ON THE NEW (LOCAL) FORMS OF THE PRINCIPLE OF LEAST DISSIPATION OF ENERGY

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The author has recently dealt in detail [1, 2] with the variational principle of thermodynamics, particularly with Onsager's principle of the least dissipation of energy [3]. He demonstrated that the traditional (local) form of the principle is

$$\delta[\sigma - T^{-1}\Phi]_x = 0, \qquad \Phi = \frac{1}{2}\sum_{i,k} R_{ik} J_i J_k \tag{1}$$

where Φ is the dissipation function of the fluxes and σ is the entropy production per unit volume and unit time. We were able to reformulate [1] also with the aid of the dissipation function Ψ given as the function of thermodynamic forces, i.e.,

$$\delta[\sigma - T^{-1}\Psi]_j = 0, \qquad \Psi = \frac{1}{2}\sum_{i,k} L_{ik} X_i X_k. \tag{2}$$

In the case of the traditional principle (1) we vary over the fluxes J_i besides the constant forces X_i, whereas here we vary over the forces X_i besides the fixed fluxes. Though the formulations (1) and (2) are in principle equivalent to each other, practically, however, the (2) is more productive. The Fourier equation of heat conduction, the Fick equation of diffusion and other (parabolic type) differential equations can be obtained from (2) only. The principles (1) and (2) can be summarized in a single universal (local) principle

$$\delta[\sigma - T^{-1}(\Psi + \Phi)] = 0, \tag{3}$$

in which we can already simultaneously vary over forces and fluxes. In the principle (3) the whole linear Onsager's theory of irreversible thermodynamics is contained, moreover also one of the non-equilibrium theories to be developed in the future, in which the potential character of the dissipation functions Ψ and Φ is ensured [2].

Now we shall demonstrate that the variational principle (3) can be brought into a form similar to the Gauss principle of mechanics. It was PRIGOGINE who already assumed that a differential principle similar to the Gauss principle is valid also in thermodynamics [4]. In the case of the diagonal form of the functions Ψ, Φ and Onsager's linear laws (considering that the entropy production σ is a bilinear expression in the fluxes and forces) from (3) we obtain

$$\delta\left[\sum_i J_i X_i - \frac{1}{2}\sum_i\left(\frac{X_i^2}{R_i} + R_i J_i^2\right)\right] = \delta\left[-\frac{1}{2}\sum_i R_i\left(J_i - \frac{X_i}{R_i}\right)^2\right] = 0. \quad (4)$$

From this it follows that

$$C \equiv \sum_i R_i\left(J_i - \frac{X_i}{R_i}\right)^2 = \text{minimum} \quad (5)$$

in the course of irreversible processes taking place in the system. In general it can be said that for *given thermodynamical free forces X_i and in the case of given restrictions only such irreversible processes can take place in the systems of which the "constraint" C is minimum.* In applications instead of the diagonalized form (5) of the principle the total form of the linear laws are to be used ($J_i = \sum_k L_{ik} X_k$, $X_i = \sum_k R_{ik} J_k$). Thus the form of the Gaussian type extremum principle which can also be used in practice is

$$C \equiv \frac{1}{2}\sum_{i,k} R_{ik}\left(J_i - \sum_s L_{is} X_s\right)\left(J_k - \sum_r L_{kr} X_r\right) = \text{minimum}. \quad (6)$$

The examinations and applications prove that the variational principle (3) and the Gauss type extremum principle (6) (equivalent to (3)) prove to be the most general principles of non-equilibrium thermodynamics. In order to avoid misunderstandings we emphasize that though some formal similarity is to be found by principle (3) with the minimum principle of ONSAGER and MACHLUP characterizing the fluctuations taking place in adiabatic aged systems [5], the difference consists in that the principle (3) is considered to be the unified form of (1) and (2), hence we vary it over the fluxes as well as over the forces! Though there are also other important differences (see [2]), the essential deviation is in the way of the variation over the forces.

Such a principle has never been suggested by ONSAGER neither is the existence of the Gauss type principle mentioned in any of his works. The existence of such a principle was suggested by PRIGOGINE [4].

Summary

Onsager's principle of least dissipation of energy was formulated in a general local form, then a new alternative form of the principle was given, which is practically more productive than the traditional one. With the aid of the two variational principles such a minimum principle was obtained, which can be written in an analogous form to the Gauss principle of mechanics. The existence of such a principle has already been assumed ten years ago by PRIGOGINE. The principle can be widely used for the solution of thermodynamical constraint problems.

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