THE ROLE OF AXIOMS AND MODELS IN THE THEORY OF PHYSICAL KNOWLEDGE II

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6. Constructive axiomatics[18]

In the foregoings (see in Paper I) the system of axioms defining the model and simultaneously determining the fundamental concepts and also the theorems of it, serves as the basis of a quantitative model theory have been dealt with. It is evident that neither by the properties of the model nor by the requirements of mathematical logics is the choice of the system of axioms unambiguously determined. Thus, we have a certain liberty for solving the problem as to which theorems should be considered as axioms in the building up of a model theory. This is the more as because only physical verities have decisive significances, independent of the internal hierarchy of these systems. Hence, the problem is within which system of determined verities from among the verities are the ones and in which sequence ought they to be considered as the most fundamental basic elements of a system of axioms, in order to deduce from them all the other true theorems is a secondary order and essentially an esthetical point of view. We have indeed some liberty in building out an axiomatic model theory, concerning the choice and grouping of axioms. This freedom enables us to seek for simplicity and conciseness in the axiomatic development of a discipline, apart for which they are ensured a priori by themselves in this respect by the properties of the genetic axiomatic method. This further simplification is related to logical and formal (computation technical) problems and finally is sustained even more by the theoretical researcher, working with a considerable mathematical apparatus.

The endeavouring to reach logical and computational simplicity leads far above the ordinary application of the axiomatic method to a particular kind of it, which is called constructive axiomatics. In order to understand the origin, and essence of constructive axiomatics and under certain circum-

[18] The ordinary axiomatic method by which, in general, the historical development of the concepts and theorems of the branch in question, the development stages of the branch in question, the development stages of the discipline, hence the inductive way of recognition, used to be called the genetic axiomatic method. In paper I. this axiomatic method has been dealt with.

stances its usefulness, let us pay attention to the following remarks connected with the properties of the ordinary axiomatic method (genetic axiomatics).

As it could be seen axioms are not considered to be correct, because their being true is evident, owing to their simplicity, however, the characterizing and general property of correct axioms is their simplicity in content and form. (Just because they are fundamental theorems and thus express the most general and essential relations between basic concepts, and moreover are often in direct relationship with the experimental investigation from which they have been generalized by abstraction.) From that it follows, that genetic axioms of an axiomatical model theory cannot be complicated expressions as regards their content and cannot be formulated in the form of cumbersome mathematical expressions, for instance, in the form of differential equations, variational principles, etc. At the same time the theories of modern physics have shown that logical simplification can be considerably improved by the use of ever more abstract and general mathematical apparatus. In order to confirm this, let us consider the following example: According to the theory of relativity the three-components of electrical and magnetic field strength transform as an antisymmetric tensor. It is known, that the simplest tensor is obtained by the rotation of a vector. In the theory of relativity it is demonstrated, that the electromagnetic field strength is the rotation of the four potentials. This logically very simple expression, which can be concluded with the aid of a fairly abstract chapter of mathematics with that of tensor analysis, includes the whole domain of phenomena of induction, summarizes all the results of the ten years research work of Faraday, moreover completing it with the fact that there is no separate magnetic pole. It is almost impossible to simplify and penetrate to a greater extent and to summarize more concisely a domain of phenomena with mathematics.

Besides of the above mentioned example so to say every chapter of theoretical physics is proved almost by every chapter of the same that the logical simplification can be considerably increased by the use of an abstract mathematical apparatus having a summarizing property, hence by the increased use of formal and quantitative elements. Just because the problems outlined display between particular quantitative frames the advantages of the increased application of formal respects. By these advantages the quick reproduction in a constructive manner of the theorems of a discipline enable and eventually contribute to the development of new branches of science.

A third point of view also refers to the advantages of constructive methods, moreover in several cases refers explicitly to its necessity. Namely, owing to the complication of real world there can be only a question of the systems of axioms of the individual physical disciplines (more exactly there could be question if these were uniformly worked out in detail at the up-to-date level of knowledge) and not of the system of axioms of the whole physics.

