ANALOGY BETWEEN HYDRODYNAMIC AND MASS-TRANSFER PROCESSES TAKING PLACE ON PERFORATED PLATES IN DIFFUSION APPARATUSES*

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(Received October 15, 1964)

To establish the suitable height of diffusion operation columns provided with perforated plates, it is necessary to know the plate efficiency values, i. e. the mass-transfer coefficients, involved. In an earlier work dealing with this matter [1], the correlation

$$\eta = 0.33\, Fr_r^{-0.1}\, Ga_l^{0.04} \left(\frac{D}{h}\right)^{0.1} \tag{1}$$

was found to be valid for a single perforated plate when the hydrodynamic, physico-chemical, and constructional parameters for the rectification of binary mixtures of different properties were considered. In another form this correlation can be expressed as follows:

$$\eta = 0.33\, \frac{h^{0.12}\, \gamma_l^{0.08}\, D^{0.1}\, g^{0.06}}{w^{0.2}\, \mu_l^{0.08}} \tag{1a}$$

This equation is valid when a single perforated plate operates normally, i. e. when there is no carrying over of liquid sprays and no liquid drops fall through the apertures of the plate (in the range of vapour velocities between 0.1 and 0.7 m/sec for the total cross section of the column).

Equation (1) accounts for all the factors which affect the mass-transfer process, and the degree of mixing of the fluid. The latter is important in view of its influence on the average driving force, and through this, on the plate efficiency. This circumstance is demonstrated by the fact that the efficiency of the plate is enhanced by the increase of the plate diameter for it is known that in columns of great diameter, where no complete mixing of the fluid can take place, the efficiency of the plate is favourably influenced by the concentration gradient.

Also the data, reported [2] for the distillation of mixtures of deuterium and hydrogen on a single perforated plate, have been worked up here. As is

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* Paper presented on the first International CHISA Congress, Brno, 1962.

shown in Fig. 1, there is a satisfactory agreement between the plate efficiency data arrived at by Equation (1), and the plate efficiency data derived from those experiments carried out under extreme conditions with a mixture of peculiar properties. The physical constants for the system H_2—D_2 were taken from the publication of Malkow et al. [3]. It can be proved that plate efficiency is identical with Stanton's criterion St which is the expression in a general form of the efficiency of diffusion processes

$$\eta = \frac{k}{w} = St = \frac{Nu}{Re\, Sc} \tag{2}$$

When dealing with heat-transfer phenomena a similar equation can be written for the characterization of the efficiency of a heat-transfer equipment.

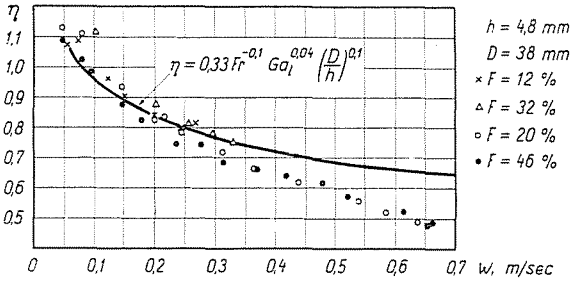

Fig. 1. Comparison of experimental plate efficiency data for single perforated plate [Timmerhaus, K. D., Weitzel, D. H., Flynn, T. M.: Chem. Eng. Progr. **54**, 635 (1958)] with equation (1) (system: H_2—D_2)

A transformation of Equation (1), and consideration of correlation (2), result in the following criterial formula

$$\eta = St = 0.33\, Re_g^{-0.2}\, Ga_g^{0.1}\, Ga_l^{0.04} \left(\frac{D}{h}\right)^{0.1} \tag{3}$$

since

$$Fr = \frac{R_{eg}^2}{Ga_g} \tag{3a}$$

The exponent — 0.2 of the Reynolds number in formula (3) points to the analogy in the dependence from the vapour velocity of the separating efficiency of columns with plates, packings, or wetted walls; further, it points to the similarity between mass-transfer processes and heat-transfer processes in these equipments.

