ON PROPERTIES OF SPECIAL INTERFEROGRAMS FOR MEASURING 3-D DISPLACEMENTS, II*

By

Z. FÜZESSY

Department of Physics, Institute of Physics, Technical University, Budapest

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I. Introduction

Independent interferograms can be recorded on a single plate by selective incoherent superposition of holograms corresponding to the undeformed and deformed states of object, respectively. The interferograms needed for unambiguous determination of 3D-displacements can be recorded on a single plate by regulated pathlength interferometry [1] or by different color interferometry [2]. In both cases the thin photosensitive medium contains several superimposed interferograms.

Properties of interferograms recorded by different techniques have been analysed in [3]. For the sake of completeness it can be worth while to deal with the properties of the interferograms from the aspect of mathematics resulting in analytical expressions for some characteristic quantities of interferograms. This is the subject of the present paper.

II

False images at plane hologram producing superimposed interferograms

In the both cases of regulated pathlengths and multifrequency interferometries those parts of the subject and reference beams which are not correlated, cannot produce the standing wave patterns required for hologram recording. Their electric field vibrate at different phases; when they are added and squared, the resulting intensity shows no interference term. The correlated subject and reference waves, of course, will record holograms. These will be regarded as superimposed independent holograms.

To generate an interferogram the superimposed holograms are illuminated with the original reference wave. In order to get different sensitivity vectors for each interferogram recorded on a single plate the number of subject illumination beams must be equal to that of the interferograms. If the spatial

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separation of reference beams is arbitrary in regulated pathlength interferometry, or holograms are illuminated with multicolor light in multifrequency interferometry, each hologram diffracts not only light for virtual image of the subject located in the original subject space; but also a great number of false images appears in the vicinity of the true image. The false reconstructions of the subject wave will emerge from the hologram at various angles. These can generate false, overlapping virtual images of the subject with magnifications dependent on reconstructing beam parameters. Here we shall be concerned only with the virtual images, although the same cross-modulation problems apply to real images as well.

A simple analysis of the cross modulation problem will be carried out. Similar analysis has been given in [4] for the discussion of the problems associated with producing multicolor images from holograms. An initial simplification is to confine all light rays to the xz plane. We assume the subject is illuminated with laser beams at M spatial frequencies producing object illuminating beams for different sensitivity vectors. The complex amplitude of the multi-spatial frequency subject wave front can be expressed at the hologram in a form

$$\mathbf{T}(x) = \sum_{p=1}^{M} T_p(x) \exp\,(2\pi\, i\, \bar{f}_{tp}\, x) \tag{1}$$

where $\bar{f}_{tp}$ is the mean spatial frequency for the pth subject wave and $T_p(x)$ is assumed to have a spatial frequency spectrum centered at zero frequency. If the reference beam consists of M plane waves at the same M wave lengths ($M = 1$ in the case of regulated pathlength interferometry and $M > 1$ using techniques of multifrequency interferometry), then the reference wavefront at the hologram is

$$\mathbf{R}(x) = \sum_{p=1}^{M} R_m \exp\,(2\pi\, i f_{rp}\, x) \tag{2}$$

where the $R_m = \text{const.}$, and $f_{rp} = (\sin\,\Theta_{rp})/\lambda_p$. Being Θ_{rp} the angle between the normal of the wave front and the x-axis and λ_p the wavelength of the light, f_{rp} determines the spatial distribution of the reference beams.

If the exposure of the photosensitive plate to the interference pattern formed by $T(x)$ and $R(x)$ has been properly controlled and the plate properly developed, and if the hologram is of the absorption type, then it can be stated that the transmittance t of the completed hologram contains a term t_E proportional to the exposure energy and hence proportional to the intensity:

$$t = t_0 - t_E = t_0 - kI \tag{3}$$

where t_0 is the transmittance of the unexposed plate and k is a constant which is of no interest here.

As a result of the interference of T and R an absorption hologram is obtained with an overall transmittance

$$t(x) = \sum_{p=1}^{M} c_p\{T_p^2 + R_p^2 + \mathbf{T}_p R_p \exp\left[2\pi i(\bar{f}_{tp} - f_{rp})x\right] + \\ + \mathbf{T}_p^* R_p \exp\left[2\pi i(f_{rp} - \bar{f}_{tp})x\right]\} \quad (4)$$

where c_p is a constant related to the spectral sensitivity of the recording material at wavelength λ_p and where $T_p = |T_p|$.

In order to reconstruct the original subject waves (the independent interferograms in double exposure hologram interferometry) the plate has to be illuminated with reference beams. The complex amplitude of the illumination at the hologram has the form:

$$\mathbf{R}'(x) = \sum_{q=1}^{M} R'_q \exp(2\pi i f_{rq} x) \quad (5)$$

which differs from the original reference illumination by the replacement of R_p with another constant R'_p. This gives the possibility to produce holograms (interferograms) of the same diffraction efficiency in both cases of regulated pathlength and multifrequency interferometries.

The virtual image waves corresponding to the third term in Eq. (4) are diffracted from the hologram

$$\mathbf{V}(x) = \mathbf{R}' t_v = \mathbf{R}' \sum_{p=1}^{M} (c_p R_p R'_p)\, \mathbf{T}_p \exp(2\pi i \bar{f}_{tp} x) + \\ + \sum_{p=1}^{M} \sum_{\substack{q=1 \\ p \neq q}}^{M} (c_p R_p R'_p)\, \mathbf{T}_p \exp\left[2\pi i(\bar{f}_{tp} + f_{rq} - f_{rp})x\right] \quad (6)$$

The first term in Eq. (6) contains the original subject wavefronts, while the second one consists of the terms corresponding to false reconstructions of the subject waves arising from the diffraction of light of spatial frequency f_q by a hologram formed with light of spatial frequency f_p.