The whole physics at the up-to-date degree of knowledge cannot be axiomaticized i.e. cannot be derived from a system of axioms obtained on the basis of a final model.[19]

This is due to the manifold and complicated structure and dynamism of the physical world, as well as from the diversity and incompleteness of models reflecting the conditions. We should not give up the study of properties of different models and their relation to one another in the purest quantitative form. By this task also the unifying of logical and computation-technical conditions is required, thus leading to constructive axiomatics as to a useful auxiliary tool. In the followings the role, advantages and backdraws of this method will be briefly demonstrated by some examples.

In an exact branch of science, owing to the above mentioned reasons, every method by which the ever more complicated and differentiated problems of modern physics are simplified and which at the same time are serving, the endeavour for logical simplification is willingly accepted by theoretical researches to facilitate their work.

Two such useful methods are mathematically known, the variational computation and the already mentioned tensor calculus, each of which is an imcomparable summarizing method. Thus, for instance, a theoretical physicist being able to compute perfectly can easily reproduce without using any auxiliary tools every detail of a discipline, if he has kept in mind only one single expression, namely, the Lagrange function of the discipline in question. Then the equations of motion of the discipline in question, are obtained as the Euler's differential equations of the variational principle referring to the Lagrange function. Thus, the details of the discipline in question can be constructed with absolute fineness and the basic equations can also be derived. Such a constructive method is very important and required from the point of view of logic as well as in order to formulate an uniform picture, either of the world or of its details.

However, in order for the method of constructive axiomatics to be placed it should be correctly shown what is the true place of the variational calculus or that one of the tensor calculus in the course of the historical development of recognition and in the course of the extension of knowledges. It can be safely stated that both methods are at the ends of a long chain

[19] The situation is a similar one in mathematics. A. N. Kolmogorov writing about this the following in an already cited place: There can be question only of the system of axioms of the individual mathematical theories and not of the system of axioms of total mathematics as a whole. Mathematics in its totality cannot be axiomaticized i.e. cannot be derived from some final system of axioms. The decisive reason for this is that the subjects and properties of the real world are ever more profoundly studied. Since this statement is also valid for physics, moreover since the endless complication of the world shows itself far more directly than in mathematics, it is quite inconceivable that there might ever be question of a uniform system of axioms of physics. Such an assumption would be in contradiction with Lenin's principle of successive and approximative recognition.

of conclusions and in such a discipline only the basic concepts and theorems of which are already known and quantitatively determined can be applied. These methods have not much to do with the formulation of those concepts and theorems to which they can be referred to, if the discipline in question should be reconstructed with their aid in a deductive way. This statement is equivalent with; that though by whose constructive methods new results may eventually be attained, their importance in this respect is particulary small, if we take the serious danger which arises from the impermissible exaggeration of the formal side into consideration. Considering the requirements of materialistic ideology the outlines of a constructive method can also gnosiologically be easily misunderstandable, as regards their essence those cannot refer to the inductive roots of their historical derivation. In another respect this view is also supported by the fact, that in a considerable part of the cases the Lagrange function of the discipline in question has no satisfactory physical meaning, apart from its being, in general, very complicated from the mathematical point of view. An example for the first case can be the Lagrange function of the electromagnetic field which can be constructed as the difference between the electric and magnetic energy densities. Though the sum of those would have a real physical sense.[20] We refer as an example for the second case to the very complicated Lagrange function corresponding to Heisenberg's new "Weltformel". Otherwise Heisenberg's new theory displays in the most plastical form all the advantages and drawbacks which are due to the constructive research method.