With the introduction of a modified form of the so-called *j*-factor [4],

$$j = St\, Ga_g^{-0.1}\, Ga_l^{-0.4} \left(\frac{D}{h}\right)^{-0.1} \tag{4}$$

we get the expression

$$j = 0.33\, Re_g^{-0.2} \tag{5}$$

This equation has the same form as the well known correlation describing the mass-, heat-, and momentum-transfer phenomena occurring in the different types of equipment (packed-, wetted wall-, and plate-columns). The perforated plate of a distillation column generally works in the turbulent range of flow owing to the intensive interaction of the phases in contact with each other, therefore mass-transfer is of a convective character. This view is also supported by the experimental fact that the plate-efficiency is practically independent of the constants of molecular diffusion.

The connexion between the mass-transfer coefficient and the efficiency of a perforated plate also points to the possibility of using the hydrodynamic analogy. It is well known [4] that the equation

$$j = \frac{\lambda}{8} \tag{6}$$

is valid only in certain cases (turbulent flow in tubes), and that quite often a deviation from this rule is to be observed. When complete analogy between mass-transfer and hydrodynamic processes can not be presumed then corrections are resorted to, and thus correlations of a more general character can be established.

As, generally,

$$\Delta p = \lambda \frac{w^2 \gamma_g}{2g} \tag{7}$$

and with turbulent flow

$$\Delta p \sim w^{1.8} \tag{8}$$

it can be written that

$$\lambda \sim w^{-0.2} \tag{9}$$

or, as it is usual when data of hydrodynamic resistance are referred to,

$$\lambda \sim Re_g^{-0.2} \tag{10}$$

The resistance of a plate in operation charged with fluid is the resultant of the resistance of the plate itself, the hydrostatic pressure of the fluid on it, and the forces necessary to overcome the surface tension during bubble-formation. Since the character of the vapour-fluid emulsion, and the extent of the contact between the phases, play an important role in a mass-transfer process, and since both are determined by the overall resistance of the plate, this magnitude should be considered first and foremost when the correlation between mass-transfer and hydrodynamics is studied.

Keeping these considerations in mind, and taking note of the similarity of Equations (5) and (10), it seems that the correlation

$$\frac{\lambda}{8} = j \cdot f(d, F, \delta \ldots) \tag{11}$$

is obtained between the mass-transfer factor and the resistance factor of the plate.

On the one hand, and as far as fundamentals are concerned, this correlation points to quite a close interdependence of mass-transfer phenomena and hydrodynamic phenomena occurring on perforated plates in distillation columns. On the other hand, after its concrete elaboration, this correlation promises to acquire practical significance in the calculations of plate-efficiencies from hydrodynamic data.

Symbols

λ = resistance factor of plates
w = vapour velocity, referred to the complete cross-section of the column, m/sec
j = modified “j-factor„
k = mass-transfer coefficient, m/sec
h = height of weir, m
D = column diameter, m
g = gravitational acceleration, m/sec^2
γ = specific gravity, kg/m^3
μ = viscosity, kg sec/m^2
D′ = diffusion coefficient, m^2/sec
Fr = Froude number, $\frac{w^2}{gh}$
Re = Reynolds number, $\frac{wh\gamma}{\mu g}$
St = Stanton number, $\frac{k}{w}$
Ga = Galilei number, $\frac{h^3\gamma^2}{\mu^2 g}$
Sc = Schmidt number, $\frac{\mu g}{D'\gamma}$
Nu = Nusselt number, $\frac{kh}{D'}$
Δp = overall resistance of plates, kp/m^2
d = diameter of apertures, m
F = free cross section of plates, m^2/m^2
σ = surface tension, kg/m
$\sim$ = proportionality symbol
η = plate efficiency
Indices
l = for liquids
g = for gases or vapours

Summary

It is shown that in general plate efficiency is identical with the Stanton number expressing as a general form the efficiency of mass- and heat transfer processes. From experimental data obtained with perforated plates there can be derived the following correlation for the modified "*j*-factor":

$$j = 0{,}33\,\mathrm{Re}_g^{-0{,}2}$$

This relationship expresses the analogy between momentum and mass transfer for perforated plates of distillation columns; furher it shows that the efficiencies of plate, packed and wetted wall columns depend in a similar way from the Reynolds number. Based on the hydrodynamic analogy hydrodynamic data can be used for plate efficiency calculations.

References

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