Periodica Polytechnica Civil Engineering

62(1), pp. 48–55, 2018
https://doi.org/10.3311/PPci.10036

RESEARCH ARTICLE

Parameter Study on Weight Minimization of Network Arch Bridges

Rimantas Belevičius[1*], Algirdas Juozapaitis[2], Dainius Rusakevičius[1]

Received 16 September 2016; Revised 13 April 2017; Accepted 17 May 2017

Abstract
The article concerns optimization of network arch bridges. This is challenging optimization problem involving even for conventional scheme of network arch bridge the identification of some topological parameters as well as shape configurations and all sizing parameters of structural members, seeking the minimum weight. Optimal bridge scheme is sought tuning a large set of design parameters of diverse character: the type of hanger arrangement, the number of hangers, their inclination angles and placement distances, the arch shape and rise, etc. Mathematically, the optimization of the bridge scheme is a mixed-integer constrained global optimization problem solved employing stochastic evolutionary algorithm. Plane heavy/ moderate/and light-deck bridges of 18, 30, 42 and 54 m spans were optimized using proposed optimization technique. The decisive design parameters and their promising ranges were revealed. Also, the influence of some simplifications is shown: changing the arch shape from elliptical to circular, placing the hangers at equal distances, etc.

Keywords
network arch bridge, weight minimization, topology, shape and sizing parameters

[1]Department of Information Technologies
Faculty of Fundamental Sciences,
Vilnius Gediminas Technical University
LT-10223 Vilnius, Saulėtekio al. 11, Lithuania
[2]Department of Bridges and Special Structures,
Faculty of Civil Engineering,
Vilnius Gediminas Technical University
LT-10223 Vilnius, Saulėtekio al. 11, Lithuania
[*]Corresponding author email: rimantas.belevicius@vgtu.lt

1 Introduction

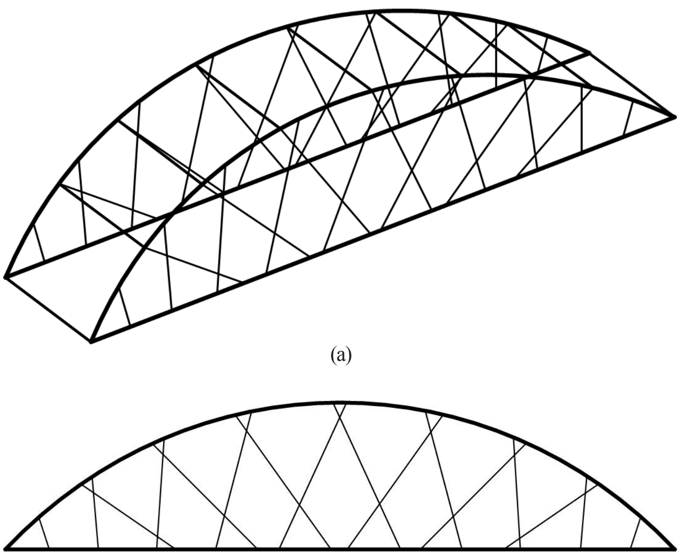
In order to reduce the costs and consequently the material consumption, the construction of recent bridges is dominated by slender structures. Taking into account a sufficient load carrying capacity and durability, reduction of the bridges' costs can be achieved only by optimizing the bridge structures. This work is dedicated to the overall optimization of network tied arch structures for pedestrian bridges of moderate span. A network arch bridge is one of the most slender bridge schemes. Intersecting incline hangers are optimal for uneven or changing external loads, leading mainly to the axial internal forces in the members of bridge. Scheme of tied arch with vertical hangers leads to a substantially heavier design.

The behaviour and design of network arch bridges were investigated in details by Tveit [1]. The main his conclusion is, an optimal network arch will remain the world's most slender bridge. Teich [2] in his dissertation tackled the development of optimal structure for network arch. Teich recommends using up to 50 hangers since their efficiency reduces significantly above 50. An essential parameter is the arrangement of hangers. Teich analysed five different arrangement types and concluded that the radial arrangement and arrangement with constantly decreasing slope of hangers are the best choices. Teich also suggests designing the arch of elliptical shape or arch with double radii, and of height 1/5 to 1/7 of the span length. On the contrary, Brito [3] asserts that for a bridge of 100 m span and of 17 m arch rise the variable hanger slope configuration is the least efficient arrangement scheme. However, in these works the influence of different parameters to the objective function (i.e., the cost of the structure or its equivalent form – the mass of structure) was studied independently.

