | 1.9   |       |       | 0.967 |       |       | 0.966 |       |       | 0.965 |       |       | 0.964 |

Table 1 (cont.)

| $p_r$ |       | 1.30  |       |       | 1.35  |       |       | 1.40  |       |       | 1.45  |       |       |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $Z_c$ | $T_r$ | 0.244 | 0.278 | 0.316 | 0.244 | 0.278 | 0.316 | 0.244 | 0.278 | 0.316 | 0.244 | 0.278 | 0.316 |
|       | 0.5   | 0.204 |       |       | 0.212 |       |       | 0.219 |       |       | 0.227 |       |       |
|       | 0.6   | 0.181 |       |       | 0.188 |       |       | 0.195 |       |       | 0.202 |       |       |
|       | 0.7   | 0.168 |       |       | 0.174 |       |       | 0.180 |       |       | 0.187 |       |       |
|       | 0.8   | 0.161 | 0.201 |       | 0.167 | 0.209 |       | 0.173 | 0.216 |       | 0.179 | 0.224 |       |
|       | 0.90  | 0.164 | 0.202 |       | 0.170 | 0.210 |       | 0.175 | 0.217 |       | 0.181 | 0.224 |       |
|       | 0.92  | 0.166 | 0.204 |       | 0.172 | 0.212 |       | 0.178 | 0.219 |       | 0.183 | 0.226 |       |
|       | 0.94  | 0.169 | 0.208 |       | 0.175 | 0.215 |       | 0.181 | 0.222 |       | 0.186 | 0.229 |       |
|       | 0.96  | 0.174 | 0.212 |       | 0.180 | 0.219 |       | 0.185 | 0.226 |       | 0.191 | 0.233 |       |
|       | 0.98  | 0.182 | 0.219 |       | 0.187 | 0.226 |       | 0.192 | 0.232 |       | 0.198 | 0.239 |       |
|       | 1.00  | 0.195 | 0.231 |       | 0.199 | 0.237 |       | 0.204 | 0.243 |       | 0.208 | 0.249 |       |
|       | 1.01  | 0.203 | 0.242 |       | 0.206 | 0.246 |       | 0.210 | 0.251 |       | 0.214 | 0.256 |       |
|       | 1.02  | 0.233 | 0.254 | 0.270 | 0.226 | 0.256 | 0.272 | 0.225 | 0.259 | 0.275 | 0.227 | 0.263 | 0.278 |
|       | 1.03  | 0.265 | 0.283 | 0.309 | 0.248 | 0.277 | 0.302 | 0.242 | 0.275 | 0.299 | 0.240 | 0.276 | 0.296 |
|       | 1.04  | 0.309 | 0.320 | 0.348 | 0.284 | 0.305 | 0.332 | 0.270 | 0.295 | 0.323 | 0.264 | 0.293 | 0.314 |
|       | 1.05  | 0.370 | 0.377 | 0.385 | 0.340 | 0.349 | 0.361 | 0.317 | 0.328 | 0.347 | 0.302 | 0.312 | 0.332 |
|       | 1.06  | 0.430 | 0.427 | 0.423 | 0.396 | 0.394 | 0.394 | 0.363 | 0.367 | 0.375 | 0.339 | 0.350 | 0.356 |
|       | 1.07  | 0.469 | 0.470 | 0.460 | 0.439 | 0.439 | 0.433 | 0.408 | 0.412 | 0.412 | 0.383 | 0.389 | 0.392 |
|       | 1.08  | 0.507 | 0.505 | 0.497 | 0.480 | 0.477 | 0.471 | 0.452 | 0.451 | 0.449 | 0.427 | 0.427 | 0.427 |
|       | 1.09  | 0.539 | 0.537 | 0.533 | 0.514 | 0.512 | 0.509 | 0.490 | 0.488 | 0.485 | 0.466 | 0.464 | 0.461 |
|       | 1.10  | 0.565 | 0.562 | 0.561 | 0.542 | 0.539 | 0.538 | 0.520 | 0.517 | 0.514 | 0.498 | 0.495 | 0.491 |
|       | 1.12  | 0.610 | 0.607 | 0.604 | 0.591 | 0.588 | 0.584 | 0.572 | 0.568 | 0.564 | 0.554 | 0.549 | 0.544 |
|       | 1.14  | 0.646 | 0.643 | 0.641 | 0.630 | 0.626 | 0.623 | 0.614 | 0.610 | 0.606 | 0.598 | 0.593 | 0.589 |
|       | 1.16  | 0.678 | 0.674 | 0.670 | 0.663 | 0.659 | 0.655 | 0.649 | 0.644 | 0.640 | 0.635 | 0.630 | 0.625 |
|       | 1.18  | 0.704 | 0.700 | 0.696 | 0.691 | 0.687 | 0.683 | 0.679 | 0.674 | 0.669 | 0.666 | 0.661 | 0.656 |
|       | 1.20  | 0.727 | 0.722 | 0.719 | 0.716 | 0.711 | 0.707 | 0.704 | 0.699 | 0.695 | 0.693 | 0.687 | 0.683 |
|       | 1.3   | 0.812 | 0.804 | 0.800 | 0.805 | 0.796 | 0.792 | 0.797 | 0.789 | 0.785 | 0.790 | 0.782 | 0.777 |
|       | 1.4   | 0.866 | 0.855 | 0.852 | 0.861 | 0.850 | 0.847 | 0.856 | 0.845 | 0.841 | 0.851 | 0.840 | 0.836 |
|       | 1.5   | 0.902 | 0.891 | 0.888 | 0.899 | 0.888 | 0.884 | 0.895 | 0.884 | 0.880 | 0.892 | 0.880 | 0.877 |
|       | 1.6   | 0.927 |       | 0.914 | 0.925 |       | 0.912 | 0.923 |       | 0.909 | 0.920 |       | 0.906 |
|       | 1.7   | 0.945 |       | 0.934 | 0.943 |       | 0.932 | 0.942 |       | 0.930 | 0.940 |       | 0.928 |
|       | 1.8   | 0.956 |       | 0.950 | 0.955 |       | 0.949 | 0.954 |       | 0.947 | 0.954 |       | 0.946 |
|       | 1.9   |       |       | 0.963 |       |       | 0.962 |       |       | 0.960 |       |       | 0.959 |

Table 1 (cont.)

