DEGREES OF FREEDOM IN THE CONTROL OF DISTILLATION COLUMNS, II.

APPLICATIONS

By

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(Received January 20, 1966)

1. Degree of freedom for the control of a distillation column with one feed, for the separation of a binary mixture, to give two products, in the case of various simplifying assumptions often employed

In the course of distillation column operation, in most cases, there are simplifying limitations. On the basis of results presented in Part I. of this discussion, and with these simplifying factors in mind, the actual degree of freedom can be determined. Some of the possible cases will be dealt with in greater detail in what follows.

1.1. Fixed pressure. Due to this single limitation the degree of freedom is 7. In atmospheric distillation when atmospheric pressure is relatively constant and variations of it can be disregarded from a point of view of column operation as very little influence on vapour/liquid equilibrium is felt, no pressure control device is needed.

However, variable pressure, and pressure control are to be carefully considered when distillation under reduced pressure is to be carried out; pressure control is important also in apparatus working under elevated pressure. In these instances pressure can be kept constant; this, of course, is achieved by regulation.

1.2. Fixed location of feed inlet, but not necessarily at the optimum location. $F = 7$.

1.3. Fixed pressure, fixed location of feed inlet. Due to the two limiting factors the degree of freedom is 6.

1.4. Fixed pressure, optimum location of feed inlet, $x_B = x'$. $F = 6$. Here, since m is not fixed, the optimum conditions of operation can be assured just by the change of the location of the feed point.

1.5. Fixed pressure, optimum and fixed location of feed inlet. Due to the three limiting conditions, $F = 5$. This special case is dealt with by BERTRAND and JONES [1] who arrive at the same result as we reached on the basis of general considerations.

1.6. Fixed pressure, optimum and fixed location of feed inlet, reflux at boiling point. From the latter condition it follows that h_D will be the function of the composition of the distillate and of pressure, therefore it is a fixed

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parameter giving one limiting factor. $F = 4$. These suppositions are also made in the paper of AN'ISIMOV and KRIVSUNOV [2], and as a result these authors also reckon with four independent variables in this special case.

1.7. Fixed pressure, feed at optimum and fixed location and at boiling point. $F = 4$. These are the suppositions in a paper by KWAUK [3] leading to the same result.

1.8. Fixed pressure, feed at optimum and fixed location and at boiling point, reflux a saturated liquid at boiling point. $F = 3$.

1.9. Fixed pressure, feed at optimum location and at boiling point, reflux at boiling point. Here, since the location of the feed inlet is not fixed, there is one less limiting factor than in the preceding case, consequently $F = 4$.

1.10. Predetermined pressure, reflux and feed at boiling point, the location of the latter fixed. $F = 4$.

1.11. Fixed pressure, reflux at boiling point, feedstock composition predetermined, location of feed inlet fixed at the optimum point. There are six limiting factors here, thus $F = 2$.

1.12. Fixed pressure, reflux at boiling point, feedstock composition predetermined, location of feed inlet fixed at the optimum point, the composition of one of the products, x_D or x_W, to be kept at a fixed value. $F = 1$.

Here we might point out that the composition of the other product cannot be fixed at the expense of the remaining degree of freedom, since x_D, x_W and x_B cannot be simultaneously fixed arbitrarily, only two are available for choice. This can be explained on the basis of what has been said in § 3.3 of Part I.

The variable freely to be chosen cannot be but some absolute quantity, this is in harmony also with the exposition in § 3.1 of Part I.

It must be further pointed out that instead of product concentrations one may fix the composition at a suitably chosen plate in the relevant column section, i.e. the composition upon a so-called regulating plate, or control plate.

It is also true in this case that but two concentration values are available for selection.

1.13. Fixed pressure, reflux at boiling point, location of the feed inlet fixed at the optimum point, composition of feed predetermined, concentration of one of the products or — what is equivalent to this — composition on one of the plates pre-determined. $F = 2$.

This case is illustrated by *Fig. 1*. It can be noted that in graphical presentation as used in distillation technique, the number of geometrical data to be chosen at will, the so-called graphical degree of freedom, is always less by one than the factual number because figures do not refer to absolute quantities but represent their ratios only. Therefore, e.g. relative quantities of substances are shown as referring to one kilomole of the feed quantity chosen at will, this means a limitation of the absolute quantity serving as reference, thus the number of variables is decreased by one. According to *Fig. 1* too, in the case of $F = 2$, only one geometrical datum may be chosen at will, the other one must be some absolute quantity not amenable to illustration in the figure. This tallies with what has been stated in § 3.1 in Part I.

