HEAT TRANSFER IN VERTICAL TUBE EVAPORATORS II

By

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Design correlations

Our experiment results (Part I) are involved in selecting correlations for their applicability in the design of reboilers and evaporators of the chemical industry, from among the numerous published and partly contradictory equations for pressure drop and for heat transfer in boiler tubes.

1. Pressure drop

Several methods have been published for the calculation of two-phase pressure drop. The method of LOCKHART and MARTINELLI [10] is the most recommended one. It has been elaborated by MARTINELLI and NELSON for the case of non-adiabatic flow of steam-water mixtures too [11].

MARTINELLI et al. define a flow parameter

$$X = \left| \left/ \frac{\left| \frac{\Delta p}{\Delta L} \right|_F}{\left| \frac{\Delta p}{\Delta L} \right|_G} \right|_G$$
(1)

where $\left(\frac{\Delta p}{\Delta L}\right)_F$ resp. $\left(\frac{\Delta p}{\Delta L}\right)_G$ is the friction pressure drop which would be pro-

duced if either the liquid or the gas phase would flow alone in the tube. In the case most frequent in boiler tubes, when both the liquid and the gas phase are in turbulent flows, the friction pressure loss of both phases may be described by the BLASIUS equation and the flow parameter becomes

$$X_{ll} = \left(\frac{w_F}{w_G}\right)^{0,9} \cdot \left(\frac{\varrho_G}{\varrho_F}\right)^{0,5} \cdot \left(\frac{\mu_F}{\mu_G}\right)^{0,1}.$$
 (2)

In two-phase flow a slip exists between the linear velocities of the gas and of the liquid phase, which must be taken into account in holdup calculations. The gas holdup (η_G) can be correlated by the flow parameter X_{tt} according to FAIR [4]:

$$\eta_G = 1 - \sqrt{\frac{1}{1 + \frac{21}{X_{tt}} + \frac{1}{X_{tt}^2}}}.$$
(3)

The pressure difference between the inlet and the outlet of the boiler tube has three constituents: frictional loss, acceleration and hydrostatic pressure difference:

$$p_1 - p_2 = \Delta p_{TP} + \Delta p_{vel} + \Delta p_{st} \,. \tag{4}$$

 Δp_{TP} is the frictional loss of the two-phase flow. MARTINELLI and NEL-SON presented a graphical correlation for Δp_{TP} in the form of

$$\frac{(\Delta p / \Delta L)_{TP}}{(\Delta p / \Delta L)_{F0}} = f(X_{tt}) = f(x_e)$$

where $(\Delta p | \Delta L)_{TP}$ is the frictional pressure drop in the boiler tube while the vapour content changes from 0 to x_e ,

 $(\Delta p | \Delta L)_{F0}$ would be the frictional loss if all the fluid were liquid, x_e is the exit vapour quality.

 Δp_{vel} is the velocity head covering the energy needed for the acceleration of the generated vapour:

$$\Delta p_{vel} = (G_F v_F + G_G v_G)_2 - (G_F v_F + G_G v_G)_1.$$
(5)

In evaporation, when the mass rates of the two phases are changing along the tube, Eq. (5) transforms into:

$$\Delta p_{vel} = G^2 \left[\left(\frac{x^2}{\eta_G \varrho_G} + \frac{(1-x)^2}{(1-\eta_G) \varrho_F} \right)_2 - \left(\frac{x^2}{\eta_G \varrho_G} + \frac{(1-x)^2}{(1-\eta_G) \varrho_F} \right)_1 \right].$$
(6)

 Δp_{st} is the weight of the fluid column contained in the boiler tube above the inlet cross section. In horizontal tubes with mass rates of the phases not changing along the tube length

$$\Delta p_{si} = g \varrho_{TP} \left(H_2 - H_1 \right) \tag{7}$$

where

$$\varrho_{TP} = \eta_G \varrho_G + \eta_F \varrho_F \,. \tag{8}$$

If the vapour content is changing between H_1 and H_2 , then the integral mean ρ_{TP} should be used, but according to FAIR [4] it is sufficient to substitute in Eq. (8) the holdup values calculated for the vapour quality

$$x = \frac{2}{3}(x_2 - x_1) + x_1.$$
⁽⁹⁾

322

The tube height B at the start of net vapour production was known in all our experiments from temperature profile calculations. Up to this level one-phase liquid flow is supposed; from level B to the exit the pressure difference is calculated by two-phase flow correlations.