| $p_r$ |       | 1.50  |       |       | 1.6   |       |       | 1.7   |       |       | 1.8   |       |       |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $Z_c$ | $T_r$ | 0.244 | 0.278 | 0.316 | 0.244 | 0.278 | 0.316 | 0.244 | 0.278 | 0.316 | 0.244 | 0.278 | 0.316 |
| 0.5   |       | 0.235 |       |       | 0.250 |       |       | 0.266 |       |       | 0.281 |       |       |
| 0.6   |       | 0.208 |       |       | 0.222 |       |       | 0.236 |       |       | 0.249 |       |       |
| 0.7   |       | 0.193 |       |       | 0.205 |       |       | 0.218 |       |       | 0.231 |       |       |
| 0.8   |       | 0.185 | 0.231 |       | 0.197 | 0.246 |       | 0.209 | 0.261 |       | 0.221 | 0.276 |       |
| 0.90  |       | 0.187 | 0.231 |       | 0.198 | 0.245 |       | 0.210 | 0.260 |       | 0.222 | 0.274 |       |
| 0.92  |       | 0.189 | 0.233 |       | 0.201 | 0.247 |       | 0.212 | 0.261 |       | 0.223 | 0.275 |       |
| 0.94  |       | 0.192 | 0.236 |       | 0.204 | 0.250 |       | 0.215 | 0.264 |       | 0.226 | 0.278 |       |
| 0.96  |       | 0.197 | 0.240 |       | 0.208 | 0.254 |       | 0.219 | 0.267 |       | 0.230 | 0.281 |       |
| 0.98  |       | 0.203 | 0.246 |       | 0.214 | 0.259 |       | 0.225 | 0.272 |       | 0.236 | 0.286 |       |
| 1.00  |       | 0.213 | 0.255 |       | 0.223 | 0.267 |       | 0.233 | 0.280 |       | 0.243 | 0.292 |       |
| 1.01  |       | 0.219 | 0.261 |       | 0.228 | 0.272 |       | 0.238 | 0.284 |       | 0.247 | 0.297 |       |
| 1.02  |       | 0.229 | 0.268 | 0.282 | 0.237 | 0.278 | 0.292 | 0.245 | 0.290 | 0.304 | 0.254 | 0.301 | 0.315 |
| 1.03  |       | 0.241 | 0.279 | 0.298 | 0.246 | 0.287 | 0.304 | 0.253 | 0.297 | 0.313 | 0.261 | 0.308 | 0.323 |
| 1.04  |       | 0.259 | 0.292 | 0.313 | 0.259 | 0.297 | 0.315 | 0.263 | 0.305 | 0.322 | 0.269 | 0.315 | 0.331 |
| 1.05  |       | 0.288 | 0.312 | 0.328 | 0.278 | 0.311 | 0.326 | 0.277 | 0.316 | 0.331 | 0.281 | 0.323 | 0.338 |
| 1.06  |       | 0.316 | 0.338 | 0.349 | 0.297 | 0.329 | 0.341 | 0.291 | 0.329 | 0.343 | 0.292 | 0.334 | 0.347 |
| 1.07  |       | 0.360 | 0.371 | 0.381 | 0.332 | 0.351 | 0.365 | 0.317 | 0.345 | 0.361 | 0.312 | 0.347 | 0.363 |
| 1.08  |       | 0.403 | 0.407 | 0.412 | 0.367 | 0.379 | 0.388 | 0.344 | 0.366 | 0.379 | 0.333 | 0.363 | 0.377 |
| 1.09  |       | 0.442 | 0.443 | 0.443 | 0.402 | 0.409 | 0.411 | 0.373 | 0.389 | 0.396 | 0.357 | 0.380 | 0.390 |
| 1.10  |       | 0.476 | 0.475 | 0.472 | 0.437 | 0.440 | 0.437 | 0.406 | 0.416 | 0.418 | 0.385 | 0.402 | 0.409 |
| 1.12  |       | 0.535 | 0.531 | 0.526 | 0.499 | 0.497 | 0.486 | 0.467 | 0.469 | 0.468 | 0.442 | 0.449 | 0.451 |
| 1.14  |       | 0.582 | 0.577 | 0.573 | 0.551 | 0.546 | 0.538 | 0.521 | 0.518 | 0.514 | 0.495 | 0.496 | 0.493 |
| 1.16  |       | 0.621 | 0.615 | 0.611 | 0.593 | 0.588 | 0.580 | 0.567 | 0.562 | 0.557 | 0.542 | 0.539 | 0.535 |
| 1.18  |       | 0.653 | 0.648 | 0.643 | 0.629 | 0.623 | 0.618 | 0.605 | 0.599 | 0.594 | 0.582 | 0.578 | 0.572 |
| 1.20  |       | 0.681 | 0.676 | 0.671 | 0.659 | 0.653 | 0.648 | 0.638 | 0.632 | 0.626 | 0.617 | 0.612 | 0.606 |
| 1.3   |       | 0.783 | 0.774 | 0.770 | 0.768 | 0.760 | 0.755 | 0.754 | 0.746 | 0.741 | 0.741 | 0.733 | 0.727 |
| 1.4   |       | 0.846 | 0.835 | 0.831 | 0.836 | 0.825 | 0.821 | 0.827 | 0.816 | 0.811 | 0.817 | 0.807 | 0.802 |
| 1.5   |       | 0.888 | 0.877 | 0.873 | 0.882 | 0.870 | 0.866 | 0.875 | 0.864 | 0.859 | 0.869 | 0.857 | 0.852 |
| 1.6   |       | 0.918 |       | 0.903 | 0.914 |       | 0.898 | 0.909 |       | 0.893 | 0.905 |       | 0.888 |
| 1.7   |       | 0.939 |       | 0.926 | 0.936 |       | 0.922 | 0.933 |       | 0.919 | 0.930 |       | 0.915 |
| 1.8   |       | 0.953 |       | 0.944 | 0.951 |       | 0.941 | 0.950 |       | 0.939 | 0.948 |       | 0.936 |
| 1.9   |       |       |       | 0.958 |       |       | 0.956 |       |       | 0.955 |       |       | 0.953 |

Table 1 (cont.)