Concerning Heisenberg's "Weltformel" further remarks should be made, because according to our experience — already its denomination is misleading. The new theory of Heisenberg — a non-linear quantum field theory is well known, — which was worked out by its author for the description and interpretation of the properties and interactions, the creation and disappearance of elementary particles. Let us assume, that by this theory or by some further modification of it, the legacies of the world of elementary particles will correctly and thoroughly be reflected (it is a pity that for the time being this assumption is not yet justified), in this case neither is it correct to call the basic equation of the theory not even between inverted commas "Weltformel". Let this not be misunderstood, we do not protest against the denomination because it is quite irrelevant what kind of denomination is given to a mathematical formula if its contents are clear to all who wish to make some declaration (mainly phylosophical declaration) about it. Thus, for instance, the denomination anti-particle or anti-material cannot be objected

[20] It should be noted that electric and magnetic vectors of field strengths simultaneously describing the electromagnetic phenomena have the two combinations which are Lorentz invariant. Due to the linearity of the Maxwell equations, however, only the combination forming the difference of their squares can be taken into consideration.

to, because the objectivity of elementary particles relating to the denomination is immediately comprehensible even to the laity. The situation is quite a different one with Heisenberg's formula. Qualified physicists and philosophers have already mentioned that after the formulation of a correct theory describing the laws of elementary particles, hence after giving a perfect "Weltformel", the task of physics will be only the solution and working out of smaller detailed problems because the legacies of macroscopic material built up of billions of elementary particles ought to be derived from a perfect Weltformel. Against such a conception objections are to be made not only on the basis of the manifoldness and inexhaustibility of the world, but also on the basis of contradiction because it is impossible to embrace by a single formula all the laws of the objective world even by some reliable approximations This follows from what has already been mentioned concerning the genetic system of axioms of physics i.e. that the theoretical model of whole physics cannot be given without axioms containing evident contradictions i.e. the totality of physics cannot be axiomatized. In respect to the constructive axiomatic methods this means from a single variational principle there never can be derived such a "Weltformel", from which to any approximation — disregarding the case of final perfection — all the basic equations of disciplines investigating the different forms of motion of the physical world could be derived.

In order to throw light on the foregoing, we refer to the relation of mechanical forms of motion related to the simple change of place, to the thermodynamic forms of motion. Considering the classical laws of mechanics and the laws of classical reversible thermodynamics as *frame legacies*, the latter never follow from the preceding ones. Hence, the uniform and common system of axioms of classical mechanics and thermodynamics can never be given. Therefore, in the case of these two disciplines the otherwise required simplification cannot be fulfilled — which is always taken as an a priori given fact by rationalists and those believing in the internal harmony of the world — namely, that by the minimum of axioms the maximum of events can be given racionally — further on the adherents of psychophysical parallelism raised by Spinoza just as instinctive followers of Leibniz's "harmonia prestabilita" do not notice or do not want to notice that in several cases the conditions of synthetizing the different disciplines owing to the contradictions and manifoldness of the objective world cannot be given. In other words: by endeavouring to summarize knowledge referring to the most possible realization by the smallest number of axioms or even by a single variational principle is correct, such efforts are not always succesful and the eventual fiasco is not due to the weakness of the human brain but to the objective contradiction in nature. In every epoch the question arises of combining the axiomatic model theories of two or more disciplines i.e. to penetrate and

describe by a certain optimal minimum of axioms the largest domain of events as far as it is possible by the conditions of theoretical models, according to the levels of knowledges. As a matter of course the objective conditions of the synthesis of different branches of sciences are different in their own epochs. In some epochs such syntheses are possible, in the case of other usually lower order of theories, there cannot be question of a synthesis at all. Moreover, it might happen and did happen in the relation of physics and chemistry, which have been separated in the development of natural science at the stage of differentiation, physics and chemistry in nuclear and molecular physics was again synthetized, because structures, interactions and processes can be classified into a branch of the quantum theory.

The fundamental concepts and theorems of the atomic shell and of molecular physics, hence its principles and axioms, are also valid in the realm of chemistry. Therefore, concerning these domains development in that sense also led to results in which the whole domain of chemistry could be described and penetrated by the laws of the quantum theory.