Mathematically, optimization of bridge scheme is constraint optimization problem with numerous discrete and continuous design parameters. For large-scale optimization problems, the high-dimensional design space contains usually numerous local minima scattered throughout the space. Common gradient optimization methods find the local minima close to the starting position, and are not suitable for larger optimization problems, and only the stochastic global optimization methods can be successfully applied to similar problems. Lute et al [6] conclude that the robust tool for optimization is genetic algorithm (GA) that can easily handle larger sets of variables, and can conveniently accommodate discrete and continuous optimization variables. Hasancebi [7] also uses GA but claims the main disadvantage of all GA is that they are computationally intensive and some approximate techniques should be considered to remedy this drawback. Rana et al. [4] and Ahsan et al [5] suggested special global optimization evolutionary algorithm EVOP and solved mixed integer optimization problem of two-span pre-stressed concrete bridge with 13 design parameters, showing that for similar problems the evolutionary algorithms can be an effective tool.

The optimal scheme of the network arch bridge can be sought by tuning a large set of parameters: starting from the cross-sections of all structural elements, the number of hangers, their arrangement type, and finishing with the hanger slope angles and the distances between adjacent hangers. The maximum effectiveness of optimization can be achieved by tuning all those parameters simultaneously since the parameters are interdependent.

The pedestrian bridges of moderate span are considered. The usual scheme of the bridge is a double arch tied by horizontal deck that is suspended by a network of hangers (Fig. 1). The arches are connected together with a lateral bracing to provide the horizontal stability. If the arches are parallel and not considering the lateral bracing, it is possible to optimize the arches separately, thus significantly diminishing the size of the optimization problem.



(a)

(b)

Fig. 1 Scheme of the network tied arch bridge (a). Single network tied arch of the bridge (b)

Thus, in this work an integrated technique for simultaneous topology, shape and sizing optimization of network arch bridges is suggested. Pure topology optimization is usually understood as optimization of continual structure seeking for optimal distribution of material over the space of the structure thus finally arriving to a discrete structure. However, in case of network arch bridges we considerably constrict the topology search space to the conventional elliptical shape of bridge but allow arbitrary number of hangers of conceivable in engineering sense schemes of their placing and inclination.

Thus the topology of the bridge is identified by the number of hangers and their arrangement type. The shape of the bridge is defined by the shape of the arch, its height and by the slopes of all hangers and distances between hangers. The sizing parameters include the cross-sections of all structural elements of the bridge. In case of moderate span, the load-bearing elements of deck and arch can be made of industrial steel profiles. In this work we consider, the deck beam is made of I-shape profile from the European IPE assortment, and the arch is a square or rectangular hollow tube from EN 10210-2:2006 assortment. The hangers are solid round strips.

The discrete nature of cross-sections of structural elements from specific assortments does not allow achieving the global minimum of optimization problem and cause non-smoothness of results. Therefore, the optimization is repeated also treating continuous welded hollow-tube shape cross-sections.

The total number of design parameters is up to 11. Some parameters are integers: the number of hangers, and the numbers of I-beam and arch cross-sections in the assortments. All remaining design parameters are treated as continuous variables.

Provided the following initial data on the structure:

- Span of the bridge;
- Material data on structural elements;
- Data on dead and traffic loadings and loading cases;
- Maximum allowable deflections at the deck of bridge;
- Lower and upper limits for the radius of hangers;
- Profile assortment tables,

the optimization programs should yield the optimal, i.e. lightest scheme of bridge with a "one button click".

In mathematical terms, the problem is a constraint mixed-integer global optimization problem, since the landscape of the objective function is complex and cannot be obtained in a closed form. The results of numerical experiments definitely point to an actually multimodal character of the problem. Since the number of design parameters is rather large, we use stochastic evolutionary optimization algorithms (EA) from MatLab that do not require the sensitivity information. Stochastic algorithms do not guarantee the global solution to the problem, but in engineering practice it is more important finding a better local solution than the solution that is currently known.