| $p_r$ |       | 1.9   |       |       | 2.0   |       |       | 2.2   |       |       | 2.4   |       |       |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $Z_c$ | $T_r$ | 0.244 | 0.278 | 0.316 | 0.244 | 0.278 | 0.316 | 0.244 | 0.278 | 0.316 | 0.244 | 0.278 | 0.316 |
|       | 0.5   | 0.297 |       |       | 0.312 |       |       | 0.343 |       |       | 0.374 |       |       |
|       | 0.6   | 0.263 |       |       | 0.277 |       |       | 0.304 |       |       | 0.331 |       |       |
|       | 0.7   | 0.243 |       |       | 0.256 |       |       | 0.280 |       |       | 0.305 |       |       |
|       | 0.8   | 0.233 | 0.291 |       | 0.245 | 0.305 |       | 0.268 | 0.334 |       | 0.291 | 0.363 |       |
|       | 0.90  | 0.233 | 0.288 |       | 0.244 | 0.302 |       | 0.267 | 0.329 |       | 0.289 | 0.356 |       |
|       | 0.92  | 0.235 | 0.289 |       | 0.246 | 0.303 |       | 0.268 | 0.330 |       | 0.290 | 0.357 |       |
|       | 0.94  | 0.237 | 0.291 |       | 0.248 | 0.305 |       | 0.271 | 0.331 |       | 0.292 | 0.358 |       |
|       | 0.96  | 0.241 | 0.294 |       | 0.252 | 0.308 |       | 0.274 | 0.334 |       | 0.295 | 0.360 |       |
|       | 0.98  | 0.246 | 0.299 |       | 0.257 | 0.312 |       | 0.278 | 0.338 |       | 0.299 | 0.363 |       |
|       | 1.00  | 0.253 | 0.305 |       | 0.264 | 0.318 |       | 0.284 | 0.343 |       | 0.305 | 0.368 |       |
|       | 1.01  | 0.257 | 0.309 |       | 0.267 | 0.321 |       | 0.288 | 0.346 |       | 0.308 | 0.371 |       |
|       | 1.02  | 0.263 | 0.313 | 0.328 | 0.273 | 0.325 | 0.340 | 0.292 | 0.350 | 0.365 | 0.312 | 0.374 | 0.390 |
|       | 1.03  | 0.269 | 0.319 | 0.334 | 0.278 | 0.330 | 0.346 | 0.297 | 0.354 | 0.370 | 0.316 | 0.378 | 0.394 |
|       | 1.04  | 0.277 | 0.325 | 0.341 | 0.285 | 0.336 | 0.351 | 0.302 | 0.359 | 0.374 | 0.321 | 0.382 | 0.398 |
|       | 1.05  | 0.287 | 0.333 | 0.347 | 0.293 | 0.343 | 0.357 | 0.309 | 0.365 | 0.379 | 0.326 | 0.387 | 0.402 |
|       | 1.06  | 0.296 | 0.341 | 0.356 | 0.302 | 0.351 | 0.364 | 0.316 | 0.372 | 0.384 | 0.332 | 0.393 | 0.406 |
|       | 1.07  | 0.312 | 0.352 | 0.367 | 0.315 | 0.359 | 0.374 | 0.326 | 0.380 | 0.392 | 0.340 | 0.400 | 0.412 |
|       | 1.08  | 0.328 | 0.365 | 0.379 | 0.328 | 0.375 | 0.384 | 0.335 | 0.393 | 0.400 | 0.348 | 0.411 | 0.418 |
|       | 1.09  | 0.348 | 0.378 | 0.390 | 0.344 | 0.382 | 0.393 | 0.347 | 0.400 | 0.407 | 0.357 | 0.417 | 0.424 |
|       | 1.10  | 0.372 | 0.396 | 0.405 | 0.364 | 0.397 | 0.407 | 0.361 | 0.413 | 0.417 | 0.368 | 0.428 | 0.432 |
|       | 1.12  | 0.421 | 0.435 | 0.441 | 0.407 | 0.431 | 0.436 | 0.394 | 0.442 | 0.439 | 0.393 | 0.452 | 0.449 |
|       | 1.14  | 0.472 | 0.478 | 0.477 | 0.455 | 0.470 | 0.468 | 0.431 | 0.475 | 0.463 | 0.422 | 0.480 | 0.469 |
|       | 1.16  | 0.520 | 0.520 | 0.517 | 0.501 | 0.509 | 0.504 | 0.472 | 0.504 | 0.492 | 0.456 | 0.505 | 0.491 |
|       | 1.18  | 0.561 | 0.559 | 0.554 | 0.542 | 0.546 | 0.539 | 0.512 | 0.542 | 0.521 | 0.492 | 0.538 | 0.515 |
|       | 1.20  | 0.597 | 0.593 | 0.589 | 0.580 | 0.581 | 0.573 | 0.549 | 0.573 | 0.552 | 0.527 | 0.566 | 0.541 |
|       | 1.3   | 0.728 | 0.720 | 0.714 | 0.715 | 0.710 | 0.702 | 0.692 | 0.696 | 0.681 | 0.671 | 0.683 | 0.665 |
|       | 1.4   | 0.808 | 0.798 | 0.793 | 0.799 | 0.790 | 0.784 | 0.782 | 0.779 | 0.769 | 0.767 | 0.768 | 0.756 |
|       | 1.5   | 0.862 | 0.851 | 0.846 | 0.856 | 0.846 | 0.840 | 0.844 | 0.837 | 0.829 | 0.832 | 0.829 | 0.819 |
|       | 1.6   | 0.900 |       | 0.884 | 0.896 |       | 0.879 | 0.888 |       | 0.871 | 0.880 |       | 0.865 |
|       | 1.7   | 0.927 |       | 0.912 | 0.925 |       | 0.909 | 0.919 |       | 0.903 | 0.913 |       | 0.899 |
|       | 1.8   | 0.947 |       | 0.934 | 0.945 |       | 0.932 | 0.942 |       | 0.928 | 0.939 |       | 0.925 |
|       | 1.9   |       |       | 0.951 |       |       | 0.950 |       |       | 0.947 |       |       | 0.945 |