The situation is a different one in the relation of the already mentioned classical mechanics and classical thermodynamics, where basic laws and basic equations have such different properties that the theories cannot be thought of at all as the synthesis of the models. Moreover if for a time we had hoped that once such a synthesis might be possible, all these hopes must be destroyed nowadays. The constructive axiomatics of the total irreversible thermodynamics is placed upon Onsager's linear laws, upon the symmetry of the matrices of coefficients occurring in these laws, and finally on the validity of the principles of the least energy dissipation or the variational principle of entropy production. The basic equations of irreversible thermodynamics and from this the whole theory can be derived, and can be reproduced from this variational principle. On the other hand it is known that the equations of motion of mechanics always proved to have rigourously reversible properties also in the forms arising from Newton, Hamilton or even from this of Lagrange. The situation is a similar one with Schrödinger's and Dirac's equation which always remain invariant against the sign change or time i.e. they have a reversible character. In recent and detailed microphysical examinations it is unambiguously shown, that in the world of microphysics, reversibility is a fundamental property. Whereas by irreversible thermodynamics at any time the irreversibility of macro-processes are emphasized, and those are described by equations which can be derived from the variational principle of the least entropy production. Without analysing the reversibility of individual micro processes, the irreversibility of macro-processes, by statistical legacies referring to the ensembles of micro-particles and in this respect to the examination of determinism and indeterminism problems, it can be stated that the originally consisting difficulty encountered in producing a uniform system of axioms

embracing mechanics and thermodynamics did not decrease but is ever increasing. It can safely be stated, whatever kind the new laws of microphysics there might be, the new "Weltformulas" (until those have a reversible character) the laws of irreversible macro-processes (included the irreversible fundamental laws of biology also) can never be derived from them.

Thus, it is evident particularly if physical phenomena are classified according to Bohm and de Broglie into macroscopic quantum mechanic and subquantum mechanic levels, that the basic equation of a theory providing a whatsoever perfect desription of the world of subquantum mechanic levels can be called "Weltformel" only if the otherwise marvelous world of elementary particles in spite of the not less admirable and similarly important world of higher dimensions is favoured on the basis of subjective feelings. No objective reasons are given for such a distinction.

In what was said in the foregoing in connection with Heisenberg's new formula — but in reality disregarding it and without any physical or phylosophical evaluation of it — might be embodied, though in a different and in a milder form, in connection with Einstein's field theory.[21]

It is indisputable that this idea seemed to be intolerable for Einstein, that between gravitational and electromagnetic fields, which display such a great similarity, qualitative divergencies are shown in other respects owing to which two separate continua ought to be dealt with in physics.

Therefore, Einstein — relying on the basis of constructive axiomatics in the most abstract regions far from every days experimentations, and starting out from the ambition to embrace by the minimum of axioms the maximum of events — postulated the existence of a uniform field theory and urged its development until the end of his life. The investigations carried out on this subject matter by Einstein as well as by others are exclusively of mathematical nature of constructive character and miss the properties characteristic for the genetic axiomatic method. Thus, particularly in the light of the unsuccessful thirty years of experimentation, and otherwise, too, no assurance is to be seen that conditions ensuring the existence of the uniform field theory might follow from the genetic axiomatic model theory of gravitational and electromagnetic fields formulated in an inductive way. Therefore, in the attempts hitherto carried out in order to develop a uniform theory of the gravitational and electromagnetic fields can only be considered as random experiments as, for instance, the satisfaction of a differential equation by some hypothetical formula without having confirmed the conditions of existance and the fulfilling of solutions. Of course, if a differential equation

[21] This problem is dealt with in detail by T. Elek on this chain of thought, to a certain extent differing from the method followed here, in his dissertation entitled "On Albert Einstein's gnosiological conception and on the philosophical content of the Theory of relativity". Budapest 1961.

is satisfied by a trial function, and thus at least a particular solution is obtained which corresponds to the requirements — at least for physicists — then a detailed study of the existing conditions of the solutions are superfluous.