Fig. 1. is a plot of the calculated pressure differences $(p_1 - p_3)_{calc}$ against the measured values $(p_1 - p_3)_{meas}$. Our results are seen to be fairly well described





by the LOCKHART-MARTINELLI correlation: the mean value of our measurements lies 2.7 per cent above the calculated mean; the measured mean value is determined with \pm 0.82 per cent error; the deviation of individual measurements around the experimental mean is \pm 6.6%, around the calculated mean somewhat greater: \pm 7.35%. The calculation of the pressure difference requires the knowledge of the following quantities: the mass flow rate, the pressure in the vapour head, the inlet and exit vapour contents and the level, where net vapour production begins.

2. Heat transfer

2.1. Correlations for nucleate boiling

In boiler tubes heat is transferred by two mechanisms:

1. by flow convection;

2. by the convection caused by bubble formation.

Flow convection heat transfer — for one-phase flow — is described by the Colburn equation:

$$Nu_b = 0.023 \operatorname{Re}_{b}^{0.8} \cdot \operatorname{Pr}_{b}^{1/3} \cdot \left(\frac{\mu_b}{\mu_s}\right)^{0.14}.$$
 (10)

Experiments made in one-phase hot water flow indicated that the constant in the Colburn equation should be taken as 0.018 instead of 0.023 in the case of water boiling. Authors suppose that the diminution of heat transfer rate is caused by a "shadowing effect" of air bubbles separating at the heating surface (see Part I. 4.). $q_{\rm conv}$ will note in the further discussion the heat flux evoked by flow convection.

Heat flux caused by nucleate boiling is denoted by q_{BO} . In a liquid pool under non flow conditions in nucleate boiling where the produced vapour phase is able to leave the heating surface, the heat flux is a function of the difference between the wall temperature and the liquid saturation temperature, rather than of the temperature difference between the wall and the liquid bulk (of importance where the liquid bulk is not at saturation temperature), that is:

$$q_{BO} = f(t_s - t_{sat}) = f(\delta t_{sat}).$$
(11)

A film coefficient for boiling heat transfer, α_{BO} may be defined, but is not a constant, this itself being dependent on δt_{sat} :

$$q_{BO} = \alpha_{BO} \cdot \delta t_{\text{sat}} \,. \tag{12}$$

Numerous correlations are published for nucleate pool boiling heat transfer. Some of these define a Nu_{*} and a Re_{*} number for the bubble heat transfer mechanism applying the bubble diameter, l_* , as the significant length and the velocity of the liquid stream replacing the departing bubbles as the significant velocity. Different authors propose diverse forms for these dimensionless numbers. The numerical coefficients comprise surface force effects, so their values are expected to change for different wall material-liquid pairs and surface roughness. The correlation of BORISHANSKI [3] is based on the theorem of corresponding states and may be used for any liquid, if the critical state variables are known.

Some of the most important correlations are found in Table I, the same correlations are evaluated for water boiled at atmospheric pressure and plotted, together with own measurements, in Fig. 2. In further calculations authors used the LABUNTZOV equation [9]:

a)
$$\operatorname{Nu}_{*} = 0.125 \operatorname{Re}_{*}^{0.65} \operatorname{Pr}_{F}^{1/3}$$
 if $\operatorname{Re}_{*} \ge 10^{-2}$
b) $\operatorname{Nu}_{*} = 0.0625 \operatorname{Re}_{*}^{0.5} \operatorname{Pr}_{F}^{1/3}$ if $\operatorname{Re}_{*} \le 10^{-2}$
c) $\operatorname{Nu}_{*} = 0.125 (\operatorname{Re}_{*} \operatorname{Pr}_{F})^{0.5}$ if $\operatorname{Re} \ge 10^{-2} \operatorname{Pr}_{F} \ll 1$
 $\operatorname{Nu}_{*} = \frac{\alpha}{\lambda_{F}} l_{*}; \operatorname{Re}_{*} = \frac{q\varrho_{F}}{\varrho_{G} \cdot r \cdot \mu_{F}} l_{*}; \quad l_{*} = \frac{c_{F}\varrho_{F}\sigma T_{\text{sat}}}{(r \cdot \varrho_{G})^{2}}.$ (13)

This equation was established considering the experimental results of many authors with $\pm 30\%$ max. deviation. We used Eq. (13a) for our experimental values Re_{*} > 0.14, Pr = 1.72; this is the equation plotted in Fig. 2.