| $p_r$ |       | 2.6   |       |       | 2.8   |       |       | 3.0   |       |       | 3.5   |       |       |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $Z_c$ | $T_r$ | 0.244 | 0.278 | 0.316 | 0.244 | 0.278 | 0.316 | 0.244 | 0.278 | 0.316 | 0.244 | 0.278 | 0.316 |
| 0.5   | 0.405 |       |       |       | 0.436 |       |       | 0.467 |       |       | 0.544 |       |       |
| 0.6   | 0.358 |       |       |       | 0.385 |       |       | 0.412 |       |       | 0.479 |       |       |
| 0.7   | 0.330 |       |       |       | 0.355 |       |       | 0.379 |       |       | 0.440 |       |       |
| 0.8   | 0.316 | 0.392 |       |       | 0.338 | 0.421 |       | 0.360 | 0.450 |       | 0.417 | 0.520 |       |
| 0.90  | 0.311 | 0.383 |       |       | 0.333 | 0.411 |       | 0.355 | 0.438 |       | 0.408 | 0.504 |       |
| 0.92  | 0.312 | 0.384 |       |       | 0.334 | 0.411 |       | 0.355 | 0.437 |       | 0.408 | 0.502 |       |
| 0.94  | 0.314 | 0.385 |       |       | 0.335 | 0.411 |       | 0.357 | 0.438 |       | 0.409 | 0.502 |       |
| 0.96  | 0.317 | 0.386 |       |       | 0.338 | 0.413 |       | 0.359 | 0.439 |       | 0.410 | 0.502 |       |
| 0.98  | 0.320 | 0.389 |       |       | 0.341 | 0.415 |       | 0.362 | 0.440 |       | 0.412 | 0.502 |       |
| 1.00  | 0.325 | 0.393 |       |       | 0.346 | 0.418 |       | 0.366 | 0.443 |       | 0.416 | 0.504 |       |
| 1.01  | 0.328 | 0.396 |       |       | 0.348 | 0.420 |       | 0.368 | 0.445 |       | 0.417 | 0.505 |       |
| 1.02  | 0.332 | 0.398 | 0.416 |       | 0.351 | 0.423 | 0.441 | 0.371 | 0.447 | 0.466 | 0.420 | 0.507 | 0.528 |
| 1.03  | 0.335 | 0.402 | 0.419 |       | 0.355 | 0.426 | 0.444 | 0.374 | 0.449 | 0.468 | 0.422 | 0.508 | 0.530 |
| 1.04  | 0.339 | 0.405 | 0.422 |       | 0.358 | 0.429 | 0.446 | 0.377 | 0.452 | 0.471 | 0.425 | 0.510 | 0.531 |
| 1.05  | 0.344 | 0.410 | 0.425 |       | 0.363 | 0.432 | 0.449 | 0.381 | 0.455 | 0.473 | 0.428 | 0.512 | 0.533 |
| 1.06  | 0.349 | 0.415 | 0.429 |       | 0.367 | 0.436 | 0.452 | 0.385 | 0.458 | 0.476 | 0.431 | 0.515 | 0.535 |
| 1.07  | 0.356 | 0.420 | 0.434 |       | 0.373 | 0.441 | 0.457 | 0.390 | 0.462 | 0.479 | 0.435 | 0.517 | 0.537 |
| 1.08  | 0.362 | 0.429 | 0.439 |       | 0.378 | 0.447 | 0.461 | 0.395 | 0.466 | 0.483 | 0.439 | 0.520 | 0.540 |
| 1.09  | 0.370 | 0.434 | 0.444 |       | 0.385 | 0.452 | 0.465 | 0.401 | 0.470 | 0.486 | 0.443 | 0.523 | 0.542 |
| 1.10  | 0.379 | 0.443 | 0.450 |       | 0.392 | 0.458 | 0.470 | 0.407 | 0.475 | 0.491 | 0.448 | 0.527 | 0.545 |
| 1.12  | 0.399 | 0.463 | 0.464 | 0.409 | 0.473 | 0.482 | 0.422 | 0.486 | 0.501 | 0.458 | 0.535 | 0.552 |       |
| 1.14  | 0.422 | 0.485 | 0.480 | 0.429 | 0.490 | 0.495 | 0.438 | 0.498 | 0.512 | 0.470 | 0.545 | 0.560 |       |
| 1.16  | 0.450 | 0.506 | 0.499 | 0.451 | 0.509 | 0.511 | 0.457 | 0.513 | 0.525 | 0.484 | 0.556 | 0.569 |       |
| 1.18  | 0.481 | 0.534 | 0.519 | 0.478 | 0.529 | 0.528 | 0.480 | 0.530 | 0.540 | 0.500 | 0.568 | 0.579 |       |
| 1.20  | 0.513 | 0.558 | 0.541 | 0.506 | 0.551 | 0.546 | 0.504 | 0.548 | 0.556 | 0.518 | 0.582 | 0.590 |       |
| 1.3   | 0.654 | 0.670 | 0.654 | 0.641 | 0.657 | 0.648 | 0.631 | 0.648 | 0.646 | 0.620 | 0.661 | 0.658 |       |
| 1.4   | 0.753 | 0.757 | 0.745 | 0.741 | 0.745 | 0.737 | 0.732 | 0.737 | 0.732 | 0.716 | 0.739 | 0.731 |       |
| 1.5   | 0.822 | 0.820 | 0.811 | 0.813 | 0.812 | 0.805 | 0.805 | 0.805 | 0.800 | 0.791 | 0.803 | 0.796 |       |
| 1.6   | 0.872 |       | 0.859 | 0.866 |       | 0.854 | 0.860 |       | 0.851 | 0.848 |       | 0.849 |       |
| 1.7   | 0.909 |       | 0.895 | 0.905 |       | 0.891 | 0.901 |       | 0.889 | 0.893 |       | 0.887 |       |
| 1.8   | 0.936 |       | 0.922 | 0.934 |       | 0.920 | 0.931 |       | 0.919 | 0.927 |       | 0.919 |       |
| 1.9   |       |       | 0.944 |       |       | 0.943 |       |       | 0.943 |       |       | 0.945 |       |

Table 1 (cont.)