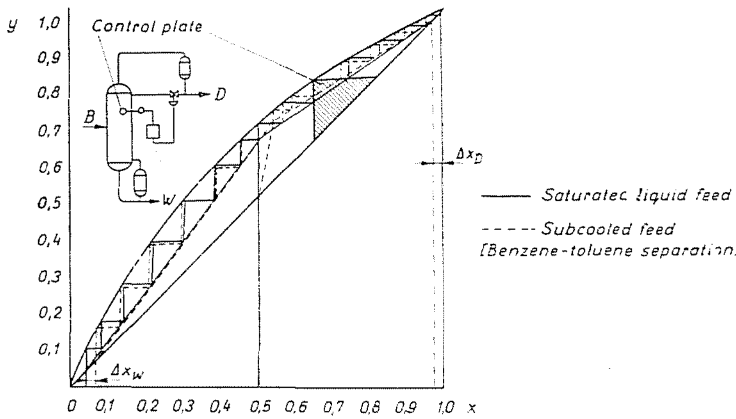

Fig. 1

2. Distillation column with several feeds and side-products for the separation of a binary mixture

2.1. The effect of side streams on the degree of freedom.

It quite often happens that the distillation column delivers more than two products; in such a case product outlets can be provided for by draining off various plates. Every side stream increases the degree of freedom by two, the location of the drawoff and the quantity of product drained there are then the additional parameters to be fixed arbitrarily. If the location of the side streams cannot be changed, only one additional degree of freedom results with each. Parameters characterizing the product, i.e. composition and thermal state, are unequivocally defined by the choice of a plate wherefrom product is to be drained off.

2.2. The effect of multiple feed upon the degree of freedom.

Each of the feed inlets increases the number of the degrees of freedom by four. These are: the location of feed inlet, the quantity of the mixture fed in, its composition, and its thermal state. Contrary to the case of product draining, generally there is no limiting factor involved with the last two parameters.

On the basis of the foregoing, the degree of freedom F for a distillation column with several feed inlets and several products, and separating a binary mixture, is represented by

$$F = 4 + 2s + 4b \tag{1}$$

where b = the total number of the feed inlets
s = the total number of the outlets.
If the location of the inlets and outlets is determined beforehand, the degree of freedom is

$$F = 4 + s + 3b\,. \tag{1a}$$

Of course, with additional limiting simplifications, e.g. feed at optimum composition and at the boiling point, the degree of freedom is reduced and can be found by the considerations discussed above.

3. Apparatus without feed inlet, working in a closed cycle

With the aid of general Equation (1), the degree of freedom of distillation apparatus without feed inlet and product outlet operated in a closed cycle, especially for experimental purposes, can be calculated; $F = 4$. This is valid

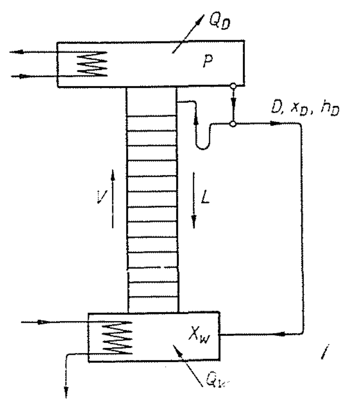

Fig. 2

for multicomponent systems too, as will be shown later by Equation (3), since concentrations and, consequently, the number of components, do not count, from the point of view of the degree of freedom, in a system filled with a substance of given composition, and operated in a closed cycle. At the expense of the maximum degree of freedom $F = 4$, the following parameters can be controlled at will:

h_D, or alternatively Q_D
V, or alternatively Q_W
reflux ratio, i.e. L, or alternatively D, and
pressure P
(see *Fig. 2*).

With pre-determined pressure and total reflux, only two characteristics i.e. V and h_D, the enthalpy of the reflux can be fixed at will.

4. Generalization of the method for cases of multicomponent mixtures

4.1. Apparatus for two products, and one feed inlet.

When this method is extended to multicomponent mixtures it is possible to proceed from considerations observed with binary mixtures in § 4 of Part I. In the case of mixtures with a number C of components, the variables characterizing the process are the following:

Table I

Characteristics	Number of variables
Flow rate of the feed	1
Flow rate of the bottom product	1
Flow rate of the distillate	1
Composition of the feed	$C-1$
Composition of the bottom product	$C-1$
Composition of the distillate	$C-1$
Composition of liquid at feed plate	$C-1$
Enthalpy of the feed	1
Enthalpy of the bottom product	1
Enthalpy of the distillate	1
Heat from steam to reboiler	1
Heat removed through condenser	1
Column plate number	1
Plate number in upper column section (location of feed inlet)	1
A value* characterizing phase equilibrium conditions	1
Pressure at column head	1
Temperature at column head	1
Total:	$4C+9$

* Phase equilibrium conditions can also be characterized by other quantities, e.g. by $C-1$ relative volatility values. However, also the number of limiting equations is equal to the number of variables, consequently, from a point of view of the degree of freedom, the result will be the same.