In this analogy which concerns the uniform field theory it should be noted, that its "trial functions" constructed in an intuitive way have not been successful, thus the conditions of the existence for a uniform field theory should be examined. The purpose of these examinations is with their help to decide whether Einstein's conceptions in this realm corresponded with the totality of the real world, or whether gravitational and electromagnetic phenomena were such diverse qualities, that owing to objective counterdictions the competence of a uniform field theory is not ensured. In order to carry out such existential investigations, first of all a genetic system of axioms of gravitational and electromagnetic field theories free of counterdictions should be developed which hitherto, at least is an uniform perspicuous form — — utilizable for the examination of such a difficult problem — is not given.

Only after these examinations, in the knowledge of their results, could a decision be brought on the correctness or incorrectness of Einstein's theoretical conceptions.[22]

The positive reply praises Einstein's admired intuition, whereas the negative result means that the diversity of gravitational and electromagnetic phenomena is non-appealable, the unity and harmony had been observed only by Einstein's rationalism.

From our own point of view we wanted to illustrate by this problem that the genetic method following the way of inductive recognition is always more reliable than are the constructive methods operating exclusively with mathematical methods. This new objective knowledge can be safely introduced into the theories during the refinement of an axiomatic model-theory only by some new knowledges obtained in an inductive way. Through this bold flight with the aid of the methods of constructive axiomatics i.e., with pure mathematical methods towards ever more abstract realms is useful and at the same time more enrapturing than genetic research is on proceeding in an inductive way. The latter is a safer method having reality as a solid basis.

[22] It can also be stated without the exact examinations carried out by the genetic method, that the genetic system of axioms of the axioms of the gravitational field is surely poorer than that of the electromagnetic field. Therefore, though it cannot be imposed as a condition, that the genetic system of axioms of the two field theories have to contain axioms of identical numbers and structures for ensuring the existence of a uniform field theoretical system of axioms, however, the differences between genetic system of axioms known at present which arise from the hitherto known physical facts are very considerable. The genetical and physical problems relating to the uniform field theory are in a very original manner summarized in the paper of J. Horváth "Classical theory of physical fields and the geometric structure of the space" (In Hungarian) Fizikai Szemle 2. X. 1960.

Therefore, the improved and more embracing development of model theories is ensured only by the correct combination of both, hence by the particular unity of the inductive and deductive research methods, during the uninterrupted development of our knowledge.

Summary

In the following the principal results which might be obtained by the deliberate application of the axiomatic method is summarized point by point.

1. The explanation and learning of any scientific discipline is facilitated by the representation built upon the exact physical model and the accuracy of axiomatization.

2. The validity limit of any axiomatic model theory is much easier to survey by the deliberate application of the axiomatic methods as it would be otherwise.

3. A possibility is given only by the deliberate application of the axiomatic model theory by comparing the properties of models at two different stages of development of a discipline and by exact quantitative means according to uniform points of view hereby measuring the extent of the development achieved.

4. The deliberation on the lack of model and of theory relies upon the system of axioms determining it, stimulates the improvement of the model for the modification of the system of axioms, eventually to its extension but in every way to the development of the theory.

5. A system of axioms determining the fundamental properties of an axiomatic model theory is the result of inductive recognition, thus it is in direct relation with reality.

6. An axiomatic model theory is correct inasmuch as it reflects the conditions of the real world. This statement first of all and directly refers to the fundamental system, whereas to what extent this statement is valid for the fundamental system, automatically and to the same extent is it fulfilled for every further theorem as a consequence which might be derived from the fundamental system.

7. The model represents an important median degree in the process of recognition from experience to theory, as well as from theory to practice. On the one hand, possibility is given for the experimental researcher, and on the other hand, for the engineer and theoretical researcher to understand each other. Finally the solid basis required by the theoretical researcher for establishing real scientific theories is ensured by the model.

8. The knowledge up to the axiomatic accuracy of a physical model theory contributes considerably to the formulation of the correct materialistical ideology. The deliberated building up of axiomatic model theories in the different disciplines of physics, though they are only methodological problems, but such, which will be indispensable in physical discipline, ever more differentiated and becoming ever more abstract.

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