A direct problem in obtaining the objective function value supposes a linear static analysis of bridge structure via the finite element method (FEM) and checking constraints. The constraint optimization problem is converted to the unconstrained one employing the static penalization technique. Slight improvement in optimization results can be obtained using hyper-heuristics combining the evolutionary algorithm and consecutive pattern search.

In this paper, the plane load-bearing structures of pedestrian network arch bridges of 18, 30, 42, and 54 m spans are designed. Also, it is shown that for different types of decks, i.e. heavy/ moderate/light-deck, very different optimal topologies of load-bearing structure should be used.

The remainder of the paper is organized as follows. Section 2 briefly describes the idealizations of the bridge structure, the mathematical model for the optimization problem, as well as the suggested optimization technique. Section 3 provides several examples of application of the proposed technique and shows the influence of some initial conditions, which are usually taken in engineering practice on the design variables. Obtained optimization results of network arch bridges of different spans are compared with corresponding results on the competing bridge scheme – under-deck stayed bridge; the comparison confirms Tveit's conclusion about the advantages of network arch scheme. The last section presents conclusions.

2 Problem formulation

2.1 Idealizations

The load-bearing structure of the network arch bridge is idealized as a plane frame system. The catenary effect, the sag of the longest hangers for the moderate spans is not relevant and is neglected. All connections between structural elements are treated as ideal contacts, i.e. all connections are perfectly rigid. The fatigue and vibration effects are not investigated – though fatigue can be controlled by the given allowable design stress. Many vibration and fatigue effects, and also risk of loss of local stability can be avoided if the relaxation of hangers is suppressed. This condition is included into the constraint system of optimization problem: if compression occurs in any hanger for any load case, the structure is treated as an infeasible. However, despite the fact that hangers receive only tension forces and are slender, all structural elements are idealized as 2-node beam finite elements. As shown in [8], the linear analysis of the structure is valid, and when compared to the non-linear analysis, the discrepancies in results are on the safe side. Finally, the outer hangers have negligible influence on the behaviour of structure and therefore can be removed.

Since the bridges of moderate span are considered, the cross-sections of the arch and tie are constant. Also all hangers have the same cross-section.

According to the Eurocode 1, the bridge structure is analysed for the self-weight of the bridge itself plus four loading cases of a traffic load: distributed loading over the whole span, half of span, middle-half of span, and over both outside quarters of span.

2.2 Optimization technique

The complete set of optimization programs consists of four independent parts: a meshing program that from the set of design parameters prepares the whole data set for the FEM analysis program, the FEM linear static analysis program, evolutionary algorithm from MatLab [9], and the program for evaluation of objective function value. The main module of program system is the EA. At the start, it randomly generates the given number of initial sets of the design parameters – in terms of EA, the population of individuals. Meshing program renders those individuals into the complete data sets for FEM program. The last program on the basis of FEM analysis results evaluates the objective function, verifies the constraints and, in case of constraint violation, penalizes the objective function so that the infeasible individuals would have lower probability to be included into the next population. Since the bottleneck in global optimization problems generally is the computational time, the fast problem-oriented meshing, original FEM analysis and post-processing programs in Fortran were employed. All Fortran programs are connected to the EA algorithm as the "black-box" programs. The short computation time is the main reason, why we cannot rely on the standard FEM packages and more precise analysis types.

On the basis of the obtained results EA generates the new improved population using special genetic operators of mutation, crossover and selection. This loop continues until one of the following criteria is reached: the maximum given number of populations is achieved, or the objective function value did not change over the given number of the last populations (called the "number of stall generations"), or the change in weighted average of objective function per stall generations is less than a given tolerance.

The best obtained (local) solutions then can be refined using other derivative-free heuristics, e.g. pattern search (PS) algorithm. For problems with a larger number of design variables it improves the solution up to 2%.