| P <sub>r</sub> |                | 4.0   |       |       | 4.5   |       |       | 5.0   |       |       | 6     |       |       |
|----------------|----------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Z <sub>c</sub> | T <sub>r</sub> | 0.244 | 0.278 | 0.316 | 0.244 | 0.278 | 0.316 | 0.244 | 0.278 | 0.316 | 0.244 | 0.278 | 0.316 |
|                | 0.5            | 0.620 |       |       | 0.696 |       |       | 0.773 |       |       | 0.924 |       |       |
|                | 0.6            | 0.545 |       |       | 0.611 |       |       | 0.677 |       |       | 0.807 |       |       |
|                | 0.7            | 0.500 |       |       | 0.560 |       |       | 0.619 |       |       | 0.737 |       |       |
|                | 0.8            | 0.473 | 0.590 |       | 0.528 | 0.659 |       | 0.583 | 0.728 |       | 0.691 | 0.863 |       |
|                | 0.90           | 0.461 | 0.569 |       | 0.513 | 0.633 |       | 0.564 | 0.697 |       | 0.664 | 0.821 |       |
|                | 0.92           | 0.460 | 0.567 |       | 0.511 | 0.630 |       | 0.562 | 0.693 |       | 0.661 | 0.815 |       |
|                | 0.94           | 0.460 | 0.565 |       | 0.511 | 0.627 |       | 0.560 | 0.689 |       | 0.658 | 0.810 |       |
|                | 0.96           | 0.461 | 0.564 |       | 0.511 | 0.626 |       | 0.560 | 0.686 |       | 0.656 | 0.805 |       |
|                | 0.98           | 0.462 | 0.564 |       | 0.511 | 0.624 |       | 0.560 | 0.684 |       | 0.654 | 0.801 |       |
|                | 1.00           | 0.464 | 0.565 |       | 0.513 | 0.624 |       | 0.560 | 0.683 |       | 0.653 | 0.798 |       |
|                | 1.01           | 0.466 | 0.565 |       | 0.514 | 0.624 |       | 0.561 | 0.682 |       | 0.653 | 0.797 |       |
|                | 1.02           | 0.468 | 0.566 | 0.590 | 0.515 | 0.624 | 0.650 | 0.561 | 0.682 |       | 0.653 | 0.795 |       |
|                | 1.03           | 0.469 | 0.567 | 0.591 | 0.516 | 0.625 | 0.651 | 0.562 | 0.682 |       | 0.653 | 0.794 |       |
|                | 1.04           | 0.471 | 0.568 | 0.591 | 0.518 | 0.625 | 0.651 | 0.564 | 0.682 |       | 0.653 | 0.793 |       |
|                | 1.05           | 0.474 | 0.570 | 0.592 | 0.520 | 0.626 | 0.651 | 0.565 | 0.682 |       | 0.654 | 0.793 |       |
|                | 1.06           | 0.476 | 0.571 | 0.593 | 0.522 | 0.627 | 0.652 | 0.566 | 0.683 | 0.709 | 0.654 | 0.792 |       |
|                | 1.07           | 0.479 | 0.573 | 0.595 | 0.524 | 0.628 | 0.652 | 0.568 | 0.683 | 0.709 | 0.655 | 0.791 |       |
|                | 1.08           | 0.482 | 0.575 | 0.597 | 0.526 | 0.630 | 0.653 | 0.570 | 0.684 | 0.710 | 0.656 | 0.791 |       |
|                | 1.09           | 0.486 | 0.577 | 0.598 | 0.529 | 0.631 | 0.654 | 0.572 | 0.685 | 0.710 | 0.657 | 0.791 |       |
|                | 1.10           | 0.490 | 0.580 | 0.600 | 0.532 | 0.633 | 0.656 | 0.575 | 0.686 | 0.711 | 0.659 | 0.791 | 0.819 |
|                | 1.12           | 0.498 | 0.585 | 0.605 | 0.539 | 0.637 | 0.659 | 0.579 | 0.689 | 0.712 | 0.662 | 0.791 | 0.819 |
|                | 1.14           | 0.507 | 0.592 | 0.610 | 0.546 | 0.642 | 0.662 | 0.586 | 0.692 | 0.715 | 0.666 | 0.792 | 0.819 |
|                | 1.16           | 0.518 | 0.599 | 0.617 | 0.555 | 0.648 | 0.667 | 0.593 | 0.696 | 0.718 | 0.670 | 0.794 | 0.819 |
|                | 1.18           | 0.530 | 0.608 | 0.624 | 0.564 | 0.654 | 0.672 | 0.601 | 0.701 | 0.721 | 0.675 | 0.796 | 0.820 |
|                | 1.20           | 0.544 | 0.618 | 0.632 | 0.575 | 0.662 | 0.678 | 0.609 | 0.706 | 0.725 | 0.681 | 0.799 | 0.822 |
|                | 1.3            | 0.625 | 0.676 | 0.684 | 0.641 | 0.709 | 0.717 | 0.663 | 0.742 | 0.755 | 0.718 | 0.819 | 0.837 |
|                | 1.4            | 0.712 | 0.743 | 0.744 | 0.717 | 0.764 | 0.766 | 0.728 | 0.787 | 0.794 | 0.766 | 0.849 | 0.861 |
|                | 1.5            | 0.785 | 0.804 | 0.800 | 0.786 | 0.818 | 0.816 | 0.792 | 0.834 | 0.836 | 0.818 | 0.882 | 0.890 |
|                | 1.6            | 0.843 |       | 0.851 | 0.842 |       | 0.861 | 0.847 |       | 0.876 | 0.866 |       | 0.919 |
|                | 1.7            | 0.888 |       | 0.891 | 0.888 |       | 0.899 | 0.892 |       | 0.912 | 0.908 |       | 0.948 |
|                | 1.8            | 0.924 |       | 0.923 | 0.925 |       | 0.930 | 0.928 |       | 0.942 | 0.943 |       | 0.973 |
|                | 1.9            |       |       | 0.948 |       |       | 0.956 |       |       | 0.967 |       |       | 0.995 |



TABLE 1 (cont.)