Between these variables the following limiting factors operate:

Table II

Characteristic	Number of limit ng fac ors
Complete material balance	1
Partial material balance referring to components	C—1
Heat balance	1
Correlation between temperature, pressure and composition	1
Correlation between enthalpy, composition and temperature at boiling point, of the bottom product	1
Correlation of the composition of the liquid on the feed plate and of the distillate	C—1
Correlation between the composition of the liquid on the feed plate and that of the bottom product	C—1
Phase equilibrium condition as a function of pressure	1
Plate number of the column	1
Total:	3C+3

After subtracting the number of limitative from the number of variables, the degree of freedom will be given as

$$F = 4C + 9 - (3C + 3) = C + 6\,. \tag{2}$$

4.2. Apparatus with side outlet, and several feed inlets.

These discussions can be extended in a similar way as was done in § 2 for binary systems, to include columns with several feed inlets and side outlet, used for the separation of multicomponent systems.

The number of components only affects the degree of freedom involved by the feed in so far as each feed means $C + 2$ degrees of freedom: $C - 1$ concentration, 1 thermal state, 1 mass, 1 location. The degree of freedom is affected by the number of outlets independently of the number of components because the concentration at the outlet is unequivocally determined by its location. On this basis, in the case of multicomponent systems, with several side outlets and several inlets the general expression of the degree of freedom is

$$F = 4 + (C + 2)b + 2s\,. \tag{3}$$

5. Comparison of this method with others; no necessity for the so-called internal variables

In order to check the correctness of the foregoing conclusions, a comparison of Equation (2), referring to an apparatus with one feed inlet for the separation of multicomponent mixtures into two product streams, with the result of GILLILAND and REED [4] seems appropriate. The work of these authors was chosen because subsequent researches have used methods closely similar, in essence, to GILLILAND and REED's fundamental notions of how to treat these questions. A common feature of all these is to be found in that starting with the concept of equilibrium units and taking into account the internal variables, out of element they put together some overall picture. In contradistinction to this so-called build-up method, we used the so-called circumscription method which takes into account only external variables presenting suitable points for intervention, and regard the internal variables

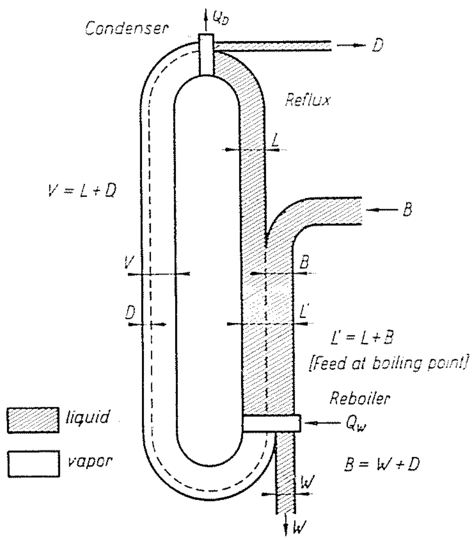

Fig. 3

or the characteristics of the several elements as being fixed by the former already at the start.

According to Equation (11) in the work of GILLILAND and REED [4], in the general case the degree of freedom is $C + 2n + 10$, as against our $C + 6$ parameters suitable for fixing at will. The authors referred to consider the function of each plate by means of two independent data, i.e. by $2n$ altogether; the same applies to the feed plate, i.e. two data. These parameters were not considered in our discussion because pressures at different places in the column are functions of other variables already considered in other contexts, neither did the independent heat balance of separate plates enter our calculations. As a further simplification in this work it was assumed that feed pressure is equal to column pressure (one datum), and the pressure in the reboiler (one datum) cannot, on the basis of the foregoing, be regarded as an independent variable. Since the elaboration of a technique of control was the aim pursued here, the number of plates in the column was taken as fixed beforehand (one datum). When design is the object of study, as it was in the case of GILLILAND and REED's paper, this limiting factor does not operate, thus the degree of freedom is higher by one. In view of these circumstances, the number of degrees of freedom given in the paper cited must be reduced by $2n + 5$ and must be increased at the same time by one, since those authors calculated with relative quantities of substances, i.e. they fixed one among the absolute quantities in our method to serve as a reference quantity. In the end we arrived to $C + 6$ as being the degree of freedom; the same figure we found in our investigations.

The correctness of our circumscription method based on external variables is supported by *Fig. 3* which represents the material turnover of a distillation column where feed is assumed at the boiling point, for the sake of simplicity. The width L of the innermost stripe, which indicates the reflux, or the liquid stream of the upper section of the column, is a function of the heat input Q_W transferred in the reboiler when product quantities W and D are given. This also determines the values of Q_D; this also, with given values of D and V, is relevant to the liquid stream of the upper section of the column. It can be seen that, from the point of view of control, external material- and heat fluxes unequivocally determine the internal variables the consideration of which, in conformity with our exposition, seems not to be necessary.

Summary

The degrees of freedom for some practical cases with simplifying factors often employed are determined. The general method is used for several feeds, side products and closed cycle. It is extended for multicomponent distillation. Our "circumscription" method is compared with that of Gilliland and Reed and no necessity for the internal variables is demonstrated.

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