A COMPUTER PROGRAM FOR SIMULATING SOME SIGNAL-TO-NOISE ENHANCEMENT METHODS USED IN HIGH PERFORMANCE INSTRUMENTS

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Abstract

A computer program for a personal computer was written to test seven noise filtering methods with simulated signals buried in noise for education. The methods tested were as follows: filtering by an RC network, accumulation, sampling with integration, smoothing by Savitzky-Golay filter, combination of accumulation and Savitzky-Golay smoothing, combination of integral sampling and Savitzky-Golay smoothing and filtering by Fourier transformation. The two combined methods have yielded the best results.

Introduction

High performance laboratory measuring and analytical instruments output their results in the form of (electrical) signals covered with noise. Sometimes the noise level is so high that the important signal is completely masked. The minimum of the detectable effect and the quality of the measurement, however, depend on the signal-to-noise ratio (S/N) of the output of the instrument.

Many techniques are available to increase this ratio during or after the measuring process. It is very useful to compare the performances of the different signal-to-noise ratio enhancement methods in common circumstances. In this paper a computer program will be outlined by which seven different methods can be tested by simulated digital samples of signals buried in noise. Additivity of the signal and the noise is supposed for the simulation. Further on, the autocorrelation functions and the power spectra of noise are also determined during the run before and after the noise suppressions to estimate the transfer functions of the different techniques.

The interactive program FILT has been written in BASIC language for a Commodore 64 desk-top computer and compiled by AUSTROCOMP compiler to increase the running speed. For similar reason the graphic routine has been written in machine language. The total running time of the program is very long,—about 3 hours—, because of many time consuming plottings. The program has been used successfully for education, too.

Some questions of the sample simulation

The flow-chart diagram of our program is shown in Fig. 1. The signals simulating the sampled and digitized output of any instrument are combined by the program itself from known signals and from some amount of noise. The noiseless signal is a sum of Lorentzian bands (four bands maximum) given by the following formula:

$$y_{(t)} = \sum_{k=1}^{4} \frac{A_k \gamma_k^2}{(t-t_k)^2 + \gamma_k^2}, \tag{1}$$

where A_k is the maximum of the k-th band, t_k is the position of this band and γ_k is its half-band width. The noise is a Gaussian broadband one. In order to save time the signals and the noise are generated previously by auxiliary programs SGEN and ZGEN, respectively. Using Eq. (1) and the parameters selected from the keyboard (t_k has to be between 1 and 256) signals containing 256 sampled values in each case can be generated. These signals are stored on a floppy diskette in sequential files labelled by file names given by the user. E.g. the signal with a file name SPEC1 is shown in Fig. 2. The noise file containing 2816 data is calculated by the program ZGEN which applies repetitively the random function of the computer. In order to decrease the correlation among the data only every fifth random number is taken into account in the twelve member averages with which Gaussian distribution of noise is approximated [1]. The distribution of noise depicted in Fig. 3 shows that this approximation is good. The correlation of noise may be characterized by the autocorrelation function $R_{zz(\tau)}$. This function is estimated here by the following expression distinguished by' from $R_{zz(\tau)}$:

$$R'_{zz(m)} = \frac{1}{M-m} \sum_{i=1}^{M-m} z_{(i)} z_{(i+m)}, \tag{2}$$

where the delay m runs from 0 to 255 and M is 256. A result determined from a random set of the noise file is depicted in Fig. 4. It can be seen that the function obtained is similar to the one describing white noise, but the fluctuation is relatively high [2]. Because of the low value of M the result is only a poor estimate of the real autocorrelation function. Thus the power spectrum of noise derived from $R'_{zz(m)}$ by Fourier transformation according to Wiener-Khinchin theorem:

$$S'_{(k)} = \mathscr{F}\{R'_{zz(m)}\}, \tag{3}$$

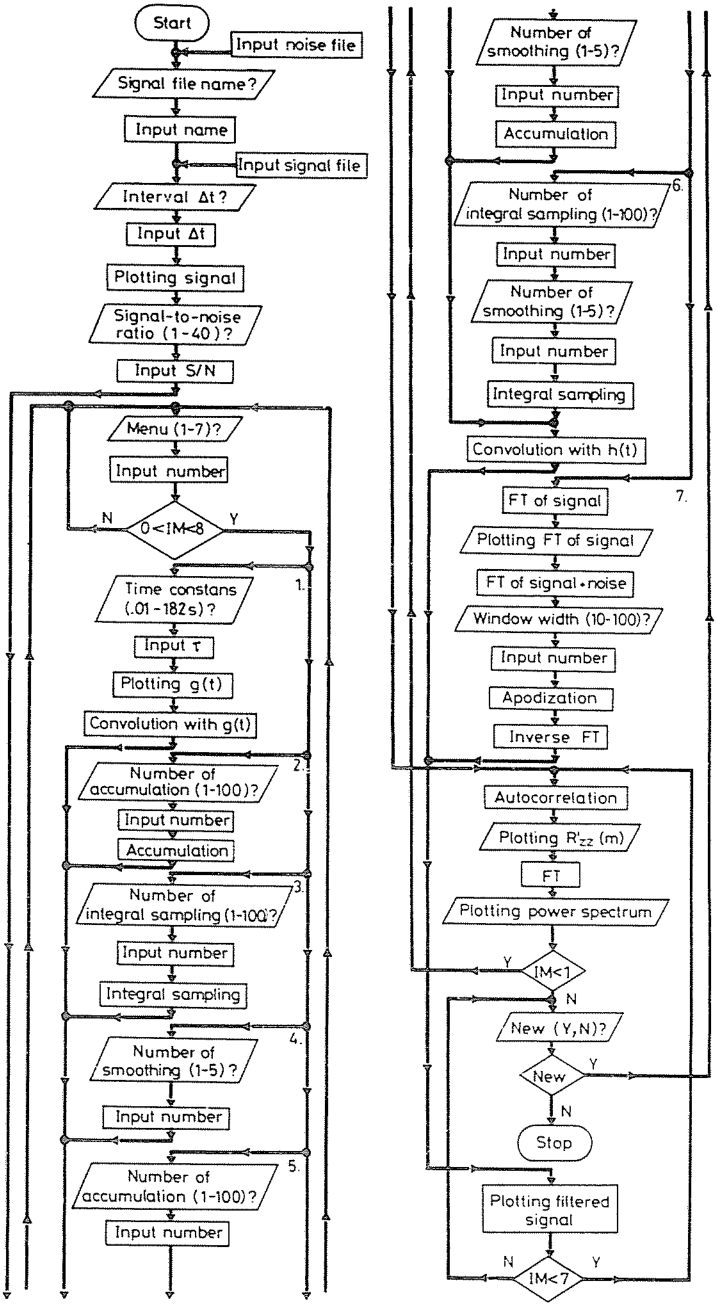

Fig. 1. Flow-chart diagram of the program FILT

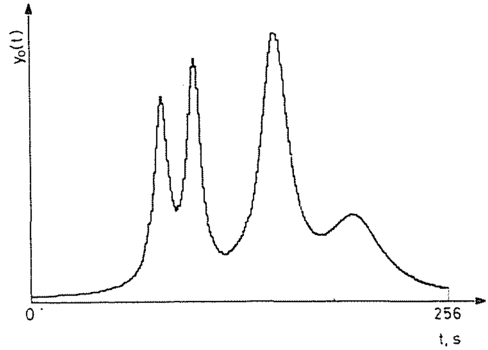

Fig. 2. The signal SPEC1 without noise. This contains 256 data points among which the max. value is 7360 and the min. one is 86. The time interval is 1 s