All the genetic parameters have to be tuned to the problem. Nevertheless, generally each numerical experiment ends up with different results therefore the optimization is repeated tens of times until the median values of objective function in final populations stabilize.

2.3 Statement of optimization problem

In mathematical terms, the optimization problem is formulated as follows:

$$f^* = \min_{x \in D} f(x) \quad (1)$$

for all load cases subject to:

- Structure equilibrium constraints;
- Strength constraints on all structural elements;
- Stability constraints on the arch elements;
- Relaxation constraints on the hanger elements;
- Displacement constraints on the deck nodes.

$f(x)$ in (1) is a nonlinear objective function of continuous and integer variables $f : \Re^n \rightarrow \Re$, n is the number of design parameters x, and $D \subset \Re^n$ is a feasible region of design parameters. The global minimum f^* at minimizers $x^* : f(x^*) = f^*$ should be found. No assumptions on unimodality are included into the formulation of the problem, i.e. a number of local minima may exist.

The total mass of the bridge is considered the objective function.

The complete set of design parameters along with their characteristics and bounds is listed in Section 2.4, Table 1.

The constraint optimization problem is converted to an unconstrained problem using static penalties proportional to the extent of constraint violation. In any trespass of any allowable value of requirement c_i, the value of objective function is penalized:

$$f := p\ (|c_i - c_{i\ allowable}|)\ /\ c_i \tag{2}$$

The penalty factor p depends on the problem; in comparable problems of bridge optimization [10] the factor $p = 2$ demonstrated the best results.

The overall structure equilibrium constraints are checked solving the static problem via the FEM program. All the strength, stability and displacement constraints are formulated according to the Eurocode 3. Any occurrence of compression force in any hanger is penalized proportionally to the average value of tension axial forces in all hangers. Finally, the vertical displacement of any node at the deck due to the traffic load is constrained to 1/400 of the span.

2.4 Design variables and bounds

The typical plane load-bearing structure of the network arch bridge is shown in Fig. 2.

Since the literature on the optimal scheme of network arch bridges provides sometimes contradictory recommendations, in this work the set of design parameters that encompasses most practical topologies and shapes of the scheme is employed.

The most discussed topic is the arrangement scheme of the hangers. Thus, we construct the complete hanger arrangement combining two anti-symmetrical sets of hangers (Fig. 2 (b), (c)). Also we treat constantly changing inclination angles of hangers and changing distances between adjacent hangers. Two hanger slopes are included in the design variables: the inclination angle of the first hanger and the angle of the last hanger α_f (Table 1, initial and final slopes).

Consequently, each hanger has different inclination angle. It should be noted that those two design parameters cover also simpler cases of hanger arrangements: the hanger placement at equal inclination angles or the vertical hanger placement. Denoting the number of sections at the tie produced by one set of hangers by n, the angle change then is

$$\Delta\alpha = (\alpha_i - \alpha_f)\ /\ (n-3) \tag{3}$$

The widths of all sections are also constantly varying. In the set of design variables we include only the width of the first section w_i measured by the average section width w_a. Then the width of each subsequent section is obtained augmenting the last width by