In forced convection boiling in tubes the produced vapour is prevented from leaving the system: the resulting flow patterns are described by different heat transfer equations. The heat flux in the entire boiler tube is obtained by integration:

$$q = D\pi \int_{0}^{L} \alpha_{F} \cdot (\delta t_{F}) \cdot dL.$$
(14)

The methods proposed by various authors to determine the integral heat flux are of the following five types:

I. Heat transfer equations for nucleate pool boiling (Table I) are applied with a factor $0.7 \ldots 1.0 [1, 2, 3, 14, 15, 17]$.

II. The convective heat transfer equation was used by PIRET and ISBIN to describe their evaporation experiments in a 1.5 m long, 25 mm i. d. vertical tube with water, aqueous solutions and organic liquids [13]:

$$\mathrm{Nu}_{PI} = 0,0086 \operatorname{Re}_{m}^{0.8} \cdot \operatorname{Pr}_{F}^{0.8} \left(\frac{\sigma_{\mathrm{water}}}{\sigma} \right)^{0.33}.$$
 (15)

In Re_m the significant velocity is the logarithmic mean from the inlet and the exit velocities:

$$\operatorname{Re}_{m} = \frac{v_{m} \cdot D \cdot \varrho_{F}}{\mu_{F}}; \quad v_{m} = \frac{v_{2} - v_{1}}{\ln \frac{v_{2}}{v_{1}}}.$$

2 Periodica Polytechnica Ch. XVII/4.



Fig. 2. Heat transfer equations (Table I) for nucleate pool boiling of water, 1 atm

Homogeneous flow was assumed to determine the inlet v_1 and the exit v_2 velocities of the two-phase vapour—liquid mixture, i.e. that both the vapour and the liquid phase have the same velocity, e. g.

$$v_2=rac{4w}{D^2\pi}\left(rac{x_2}{arrho_G}+rac{1-x_2}{arrho_F}
ight).$$

A reboiler design program for a digital computer, based upon the Piret – Isbin equation has been prepared by HUGHMARK [5].

III. The convective heat transfer coefficient has been applied by MUMM, SCHROCK and GROSSMANN, WRIGHT and co-workers. They consider the velocity increase due to vaporization by the Martinelli flow parameter (i.e. by the vapour content x):

$$X_{tl} = \left(\frac{1-x}{x}\right)^{0.9} \cdot \left(\frac{\varrho_V}{\varrho_F}\right)^{0.5} \cdot \left(\frac{\mu_F}{\mu_V}\right)^{0.1}$$
(16)

Table I

Nucleate pool boiling heat transfer correlations

(Fig. 2)

$\frac{c_F \delta_{\text{sat}}}{r} = \text{const} \left[\frac{q}{\mu_F r} \left(\frac{\sigma}{g(\varrho_F - \varrho_G)} \right)^{0.5} \right]^{0.33} \Pr_F^{1.7}$ const = 0,013 (boiling water on $D = 0,061$ cm Pt wire)
const = 0.010 (boiling benzene on polished chromium plate)
$rac{lpha l_{st}}{\lambda_F}=0,\!125\left(rac{q l_{st}}{arrho_G r^{p}_F} ight)^{\!0,65} \mathrm{Pr}_F^{1/3}$
$l_{*} = \frac{c_{F} \varrho_{F} \sigma T_{\text{sat}}}{(r \varrho_{G})^{2}}$ $\frac{\alpha l_{*}}{\lambda_{F}} = 74 \left(\frac{q l_{*}}{r \mu_{F}}\right)^{0.7} \Pr_{F}^{1/3}$ $l_{*} = \left(\sigma \right)^{0.5}$
$ \begin{aligned} & \star = \left(\frac{1}{g(\varrho_F - \varrho_G)} \right) \\ & \alpha = 3(p^{0,14} + 1.82 \cdot 10^{-4} p^2)q^{0,7} \\ & n[kn](m^2], \alpha [kca]/m^2 h \circ Cl, \alpha [kca]/m^2 h] \end{aligned} $
$lpha = 600 \frac{p_{cr}^{1/3}}{T_{cr}^{5/6}M^{1/6}} \left(0.37 + 3.15 \frac{p}{p_{cr}}\right) q^{2/3} p[kp/cm^2], \ lpha[kcal/m^2 h \ ^{\circ}C], \ q[kcal/m^2h]$

and the effect of nucleation by the boiling number

$$Bo = \frac{q}{G_{tot} \cdot r} \,. \tag{17}$$

The equation of MUMM for the boiling of water is [12]