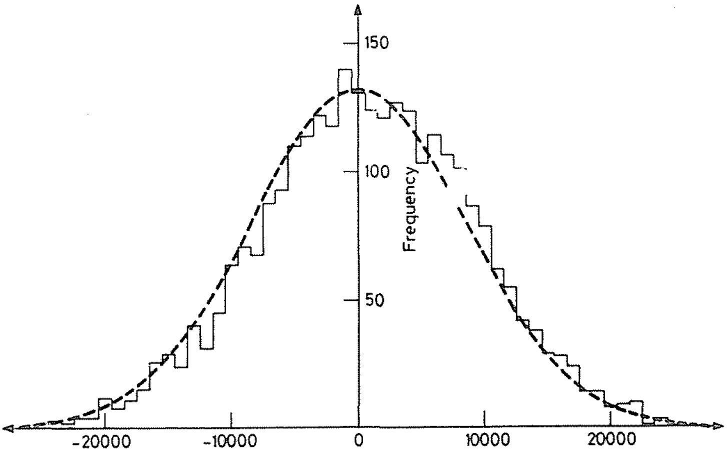

Fig. 3. Distribution of the simulated noise. The dashed line denotes Gaussian distribution

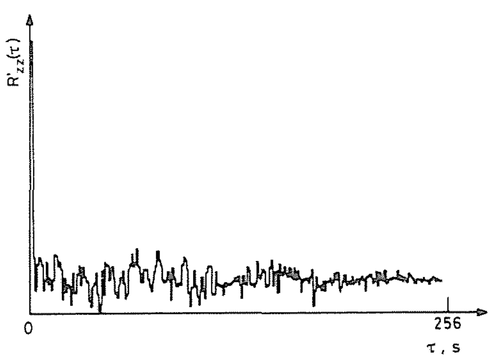

Fig. 4. The autocorrelation function of unfiltered noise estimated by Eq. (2). The maximum delay is 256 s. The variance from $R'_{zz(0)}$ is 1180.8 units

has also a high variance level as it can be seen in Fig. 5. Unfortunately it was impossible to decrease this high variance level in our case. Decreasing the variance by a factor $\sqrt{n}$ requires increasing the value M by n [2]. E.g. if one desires to obtain the power spectrum with 20 times lower variance than at present the value M has to be increased to 102 400. We have no memory and no time for simulating the quality of power spectra determined by very fast

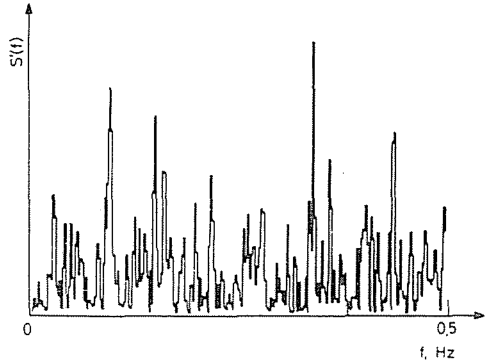

Fig. 5. The power spectrum of unfiltered noise. The maximum of the frequency domain is 0.5 Hz. The max. value in the power spectrum is 8 088 219

multichannel real-time correlators [2, 3]. The variance σ_z of noise, however, can be well estimated by the square root of the value of $R'_{zz(0)}$, thus it can be used successfully for calculating the signal -to-noise ratio. S/N is defined to be the ratio of maximum peak height in the signal to the value σ_z. A typical value of 6.23 for S/N is used for starting data in this paper. (See Fig. 6).

It is clear that the frequency of sampling in the presence of broadband (white) noise may not be in accordance with the sampling theorem. If we want to represent accurately the spectral distribution of noise, an enormously high sampling rate has to be used. The minimum of the sampling frequency is given by the frequency limit of the noiseless signal, i.e. $f_s > 2f_m$, where f_m is the frequency limit or Nyquist folding frequency. In the case of digital noise filtering some oversampling is recommended relative to the minimum one [4]. This is realized in our example where $f_s > f_s$. (See Fig. 2). This oversampling, however, is undersampling for noise. Thus the noise spectrum transformed from the sampled data is distorted by the effect of aliasing [4]. From this it follows that the noise function reconstructed from the sampled data by the following expression:

$$z_{(t)} = \sum_{l=-\infty}^{+\infty} z_{(l\Delta t)} \frac{\sin \pi f'_m (t - l\Delta t)}{\pi f'_m (t - l\Delta t)}, \tag{4}$$