It is seen from the phase factor that the false image wavefront is placed on a carrier wave of spatial frequency $\bar{f}_{tp} + f_{rq} - f_{rp}$ varying according to the wavelength and spatial distribution of reference lights.

Several ways have been devised to eliminate or diminish the effect of the false images generated in plane superimposed holograms. One of them is the separation of the spectra [5, 6, 7], another the coded reference beam method [8].

From the point of view of practical applications, especially for holographic measuring systems, the most reliable method of diminishing the effect of false reconstructions is to separate the spectrum of the true image from the spectra of the false images for all spatial frequencies used in the reconstruction process.

According to the phase factor under the double summation sign in Eq. (6) the complete separation of true and false spectra is achieved by satisfying the condition for all q and p

$$\frac{1}{2}|f_{rq} - f_{rp}| > \Delta f_{tp} \tag{7}$$

where Δf_{tp} is the false reconstruction spatial frequency bandwidth on either side of the mean. The inequality in relation (7) can be satisfied by proper choice of the direction and wavelength of each reference beam.

III

Diffraction efficiency of plane hologram producing superimposed interferograms

The diffraction efficiency of a hologram formed with light beams of several mean spatial frequencies and illuminated with beams having the same characteristics is defined as the ratio of the total optical power in reconstructed subject waves (interferogram, or interferograms) and of that in the incident light. The definition is consistent with that usually applied for plane hologram generating simple images.

The reduction in diffraction efficiency is significant when increasing the number of holograms superimposed in a single plate even if the cross-modulation spectra are separated.

In the following analysis M superimposed holograms will be considered, each formed with one of M beams and recorded linearly on thin absorptive material. Let us suppose the identity of M subject and M reference plane waves respectively. Each hologram diffracts with the same, maximum possible efficiency.

In that case the term in Eq. (3) dependent on exposure is

$$t_E = t_{E0} + t_1 \cos(2\pi f x) \tag{8}$$

The highest efficiency for a given exposure is obtained when

$$t_1 = T_{E0} \tag{9}$$

M superimposed holograms correspond to M holographic exposures on the same thin recording material. Each exposure is characterized by own spatial frequency light field. After developing the hologram has the amplitude transmittance

$$t = t_0 - \sum_{p=1}^{M} \left[t_p + \frac{1}{2} t_p \{ \exp(2\pi i f_p x) + \exp(-2\pi i f_p x) \} \right] \tag{10}$$

where the equalities (8, 9) are valid. In general cases, each t_p may have different values and arbitrary phase constant. We assume that $t_p = t_c =$ constant for all p.

When the hologram of transmittance (10) is illuminated by original M reference waves each of unit amplitude then each hologram component of the superimposed set reconstructs its original subject wave with an efficiency: $(t_c/2)^2$. It follows that the superimposed hologram as a whole has a collective efficiency given by

$$\eta_M = \left(\frac{t_c}{2} \right)^2 \tag{11}$$

Because the holograms are linear recordings we assume that in the worse case there is some point in the recording material where the M sinusoidal transmittance modulations add their maximum amplitudes t_c. In that case Eq. (10) becomes

$$t = t_0 - 2Mt_c \tag{12}$$

The maximum value of t_c is given by

$$t_c = \frac{t_0}{2M} \tag{13}$$

Using Eq. (11) the maximum value of the collective efficiency can be expressed as follows:

$$\eta_M = \left| \frac{t_0}{2M} \right|^2 \tag{14}$$

Denoting the efficiency of a single exposed hologram by η_1

$$\eta_M = \frac{\eta_1}{M^2} \tag{15}$$

Thus the efficiency reduction factor for holograms superimposed on a thin recording medium is M^2. Our attention was turned to the holographic measuring systems where the information storage is realized by double-exposure interferograms providing an image 1/4 as bright as the comparable single image. Comparing the diffraction efficiencies of the double-exposed interferograms, furthermore, taking into account the limited number of superimposed interferograms needed for determining the three components of the displacement vector, one would expect a hologram, recorded with three or four object and reference beams, to provide interferograms 1/9 or 1/16 as bright as the comparable single double exposed ones.

Conclusions

Some characteristics of superimposed interferograms have been analysed. In both cases of regulated pathlength and multifrequency interferometries false images are appearing at the reconstruction when holograms are recorded on thin recording material. The most reliable method of diminishing the effect of false reconstructions is the separation of the spectrum of the true image from the spectra of the false images.

The diffraction efficiency of the superimposed interferograms is reduced by reduction factor M^2, where M is the number of the superimposed interferograms. Because of the limited number of interferograms made on thin recording material needed for determining the displacement vector, the reduction in diffraction efficiency doesn't mean an impediment to the application of superimposed interferograms for measuring the three dimensional displacement.

Summary

Properties of interferograms recorded by regulated pathlength and multifrequency interferometries have been analyzed from the aspect of mathematics. Analytical expressions have been derived for the false images at plane holograms producing superimposed interferograms and for the diffraction efficiency of superimposed holograms recorded on thin photosensitive medium. The results are discussed from the point of view of practical application.

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dr. Zoltán FÜZESSY H-1521 Budapest