Nu = Re^{0.808}_{FO} · Bo^{0.464}
$$\left[4.3 + 5 \cdot 10^{-4} \left(\frac{\varrho_F}{\varrho_G} - 1 \right)^{1.64} \cdot x \right].$$
 (18)

The equation of SCHROCK and GROSSMANN [16]:

$$\frac{\mathrm{Nu}}{\mathrm{Re}_{FO}^{0.8} \cdot \mathrm{Pr}_{F}^{1/3}} = 170 \left[\mathrm{Bo} + 1.5 \cdot 10^{-4} \cdot X_{tt}^{-\frac{3}{2}} \right].$$
(19)

The equation of WRIGHT and co-workers [19]:

$$\frac{c_F \delta t_{\text{sat}}}{r} = 0.9 \, \frac{X_{tt}^{0.292} \cdot \mathbf{B} \mathbf{0}^{0.191}}{\mathbf{R} \mathbf{e}_{FO}^{0.236} \cdot \mathbf{P} \mathbf{r}_{F}^{0.233}}, \qquad (20)$$

where ReFO is the Reynolds number, provided all the fluid in the tube is liquid.

IV. ROHSENOW [6] simply summarized the convective and the nucleate boiling heat flux:

$$q = q_{\rm conv} + q_{BO}.\tag{21}$$

V. LABUNTZOV [9] found, that the convective and the nucleate boiling heat flux summarize only, if they are of the same order of magnitude, otherwise the more intensive convection mechanism prevails. The correlations are:

 $q = q_{\text{conv}} \quad \text{if} \quad 2q_{BO} \le q_{\text{conv}} \quad (22)$ $q = q_{BO} \quad \text{if} \quad q_{BO} \ge 2q_{\text{conv}}$ $q = q_{\text{conv}} \frac{4q_{\text{conv}} + q_{BO}}{5q_{\text{conv}} + q_{BO}} \quad \text{if} \quad 0.5 < \frac{q_{BO}}{q_{\text{conv}}} < 2.$

2.2. Comparison of boiling heat transfer correlations with own experiments

Experimental conditions, for which the fit of the listed correlations were examined, were the following: distilled water was boiled under atmospheric pressure in a vertical, 1500 mm long, 20 mm i. diameter stainless steel (KOR 5) tube with forced circulation. The range of operation variables were:

inlet velocity	0.08	2.08 r	n/s
heat flux	3600	72000	$\frac{\text{kcal}}{m^2h}$
mean wall to liquid bulk temperature	difference 2	9°C	
exit vapor content	0-13 weight%;	0-90	$\mathrm{vol}\%$.

Correlations of group I are not applicable to our experiments. This is evident from Fig 2., where the measured points lie *above* the lines representing the heat flux equations for nucleate pool boiling and can by no means fit correlations giving still *smaller* heat flux.

The PIRET-ISBIN equation (group II., Eq. 15) has been obtained for experiments with natural convection in an evaporator similar to that used in present work, so it was expected to describe our results. In Fig. 3., we plotted the Piret-Isbin equation together with our data. A great deviation is seen, but remarkably, data obtained in the operation range of natural circulation ($v_0 = 0.08 \dots 0.36$ m/s, $\Delta t > 10^{\circ}$ C) lie on the equation line or in its vicinity. This indicates the formula to describe natural circulation evaporators but only of the same dimensions as that used by Piret and Isbin. The data of TOBILEVITSH and EREMENKO obtained in a natural circulation L = 4.9 m, D = 3 cm evaporator tube marked by * in Fig. 3. deviate strongly.

Among the correlations of group III. Eqs (19) and (20) do not apply within the range of our experimental conditions, because they yield unlikely results for zero exit vapor content.

The MUMM equation fits satisfactorily our experiments for all measured exit vapor contents. Correlation is shown in Fig. 4. The average of our measure-



Fig. 3. Boiling in tube. Piret-Isbin correlation

ments is by 2.2% lower than the calculated mean, the standard deviation of individual points from the calculated mean is $\pm 25.1\%$

Groups IV and V.: In Eqs (21) and (22) we used the following substitutions:

$$q_{conv} = \alpha_{conv} \ (\bar{t}_s - \bar{t}_F)$$

 α_{conv} from Eq. (10) with 0.018 as factor
 q_{BO} from Eq. (13a) as a function of $\delta t_{sat} = \bar{t}_s - \bar{t}_{sat}$ (cf. Fig. 2 line 3).