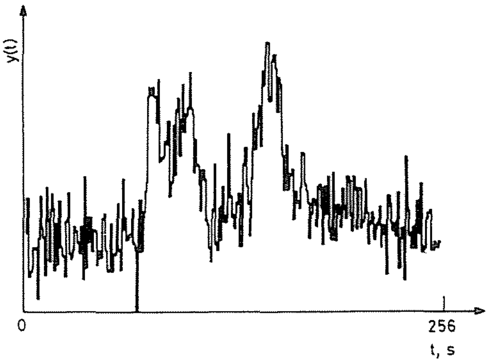

Fig. 6. The signal buried in noise. S/N is 6.23

where $\Delta t = 1/f'_s$ according to Shannon's sampling theorem, can not give real information about the noise fluctuation between the sampled points [4].

This is the general situation in the case of the sampling of output signals of any instrument for digital filtering. Fortunately, no effect of aliasing can be observed in our case when the noise is a broadband one.

Treatment of the signal-to-noise enhancement methods applied

The signal-to-noise enhancement methods tested by our program are treated in the order of the program branches. (See Fig. 1).

1. This choice corresponds to a filtering by a low-pass RC network. Noise suppression by a low-pass RC filter is a typical analog method used in its simple form or as a part of a lock-in amplifier. In both cases the important signal is also filtered. The digital simulation of the effect of an RC filter can be represented mathematically as a convolution of the sampled data by a weighting function $g_{(t)}$:

$$y_{f(i)} = \sum_{l=0}^{L} g_{(i-l)} y_{(i)}, \tag{5}$$

where $y_{f(i)}$ denotes the smoothed data and the maximum of L is 100. The function $g_{(t)}$ is asymmetric in time having a value of zero for $t > 0$. Its form is:

$$g_{(t)} = \exp(+t/\tau) / \int_{-\infty}^{0} \exp(+t/\tau)\, dt, \tag{6}$$

where τ is the time constant and $\tau = RC$. This weight function is proportional to the inverse Fourier transform of the complex transfer function of the RC filter.

In our example the value chosen for the time constant is 15 s and the sampling interval is 1 s. This yields a very strong asymmetric distortion in the observed signal and a moderate S/N enhancement. (See Fig. 7). It can be seen that the analog filtering may be applied only to the noise suppression in the frequency domain for $f>f_m$. However, such a filtering is very useful before the sampling.

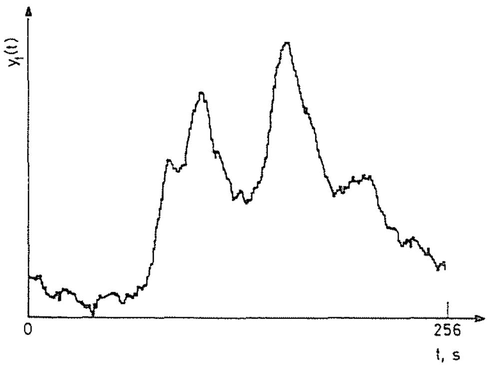

Fig. 7. The signal smoothed by a low-pass RC filter. The time constant is 15 s. S/N enhancement is 3.95

2. This is the accumulation or ensemble averaging which can be applied if the signal is repeatable and the instrument has any hardware for the coherent output and sampling the signals. The advantage of this method is that only the noise is filtered.