| $P_r$ |       | 7     |       |       | 8     |       |       | 9     |       |       | 10    |       |       |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $Z_c$ | $T_r$ | 0.244 | 0.278 | 0.316 | 0.244 | 0.278 | 0.316 | 0.244 | 0.278 | 0.316 | 0.244 | 0.278 | 0.316 |
|       | 0.5   | 1.076 |       |       | 1.226 |       |       | 1.382 |       |       | 1.534 |       |       |
|       | 0.6   | 0.936 |       |       | 1.064 |       |       | 1.191 |       |       | 1.317 |       |       |
|       | 0.7   | 0.852 |       |       | 0.967 |       |       | 1.079 |       |       | 1.191 |       |       |
|       | 0.8   | 0.797 | 0.995 |       | 0.902 | 1.126 |       | 1.004 | 1.255 |       | 1.105 |       |       |
|       | 0.90  | 0.762 | 0.943 |       | 0.859 | 1.063 |       | 0.953 | 1.181 |       | 1.046 |       |       |
|       | 0.92  | 0.757 | 0.935 |       | 0.852 | 1.053 |       | 0.945 | 1.169 |       | 1.036 |       |       |
|       | 0.94  | 0.753 | 0.928 |       | 0.846 | 1.044 |       | 0.938 | 1.158 |       | 1.028 |       |       |
|       | 0.96  | 0.749 | 0.922 |       | 0.841 | 1.036 |       | 0.931 | 1.148 |       | 1.020 |       |       |
|       | 0.98  | 0.747 | 0.916 |       | 0.837 | 1.023 |       | 0.926 | 1.138 |       | 1.012 | 1.247 |       |
|       | 1.00  | 0.744 | 0.911 |       | 0.833 | 1.022 |       | 0.920 | 1.130 |       | 1.006 | 1.237 |       |
|       | 1.01  | 0.743 | 0.908 |       | 0.831 | 1.018 |       | 0.918 | 1.126 |       | 1.003 | 1.232 |       |
|       | 1.02  | 0.742 | 0.906 |       | 0.830 | 1.015 |       | 0.916 | 1.122 |       | 1.000 | 1.227 |       |
|       | 1.03  | 0.742 | 0.904 |       | 0.829 | 1.012 |       | 0.914 | 1.118 |       | 0.997 | 1.222 |       |
|       | 1.04  | 0.741 | 0.902 |       | 0.827 | 1.009 |       | 0.912 | 1.114 |       | 0.994 | 1.218 |       |
|       | 1.05  | 0.741 | 0.901 |       | 0.826 | 1.007 |       | 0.910 | 1.111 |       | 0.992 | 1.214 |       |
|       | 1.06  | 0.741 | 0.899 |       | 0.825 | 1.004 |       | 0.908 | 1.108 |       | 0.990 | 1.209 |       |
|       | 1.07  | 0.741 | 0.898 |       | 0.825 | 1.002 |       | 0.907 | 1.104 |       | 0.987 | 1.205 |       |
|       | 1.08  | 0.741 | 0.896 |       | 0.824 | 1.000 |       | 0.905 | 1.102 |       | 0.985 | 1.202 |       |
|       | 1.09  | 0.741 | 0.895 |       | 0.823 | 0.998 |       | 0.904 | 1.099 |       | 0.983 | 1.198 |       |
|       | 1.10  | 0.742 | 0.894 |       | 0.823 | 0.996 |       | 0.903 | 1.096 |       | 0.982 | 1.195 |       |
|       | 1.12  | 0.743 | 0.893 |       | 0.823 | 0.993 |       | 0.901 | 1.091 |       | 0.979 | 1.188 |       |
|       | 1.14  | 0.745 | 0.892 | 0.922 | 0.823 | 0.990 |       | 0.900 | 1.086 |       | 0.976 | 1.182 |       |
|       | 1.16  | 0.748 | 0.891 | 0.920 | 0.824 | 0.987 |       | 0.900 | 1.082 |       | 0.974 | 1.176 |       |
|       | 1.18  | 0.751 | 0.891 | 0.919 | 0.826 | 0.986 | 1.017 | 0.900 | 1.079 |       | 0.973 | 1.171 |       |
|       | 1.20  | 0.754 | 0.892 | 0.919 | 0.827 | 0.984 | 1.015 | 0.900 | 1.076 |       | 0.972 | 1.167 |       |
|       | 1.3   | 0.780 | 0.901 | 0.922 | 0.843 | 0.984 | 1.009 | 0.908 | 1.067 | 1.095 | 0.973 | 1.150 | 1.181 |
|       | 1.4   | 0.814 | 0.919 | 0.934 | 0.868 | 0.992 | 1.011 | 0.924 | 1.067 | 1.089 | 0.982 | 1.142 | 1.168 |
|       | 1.5   | 0.854 | 0.941 | 0.952 | 0.898 | 1.005 | 1.020 | 0.946 | 1.072 | 1.089 | 0.997 | 1.141 | 1.161 |
|       | 1.6   | 0.894 |       | 0.973 | 0.930 |       | 1.032 | 0.972 |       | 1.095 | 1.016 |       | 1.159 |
|       | 1.7   | 0.932 |       | 0.994 | 0.963 |       | 1.046 | 0.998 |       | 1.102 | 1.038 |       | 1.161 |
|       | 1.8   | 0.965 |       | 1.013 | 0.993 |       | 1.060 | 1.025 |       | 1.110 | 1.060 |       | 1.164 |
|       | 1.9   |       |       | 1.031 |       |       | 1.073 |       |       | 1.119 |       |       | 1.168 |

Table 1 (cont.)

