POLARIZATION PROPERTY CHANGING DUE TO SCATTERING OF PLANE-POLARIZED LIGHT

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Summary

The plane-polarized incident beam may be plane-polarized or elliptically polarized, when reflected from the edge surfaces of the optically less dense medium. The geometrical state of the polarization plane depends on its orientation relative to surface and angle of incidence for incident beam. According to van de Hulst approximation a theoretical model is presented for calculating the depolarization ratio of reflected light.

Plane-polarized light will be depolarized by scattering from substances having optical inhomogeneities [1]. It is verified experimentally [2] that depolarization is selective, i.e. it is strong in scattering from a nonabsorbing painted surface and it is very weak from a highly absorbing one. For scattering samples the degree of depolarization as the information, coded in the change of polarization state of light can be used to improve the signal to noise ratio containing the spectral information of analytical value [3].

In this paper we wish to interpret the process of depolarization in the scattering of polarized light. For this a theoretical scattering model is constructed and the degree of depolarization is calculated for some cases.

Theory

If the geometrical size of optical inhomogeneities is very large compared to the wavelength of light, then the scattered radiation can be described by the laws of geometrical optics. Van de Hulst [4] described what happened to the rays that hit a very large sphere. The incident light is a narrow beam isolated from the part of the wave that gets to the surface of the sphere. The width of the beam is much larger than the wavelength and small compared to the radius a of the sphere. The sphere is transparent and the refractive index n is real and differs sufficiently from its surroundings. The incident beam hitting the surface gives a reflected and a refracted ray with directions obeying Snell's law (Fig. 1). When the refracted ray reaches the surface again part of it is refracted and leaves the sphere and the other part is reflected internally. This goes on ad infinitum. If p is the number of refractions (for $p=0$ the ray is externally reflected whithout any refraction, for $p=1$ the ray is refracted once and leaves
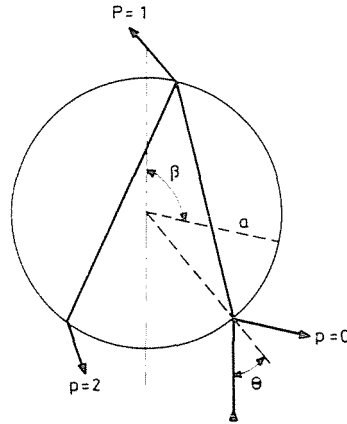
Fig. 1. Reflected, twice refracted and refracted-reflected-refracted part of a single incident beam. Symbols are explained in the text

the sphere, etc.) the intensity divided by successive reflection of a single ray at a large distance r from the sphere:

$$I_{1,2}(p, \theta) = \frac{k^2 * a^2}{r^2} I_0 * \varepsilon_{1,2}^2 * D(\theta, \beta) \tag{1}$$

where θ is the angle of incidence; β is the angle of scattering; k is the wave number defined as $k = \frac{2 * \pi}{\lambda}$; I_0 is the intensity of the incident light; $\varepsilon_{1,2}$ are the Fresnel coefficients, where 1 and 2 refer to the reflected light, plane-polarized in the incident plane and perpendicular to it, respectively; $D(\theta, \beta)$ is called the divergence. For a ray with $p = 0$ we obtain:

$$I_{1,2}(\theta) = \frac{k^2 * a^2}{2 * r^2} I_0 * \varepsilon_{1,2}^2 \tag{2}$$

The phase of the orthogonal components can be expressed by the Fresnel coefficients.

Theoretical model

The hypothetical sample being used as a scattering model has two constituents with real refractive indices n_1 and n_2. They differ from each other in such a way that $n_1 < n_2$, and they are nonabsorbing, transparent, isotropic dielectrics.

The curvature radii of the edge surfaces of the components tend to infinity $(a \to \infty)$ and their geometrical sizes are much larger than the width of the incident beam. A unit volume of the sample contains all edge surfaces of our choice with the same probability.

Calculation

The conditions we take into consideration are as follows. First, we are not interested in diffraction, since the amplitudes and phases of two orthogonal diffracted components are equal each other. Second, the case is not discussed here when $n_2 = n_1 + \mu$ where $\mu \to 0$, the Fresnel coefficients become unity ($\varepsilon_1 = \varepsilon_2 = 1$). Third, we examine the effects when a light beam arrives from an optically dense medium at the surface of one optically less dense. Fourth, to determine depolarization we take into account the reflected part of light only. Fifth, the vibration plane of the electric vector of incident light is defined as observed from the direction of propagation of the incident light, while that of reflected light is defined as observed from a direction opposite to the direction of propagation. Let the incident beam be plane-polarized with a vibration plane of the electric vector in the xy plane (Fig. 2). Then the incident beam with its propagation vector in the negative z direction hits an arbitrarily oriented edge surface of optical inhomogeneities (its geometrical location is defined by the normal $\vec{q}_i$ to the edge surfaces) with an angle of θ_i incidence. The angle between the plane of incidence and the vibration plane of incoming wave is φ_j. To obtain the change of the vibration plane we have to resolve the amplitude of the
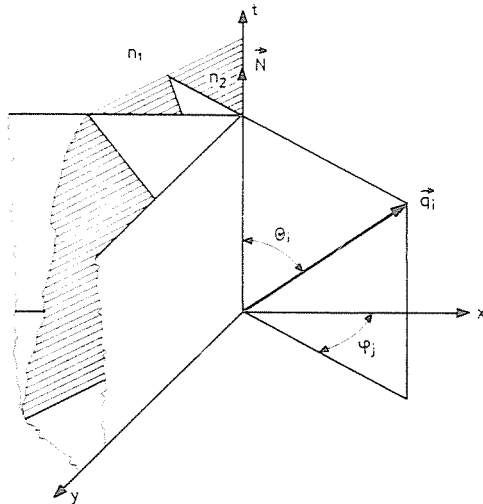