In order to simulate the effects of this method on the noise, the program adds N different noise records each containing 256 data, the records being randomly selected from the noise block. Because of the low correlation a small shift of about ten positions is enough between two records. Following Ernst's idea [5] the autocorrelation function of the accumulated noise can be derived. The result is:

$$R'^{(N)}_{zz(m)} = \frac{1}{N^2} \sum_{l=-N+1}^{N-1} (N - |l|) R'_{zz(m+lM)}, \tag{7}$$

where $R'_{zz(m+lM)}$ denotes an original autocorrelation function with a delay lM. Since practically no correlation can be found between two functions shifted by M, a simplified form can be used as follows [5]:

$$R'^{(N)}_{zz(m)} = \frac{1}{N} R'_{zz(m)}. \tag{8}$$

Thus the features of the autocorrelation function and that of the power spectrum are very similar to the original ones. The method produces a uniform

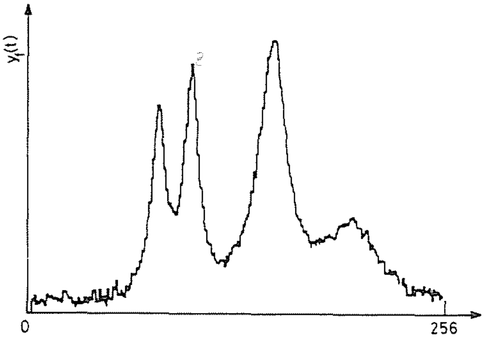

Fig. 8. The signal after 100 accumulations. S/N enhancement is 10.57. The max. value in the signal is 7369 and the min. one is −143

filtering for noise. The result is shown in Fig. 8. The number of accumulations N is typically 100. The S/N enhancement obtained is in accordance with the theoretical value $\sqrt{N}$ within an error of 10%. This error comes from the low value of M, but the reproducibility is better, being about 3%.

3. This is the sampling with integration which can be used if the measurement can be executed step by step using some hardware which stops the recording periodically. The sampling and the integration of the N samples are done during these stopping times. Also in the case of this method the filtering is limited only to noise. Many times, however, the digital filtering by any integration is applied to data sampled from continuously recorded signals (boxcar average, moving window average, ect.). Thus not only the noise is filtered. These methods are not discussed here.

The program simulates the effects of sampling with integration on the noise by averaging N samples of noise records 256 times randomly selected from the noise array. The autocorrelation function of the integrated noise is not given in the literature, but in a similar way as in Eq. (7) the following expression can be derived:

$$R'^{(N)}_{zz(m)} = \frac{1}{N} \sum_{l=-N+1}^{N-1} (1 - |l|/N) R'_{zz(m+l/N)}, \tag{9}$$

in which not large lM delays as in Eq. (7) but small l/N ones appear. So the correlation cannot be neglected and no simple expression for S/N enhancement can be obtained. It is easy to derive the transfer function of the method. From Eq. (9) it is clear that $R'^{(N)}_{zz(m)}$ is obtained by convoluting $R'_{zz(m)}$ with a triangle function. The square of the transfer function is the Fourier transform of the triangle one:

$$G^2_{(f)} = \frac{1}{N} \left\{ \frac{\sin \pi \Delta t f}{\pi \Delta t f} \right\}^2 . \tag{10}$$

In our case, when the noise is very broadband, the side-lobes of this function are damping slowly and its first zero value occurs at $1/\Delta t$, i.e. at 1 Hz. The autocorrelation function and the power spectrum have a feature similar to accumulation. The S/N enhancement shows only relatively small deviation from the $\sqrt{N}$ rule. The filtered signal is depicted in Fig. 9.

4. This choice is the smoothing by a Savitzky-Golay polynomial filter which is a typical example of digital filtering. No hardware supplement for the

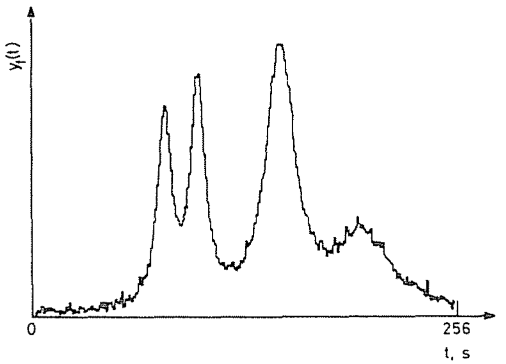

Fig. 9. The signal recorded by integral sampling with 100 averages. S/N enhancement is 12.04. The max. value in the signal is 7243 and the min. one is -11