The mean temperature difference was determined by the calculation method described in Part I, 3.1 and 3.2.

Correlation between the ROHSENOW equation (Eq. 21) and measured data is plotted in Fig. 5. The average of measurements has $\pm 2.5\%$ error and is by 4.25% above the calculated mean.

Correlation with the LABUNTZOV equation (Eq. 22) is shown in Fig. 6. The mean of measurements has $\pm 2.9\%$ error and is by 22% higher than the

calculated mean. The standard deviation of individual points from the calculated mean is \pm 36%, which could be diminished to \pm 23,5% by multiplying the factor of Eq. (22) by 1.22, but the deviation would still exceed that from the Rohsenow correlation.



Fig. 4. Forced convection boiling in tube. Mumm correlation

3. Conclusion

Relying upon the above analysis, the LOCKHART-MARTINELLI correlation can be recommended to calculate the pressure drop, and the correlation of ROHSENOW to calculate the heat flux within the range of operating conditions examined here.

Since both recommended correlations are established for a very wide range of variables, the good agreement of the measurements of this work with the predictions furnishes indirect justification of the calculation method described in Part I to estimate temperature profiles in the regime of boiling







Fig. 6. Forced convection boiling in tube. Labuntzov correlation

heat transfer. (In the regime of one-phase flow the calculation method was verified directly by own experiments.)

Evaluating our results the fact needs consideration that our experiments were performed with distilled water which absorbes air more readily than a concentrated aqueous solution. The "shadowing effect" of separation air bubbles may hence diminish or even be absent with solutions, and in this case convective heat flux should be calculated using the factor 0.023, rather than 0.018 in Eq. (10).

Reviewing the correlations for boiling in tubes, the equations of group I. are found not to be applicable in the operating range of evaporators in the chemical industry. The PIRET-ISBIN equation (Eq. 15) is valid only for natural circulation in boiler tubes of 1.5 m length, 20-25 mm i. diameter. From the correlations of group III, Eqs (19) and (20), containing the Martinelli flow parameter, are not applicable, Eq. (18), superposition of convective and boiling heat flux, fits but with great standard deviation. The same is found for Eq. (22) of LABUNTZOV, but while this latter is based upon measured data of several authors in various conditions, the MUMM equation (Eq. 18) was established only for the boiling of water.

Summary

Evaporator design methods suggested in literature are tested in the natural and the forced circulation operating regimes on the basis of own heat transfer and pressure drop measurements with water under atmospheric pressure. Pressure drop is readily computed by a modified method of LOCKHART – MARTINELLI. The transferred heat is found to be the sum of the heat fluxes transferred by the flow convection and by the nucleate boiling mechanism

$$q = q_{\rm conv} + q_{BO}$$

as proposed by ROHSENOV. Some of the correlations published in the literature have proved to be inapplicable under the conditions examined.

Symbols

- specific heat, kcal/kg °C
- $c DG H l_*L P q Q r t T$ tube diameter. m
- mass velocity, kg/m²s or kg/m² h
- height, m
- bubble diameter, m
- tube length, m
- pressure
- heat flux, kcal/m² h
- heat transfer rate, kcal/h
- latent heat of evaporation, kcal/kg
- temperature, °C temperature, °K
- $\overline{\delta t}$ film temperature difference, °C
- overall temperature difference, °C Δt
- linear velocity, m/s v

- mass flow rate, kg/s or kg/h w
- vapor content, kg/kg x
- XMartinelli flow parameter, cf. Eq (11), dimensionless
- X_{tt} Martinelli flow parameter for tubrulent flow in both phases, cf. Eq (2), dimensionless
- film coefficient of heat transfer, kcal/m²h °C α
- $\frac{\eta}{\lambda}$ holdup, m³/m³
- thermal conductivity, kcal/m h °C
- μ dynamic viscosity, kg/m s
- v kinematic viscosity, m²/s
- density, kg/m³ Q
- surface tension, newton/m σ

Subscripts

b	bulk
BO	boiling
conv	convective
e	exit
cr	critical state
\mathbf{F}	liquid
\mathbf{FO}	if all fluid were liquid
G	gas or steam
m	mean
s	surface
sat	saturation
st	static
tot	total
TP	two-phase

- vel velocity
- nucleate boiling

average

Superscripts

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