Fig. 2. Geometry for an arbitrarily oriented edge surface. The xyz coordinate system is fixed to the sample; other symbols are explained in the text

Table 1

Fresnel reflection coefficients and phase differences for orthogonal components

In the plane	Amplitudes and phase differences		
	$\theta_i < \theta_B$	$\theta_B < \theta_i < \theta_c$	$\theta_i > \theta_c$
of incidence	0	π	$\tan\dfrac{\delta_1}{2} = -\dfrac{\sqrt{\sin^2\theta_i - n^2}}{n^2 - \cos\theta_i}$
		$R1 = -\dfrac{\tan(\theta_t - \theta_i)}{\tan(\theta_t + \theta_i)} A1$	$|R1| = |A1|$
perpendicular to the plane of incidence	0	0	$\tan\dfrac{\delta_2}{2} = -\dfrac{\sqrt{\sin^2\theta_i - n^2}}{\cos\theta_i}$
		$R2 = \dfrac{\sin(\theta_t - \theta_i)}{\sin(\theta_t + \theta_i)} A2$	$|R2| = |A2|$

A1, A2 and R1, R2 are electric field amplitudes of incident and reflected beams in the plane of incidence and perpendicular to it; θ_i, θ_t are the angles of incidence and refraction; δ_1, δ_2 are extra phase differences for A1, R1 and A2, R2 at total internal reflection.

electric vector into two components polarized in the plane zq_i (A1) and perpendicular to this plane (A2). After reflection the reflected part of components (R1 and R2) would be added. Thus, the vibration plane of reflected light is defined by the interference of two components, the result of which depends on their amplitudes and phases.

Expressions for Fresnel coefficients and phase differences depending on the angle of incidence are shown for our case in Table 1 [5, 6].

For computer modelling of depolarization we assume a whole range of reflections generated by beams of unit intensity. For this, several incident beams are given differing from each other in angles of incidence (θ_i) and in angles between the planes of incidence and vibration (φ_j). For symmetry reasons the intervals range for both angles from 0 to $\pi/2$.

Intensity and position of vibration plane of reflected beam for every single observed direction ($\beta_i = 2 * \theta_i$):

$$I_{i,j} = \varepsilon_1^2(\theta_i) * \cos^2(\varphi_j) + \varepsilon_2^2(\theta_i) * \sin^2(\varphi_j) \tag{3}$$

$$\tan(\varphi_{Rj}) = \tan(\varphi_j) * \varepsilon_2^2(\theta_i)/\varepsilon_1^2(\theta_i) \tag{4}$$

where $\varepsilon_1 = R1/A1$; $\varepsilon_2 = R2/A2$; φ_{Rj} is the angle between the planes of incidence and vibration for reflected beam.

Having the intensities and vibration direction for every single reflected beam we can obtain the angular (φ_R) distribution of the averaged intensity:

$$\bar{I}(\varphi_{Rj}) = \sum_i \sum_l I_{i,l} \tag{5}$$

where l, the number of beams with the same φ_{Rj}, is an integer.

Using the assembled values of averaged intensity the degree of polarization can be expressed as:

$$\text{Dep.} = 1 - \frac{\Sigma\bar{I}(\varphi_{Rj}) * \cos^2(\varphi_{Rj}) - \Sigma\bar{I}(\varphi_{Rj}) * \sin^2(\varphi_{Rj})}{\Sigma\bar{I}(\varphi_{Rj})} \tag{6}$$

Table 2

Computed degree of reflecting depolarization

Refractive indices		Degree of depolarization		
n_1	n_2	$0 \le \theta_i \le \theta_B$	$0 \le \theta_i \le \theta_c$	$0 \le \theta_i \le \pi/2$
1	1.1	0.0943	0.609	0.861
1	1.2	0.0925	0.483	0.814
1	1.3	0.0917	0.388	0.780
1	1.4	0.0902	0.315	0.754
1	1.5	0.0886	0.286	0.735
1	1.6	0.0870	0.243	0.719
1	1.7	0.0830	0.218	0.707
1	1.8	0.081	0.186	0.696

The degree of depolarization is examined for the incident angle series: first, 0 to θ_B (rewster) second, 0 to θ_C (ritical) and third, 0 to $\pi/2$.

Reflecting depolarization values computed for some pairs of refractive indices are shown in Table 2.

Conclusion

We have developed a theoretical scattering model for arbitrarily oriented edge surfaces of optical inhomogeneities contained by the sample. The scattering properties are described by the van de Hulst approximation, on the basis of the laws of geometrical optics. For depolarization information we have taken into calculation the externally reflected (from n_1 into n_2) part of light only. Furthermore, the actual value of the calculated degree of depolarization depends, in our case, on the viewing direction. The degree of depolarization for samples with different refractive indices decreases with decreasing n_1/n_2 in three different regions of incident angle. In the first region depolarization is weak and in the third one it is strong. In the second region this degree of reflected light changes with the variation of the interval $(\theta_C - \theta_B)$. It seems that

depolarization is effected mainly by totally reflected rays, with a polarization orientation different from the plane of incidence and the plane perpendicular to it.

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