instrument is necessary to use this method, the smoothing is carried out after the measurement. This type of smoothing is mathematically a convolution of the stored data in the time domain with a symmetric weighting function, i.e.:

$$y_{f(i)} = \sum_{l=-N}^{N} h_{(i-l)} y_{(i)} \,. \tag{11}$$

The symmetry of weight function means an important difference from the analog RC filters which are equivalent to a convolution with an asymmetric function. According to the idea of Savitzky and Golay [3] the determination of the coefficients $h_{(k)}$ is based on the least squares fitting. A useful method for obtaining formulae for the coefficients $h_{(k)}$ has been elaborated by Ernst (see e.g. [4]). The polynomial forms of formulae containing only the number of coefficients $2N+1$ and the index k, depend on the order of the polynomial. In this program a weighting function having 31 coefficients is used. The coefficients are quartic polynomials as follows:

$$h_{(k)} = \frac{15(15N^4+30N^3-35N^2-50N+12)-35(2N^2+2N-3)k^2+63k^4}{4(2N+5)(2N+3)(2N+1)(2N-1)(2N-3)}, \tag{12}$$

where index k runs from $-N$ to $+N$ and $N=15$.

The transfer function $G_{(f)}$ of our Savitzky–Golay filter has a sharp frequency cut-off at about 0.07 Hz, but the high frequency rejection is not perfect, the side-lobes are damping slowly [4]. According to Bromba and Ziegler the S/N enhancement can be estimated by $(h_{(0)})^{-1/2}$ [7]. The deviation of the experimental value (2.48) from the estimated one (2.96) is relatively high.

In order to eliminate the residuum of the high frequency noise, multiple smoothing is recommended. The result of smoothing four times as an optimum is shown in Fig. 10. The S/N enhancement is moderate and the bandwidth of the transfer function decreases from 0.07 Hz to 0.06 Hz (see Fig. 11). The shape of the filtered signal shows that the method does not decrease the low frequency noise, it perfectly eliminates, however, the high frequency one. The transfer function approximates a boxcar window [4]. Theoretically this method can

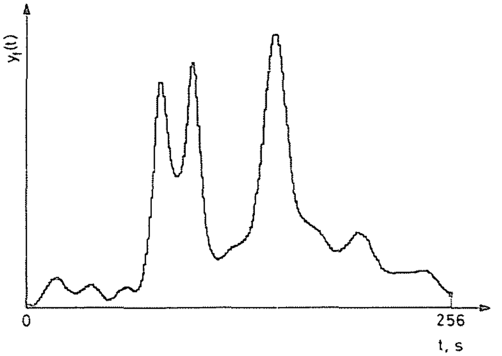

Fig. 10. The signal smoothed 4 times with a Savitzky–Golay polynomial filter. The filter is a 31 point quartic one. S/N enhancement is 2.79. The max. value in the signal is 7905 and the min. one is −64

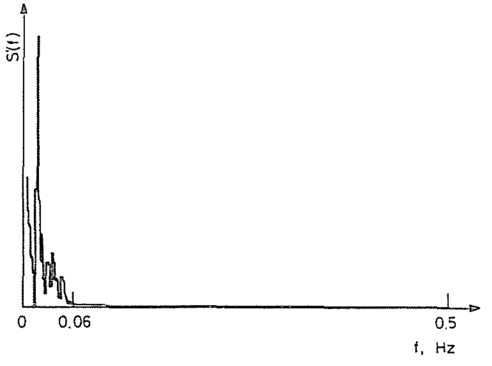

Fig. 11. The power spectrum of noise smoothed 4 times with a Savitzky–Golay filter

also yield a distortion in the signal, but in the case of a well oversampled signal it is easy to construct a Savitzky–Golay filter which eliminates the noise only.

5. This is a combination of accumulation and Savitzky–Golay smoothing. The idea is based on the fact that the accumulation uniformly decreases the noise level in the total frequency domain, a well chosen Savitzky–Golay filter, however, eliminates the residuum of the high frequency noise. The filtered signal is shown in Fig. 12. Perfect reconstruction and an enormous S/N enhancement can be observed. The power spectrum is depicted in Fig. 13.