| $p_r$ |       | 15    |       |       | 20    |       |       | 25    |       |       |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $Z_c$ | $T_r$ | 0.244 | 0.278 | 0.316 | 0.244 | 0.278 | 0.316 | 0.244 | 0.278 | 0.316 |
| 0.6   |       | 1.935 |       |       |       |       |       |       |       |       |
| 0.7   |       | 1.732 |       |       |       |       |       |       |       |       |
| 0.8   |       | 1.595 |       |       | 2.060 |       |       |       |       |       |
| 0.90  |       | 1.493 |       |       | 1.916 |       |       |       |       |       |
| 0.92  |       | 1.476 |       |       | 1.892 |       |       |       |       |       |
| 0.94  |       | 1.460 |       |       | 1.868 |       |       |       |       |       |
| 0.96  |       | 1.445 |       |       | 1.846 |       |       |       |       |       |
| 0.98  |       | 1.431 | 1.771 |       | 1.825 |       |       |       |       |       |
| 1.00  |       | 1.417 | 1.752 |       | 1.805 |       |       |       |       |       |
| 1.01  |       | 1.411 | 1.742 |       | 1.795 |       |       |       |       |       |
| 1.02  |       | 1.404 | 1.733 |       | 1.785 |       |       |       |       |       |
| 1.03  |       | 1.398 | 1.725 |       | 1.776 |       |       |       |       |       |
| 1.04  |       | 1.392 | 1.716 |       | 1.767 |       |       |       |       |       |
| 1.05  |       | 1.386 | 1.708 |       | 1.758 |       |       |       |       |       |
| 1.06  |       | 1.381 | 1.700 |       | 1.749 |       |       |       |       |       |
| 1.07  |       | 1.375 | 1.692 |       | 1.741 |       |       |       |       |       |
| 1.08  |       | 1.370 | 1.684 |       | 1.733 |       | 2.082 |       |       |       |
| 1.09  |       | 1.365 | 1.676 |       | 1.724 |       | 2.071 |       |       |       |
| 1.10  |       | 1.360 | 1.669 |       | 1.717 |       | 2.060 |       |       |       |
| 1.12  |       | 1.350 | 1.655 |       | 1.701 | 2.098 |       | 2.039 |       |       |
| 1.14  |       | 1.342 | 1.641 |       | 1.687 | 2.078 |       | 2.019 |       |       |
| 1.16  |       | 1.333 | 1.629 |       | 1.673 | 2.058 |       | 2.000 |       |       |
| 1.18  |       | 1.325 | 1.617 |       | 1.660 | 2.040 |       | 1.982 |       |       |
| 1.20  |       | 1.318 | 1.605 |       | 1.647 | 2.022 |       | 1.964 |       |       |
| 1.3   |       | 1.290 | 1.555 |       | 1.593 | 1.942 |       | 1.886 |       |       |
| 1.4   |       | 1.271 | 1.516 |       | 1.551 | 1.876 |       | 1.823 |       |       |
| 1.5   |       | 1.261 | 1.485 | 1.518 | 1.520 | 1.821 |       | 1.772 |       |       |
| 1.6   |       | 1.256 |       | 1.489 | 1.497 |       |       | 1.732 |       |       |
| 1.7   |       | 1.255 |       | 1.466 | 1.480 |       | 1.771 | 1.700 |       | 2.067 |
| 1.8   |       | 1.257 |       | 1.446 | 1.467 |       | 1.732 | 1.675 |       | 2.012 |
| 1.9   |       |       |       | 1.430 |       |       | 1.699 |       |       | 1.964 |

**Table 1 (cont.)**

| $p_r$ | 0.01  | 0.1   | 0.2   | 0.3   | 0.4   | 0.5   | 0.6   | 0.7   | 0.8   | 0.9   | 1.0   | 1.05  | 1.10  |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $Z_c$ | 0.316 |       |       |       |       |       |       |       |       |       |       |       |       |
| $T_r$ |       |       |       |       |       |       |       |       |       |       |       |       |       |
| 2.0   | 1.000 | 0.997 | 0.994 | 0.992 | 0.990 | 0.988 | 0.986 | 0.983 | 0.981 | 0.979 | 0.978 | 0.977 | 0.976 |
| 2.5   | 1.000 | 1.000 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 | 1.000 | 1.000 | 1.000 |
| 3.0   | 1.000 | 1.001 | 1.001 | 1.002 | 1.003 | 1.004 | 1.005 | 1.006 | 1.006 | 1.007 | 1.008 | 1.009 | 1.009 |
| 4     | 1.000 | 1.001 | 1.003 | 1.004 | 1.005 | 1.007 | 1.008 | 1.009 | 1.011 | 1.012 | 1.013 | 1.014 | 1.015 |
| 5     | 1.000 | 1.001 | 1.003 | 1.004 | 1.006 | 1.007 | 1.009 | 1.010 | 1.012 | 1.013 | 1.014 | 1.015 | 1.016 |
| 6     | 1.000 | 1.001 | 1.003 | 1.004 | 1.005 | 1.007 | 1.008 | 1.010 | 1.011 | 1.013 | 1.014 | 1.014 | 1.015 |
| 8     | 1.000 | 1.001 | 1.002 | 1.004 | 1.005 | 1.006 | 1.007 | 1.008 | 1.010 | 1.011 | 1.012 | 1.013 | 1.013 |
| 10    | 1.000 | 1.001 | 1.002 | 1.003 | 1.004 | 1.005 | 1.006 | 1.007 | 1.008 | 1.010 | 1.011 | 1.011 | 1.011 |

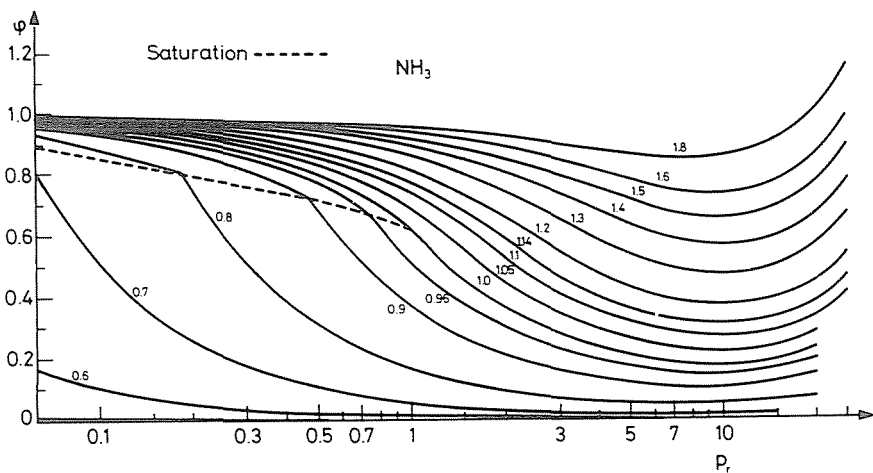
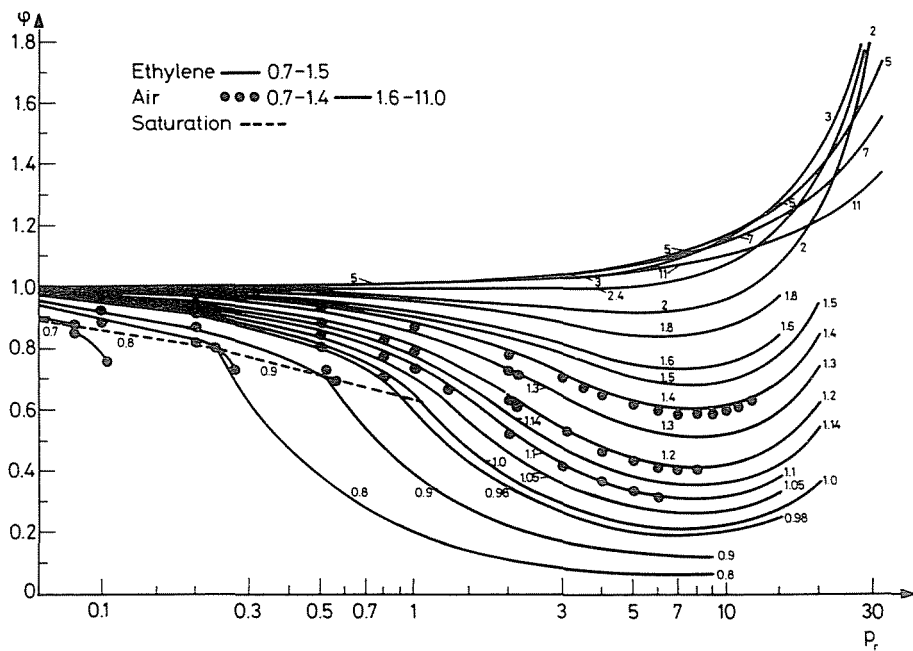
  