6. This is a combination of integral sampling and Savitzky–Golay smoothing. The result depicted in Fig. 14 is similar to the previous one.

7. This is a Fourier transform method for noise suppression which is also a purely numerical one. The sampled data are Fourier transformed after the

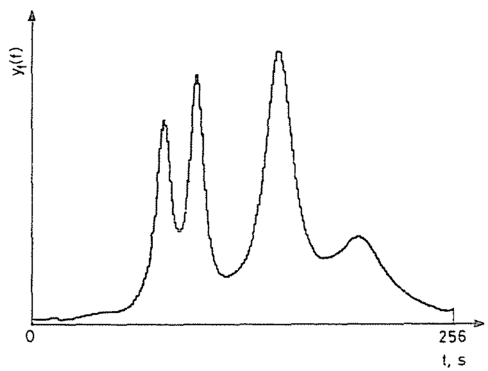

Fig. 12. The signal after 100 accumulations and smoothed 4 times by a Savitzky–Golay filter. S/N enhancement is 45.01. The max. value in the signal is 7407 and the min. one is 75

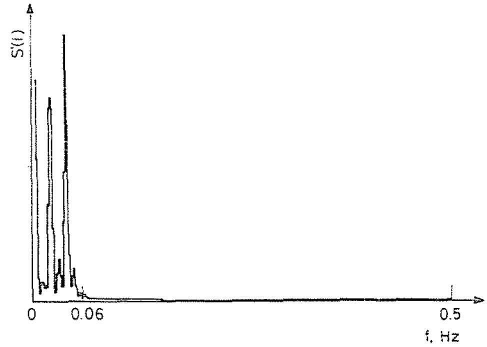

Fig. 13. The power spectrum of noise after 100 accumulations and smoothed 4 times by a Savitzky–Golay filter

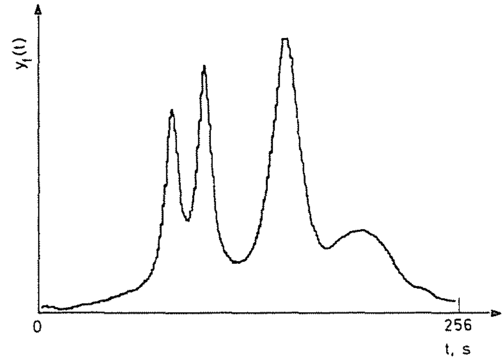

Fig. 14. The signal recorded by integral sampling with 100 averages and smoothed 4 times by a Savitzky–Golay filter. S/N enhancement is 42.42. The max. value in the signal is 7371 and the min. one is 28

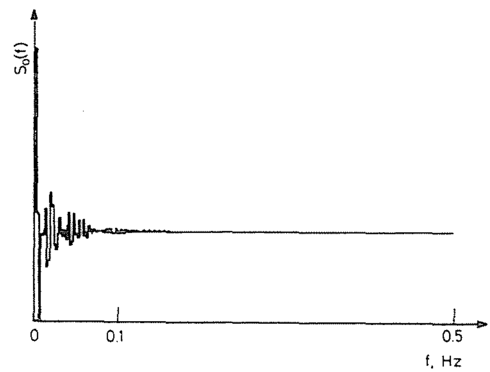

Fig. 15. Fourier transform of the signal without noise

complete measuring process and the filtering is executed in the frequency domain by apodization. In the course of apodization the Fourier transform is multiplied by an appropriate damping function which rejects the higher frequency part of the spectrum. After inverse Fourier transformation a noise filtered signal can be reconstructed [8].

The Fourier transform of our signal and the transform of this signal with noise are shown in Fig. 15. and in Fig. 16, respectively. For apodization a boxcar window having about 0.1 Hz width is used. Other damping functions have been tested, but the boxcar window has been found to be the best. The reconstructed signal after apodization is depicted in Fig. 17. Owing to the characteristics of the apodization (filtering) function the method does not decrease the low frequency noise.

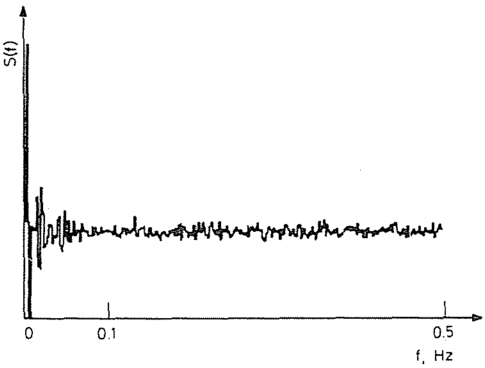

Fig. 16. Fourier transform of the unfiltered signal

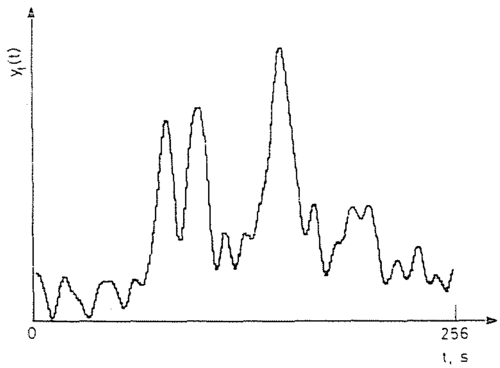

Fig. 17. Inverse Fourier transform of the signal multiplied in the frequency domain by a boxcar window having 0.09766 Hz width

Discussion

From the results given by our program it may be seen that S/N enhancement does not characterize perfectly the performance of a filtering method. The distortion of signal caused by the filtering and the spectrum of the residual noise are also very important pieces of information. Comparing the seven S/N enhancement methods tested the following conclusions can be drawn.

i. Only a moderate filtering by an RC network is practical to use because of the strong asymmetric distortion.

ii. Smoothing by Savitzky–Golay filter and filtering by Fourier transformation are appropriate to eliminate only high frequency noise. A stronger filtering causes a symmetric distortion of the signal.

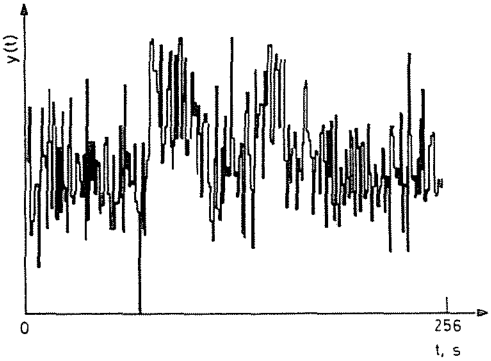

Fig. 18. The signal buried in noise. S/N is 1.87

iii. Accumulation and sampling with integration decrease the noise in the total frequency domain. Unfortunately, these methods require some supplementary hardware.

iv. The best methods are the combined ones. In order to show the performance of these methods, the filtering of a very noisy signal has been carried out. The signal with noise is depicted in Fig. 18. S/N is lower than 2. The result after filtering is shown in Fig. 19. A similar enhancement can be obtained by the other combined method.

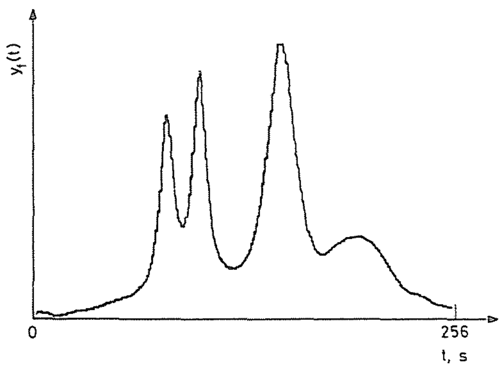

Fig. 19. The signal after 100 accumulations and smoothed 4 times by a Savitzky–Golay filter. S/N enhancement is 47.97 from the original 1.87. The max. value is 7477 and the min. one is 21

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