| $p_r$ | 1.15  | 1.20  | 1.25  | 1.30  | 1.35  | 1.40  | 1.45  | 1.50  | 1.6   | 1.7   | 1.8   | 1.9   | 2.0   |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $Z_c$ | 0.316 |       |       |       |       |       |       |       |       |       |       |       |       |
| $T_r$ |       |       |       |       |       |       |       |       |       |       |       |       |       |
| 2.0   | 0.975 | 0.974 | 0.973 | 0.972 | 0.972 | 0.971 | 0.970 | 0.970 | 0.968 | 0.967 | 0.966 | 0.965 | 0.964 |
| 2.5   | 1.000 | 1.000 | 1.000 | 1.001 | 1.001 | 1.001 | 1.001 | 1.001 | 1.002 | 1.003 | 1.003 | 1.004 | 1.005 |
| 3.0   | 1.010 | 1.010 | 1.011 | 1.012 | 1.012 | 1.013 | 1.013 | 1.014 | 1.015 | 1.017 | 1.018 | 1.019 | 1.020 |
| 4     | 1.016 | 1.017 | 1.017 | 1.018 | 1.019 | 1.020 | 1.021 | 1.021 | 1.023 | 1.024 | 1.026 | 1.028 | 1.029 |
| 5     | 1.017 | 1.018 | 1.018 | 1.019 | 1.020 | 1.021 | 1.021 | 1.022 | 1.024 | 1.025 | 1.027 | 1.028 | 1.030 |
| 6     | 1.016 | 1.017 | 1.018 | 1.018 | 1.019 | 1.020 | 1.020 | 1.021 | 1.023 | 1.024 | 1.026 | 1.027 | 1.029 |
| 8     | 1.014 | 1.015 | 1.015 | 1.016 | 1.017 | 1.017 | 1.018 | 1.018 | 1.020 | 1.021 | 1.022 | 1.024 | 1.025 |
| 10    | 1.012 | 1.013 | 1.013 | 1.014 | 1.014 | 1.015 | 1.015 | 1.016 | 1.017 | 1.018 | 1.019 | 1.020 | 1.022 |

| $p_r$ | 2.2   | 2.4   | 2.6   | 2.8   | 3.0   | 3.5   | 4.0   | 4.5   | 5.0   | 6     | 7     | 8     | 9     |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $Z_c$ | 0.316 |       |       |       |       |       |       |       |       |       |       |       |       |
| $T_r$ |       |       |       |       |       |       |       |       |       |       |       |       |       |
| 2.0   | 0.963 | 0.962 | 0.961 | 0.961 | 0.961 | 0.964 | 0.969 | 0.976 | 0.987 | 1.013 | 1.047 | 1.085 | 1.127 |
| 2.5   | 1.006 | 1.008 | 1.010 | 1.012 | 1.014 | 1.021 | 1.028 | 1.037 | 1.047 | 1.068 | 1.096 | 1.125 | 1.156 |
| 3.0   | 1.023 | 1.026 | 1.029 | 1.032 | 1.035 | 1.043 | 1.051 | 1.061 | 1.071 | 1.092 | 1.115 | 1.140 | 1.166 |
| 4     | 1.032 | 1.036 | 1.039 | 1.043 | 1.046 | 1.055 | 1.063 | 1.072 | 1.082 | 1.100 | 1.122 | 1.141 | 1.161 |
| 5     | 1.033 | 1.036 | 1.040 | 1.043 | 1.046 | 1.054 | 1.063 | 1.071 | 1.079 | 1.096 | 1.113 | 1.131 | 1.148 |
| 6     | 1.032 | 1.035 | 1.038 | 1.041 | 1.044 | 1.051 | 1.059 | 1.067 | 1.074 | 1.089 | 1.105 | 1.120 | 1.135 |
| 8     | 1.027 | 1.030 | 1.033 | 1.035 | 1.038 | 1.044 | 1.051 | 1.057 | 1.063 | 1.076 | 1.089 | 1.102 | 1.114 |
| 10    | 1.024 | 1.026 | 1.028 | 1.030 | 1.033 | 1.038 | 1.044 | 1.049 | 1.055 | 1.066 | 1.077 | 1.087 | 1.098 |

Table 1 (cont.)

| $p_r$ | 10    | 15    | 20    | 25    |
|-------|-------|-------|-------|-------|
| $Z_c$ | 0.316 |       |       |       |
| $T_r$ |       |       |       |       |
| 2.0   | 1.172 | 1.417 | 1.671 | 1.922 |
| 2.5   | 1.189 | 1.371 | 1.567 | 1.764 |
| 3.0   | 1.193 | 1.342 | 1.501 | 1.662 |
| 4     | 1.182 | 1.296 | 1.416 | 1.537 |
| 5     | 1.166 | 1.259 | 1.357 | 1.456 |
| 6     | 1.151 | 1.230 | 1.313 | 1.397 |
| 8     | 1.127 | 1.189 | 1.251 | 1.315 |
| 10    | 1.109 | 1.160 | 1.212 | 1.263 |



separate Figure (Fig. 9). Fugacity coefficients have been calculated from the following formula:

$$\ln \varphi = Z_{0.01} - 1 + \int_{0.01}^{p_r} (Z - 1) d \ln p_r \quad (5)$$

where  $Z_{0.01}$  stands for the compressibility factor at  $p_r = 0.01$ .

It is true that nowadays equations of states are rather used employing computers but a quick orientation and calculations not requiring very accurate result can take advantage of the tables and charts enclosed in the present paper.

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