$$\Delta w = -(2w_i - 2)\ /\ (n-1) \tag{4}$$

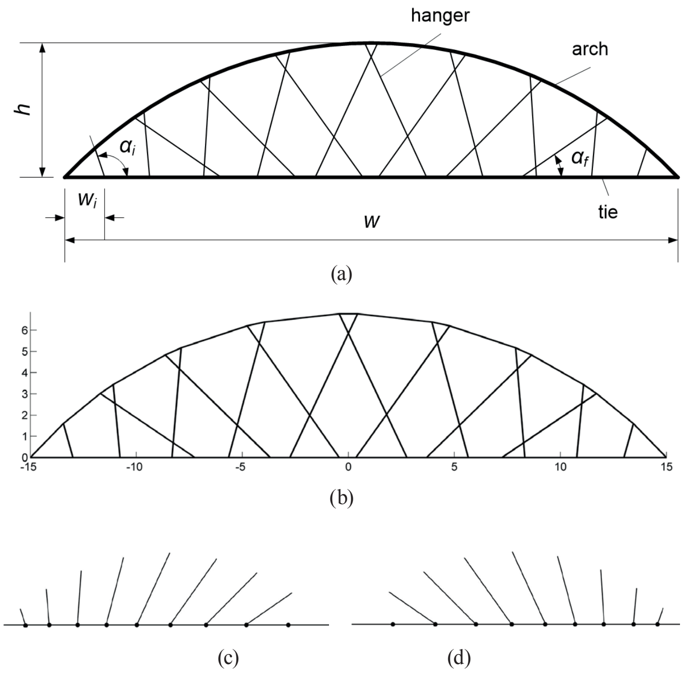


Fig. 2 Scheme of the arch bridge and the design variables (a). Hanger arrangement (b). Hanger sets 1 (c) and 2 (d)

At this type of hanger arrangement, some hangers' attachment points at the arch and tie may approach each other. When the distance between adjacent hanger attachment nodes is less than a given allowable value (it is an input for the meshing program; 0.1 m for all calculations in this work), the meshing program merges both nodes.

One of the crucial design parameters is the rise of the arch and its shape. We treat the arch as a part of ellipse and include into the design parameters the rise h and the ratio between ellipse radii $k = R_{horizontal} / r_{vertical}$. The $k = 1$ gives the circular shape, and the $k < 1/2$ and the small height of the arch resembles the section of parabola.

The section properties of the tie element – the I-shape beam are taken from the European IPE assortment (50 different profiles), and of the arch – from the European square and rectangular hollow section assortment (76 and 73 different profiles, 149 in total). The main problems associated with the use of those assortments in the optimization are that the discrete values of sectional geometrical properties are distributed in the range unevenly: for some table profiles of the same height the differences between adjacent values of the second area moments reach 25%, while for other profiles – up to 13–15%. Evidently, the rectangular cross-sections are better suited for absorbing bending in the arch, however, for some variants of the bridge, the EA results with a square hollow cross-section. Also with increasing numbers of profiles in the assortment table, the values of section properties, e.g. the section area or the second moment of the area, sometimes decrease. This hinders EA taking proper decisions on a needed number of the profile.

The complete set of the design parameters along with their bounds is provided in the Table 1.

Table 1 Set of design parameters

No. of a parameter in the genotype	Parameter	Type	Lower bound	Upper bound
1	Height of the bridge h	Continuous	$w/10$	$w/2$
2	Ratio of the ellipse radii k	Continuous	0.1	1.5
3	Number of tie sections in one set of hangers n	Integer	8	50
4	Width of the first section w_i	Continuous	$0.2\ w_a$	$1.8w_a$
5	Slope of the first hanger in a set α_i, in degrees	Continuous	70	110
6	Slope of the last hanger in a set α_f, in degrees	Continuous	20	90
7	Number of the I-beam profile in the IPE assortment n_b	Integer	1	50
8	Number of the hollow section in the hollow tube assortment n_t	Integer	1	149
9	Radius of the hanger r_h, in m	Continuous	0.005	0.02

3 Numerical results and discussion

As an example, the plain load-bearing structures of network arch pedestrian bridges of 18, 30, 42 and 54 m spans are optimized. After the Eurocode 1, the load-bearing structure of the bridge should pass all the strength, stability, and slenderness constraints for 4 loading cases of live traffic load (clarified in Section 2.1); in all cases also the dead weight of the load-bearing structure and the deck must be appended. The intensity of the live loading is 13 kN/m, while the intensity of dead loading depends on the type of deck: 13 kN/m for the light bridge, 26 kN/m for moderate deck, and 39 kN/m for the heavy one. For the sake of simplicity, these dead loading cases will be denoted by L1, L2, and L3, correspondingly. Thus, 12 different load-bearing structures were optimized in total.

To summarize, the following specific data on the bridge do not vary in the optimization process and remain constant:

- Span l – 18, 30, 42, or 54 m;
- Steel yield strength – 355 N/mm^2;
- Specific weight of steel – 7.70x10^4 N/m^3;
- Young's modulus of steel elements – 2.10x10^{11} N/m^2;
- Dead loading – 13, 26, or 39 kN/m;
- Traffic loading – 13 kN/m.

The following genetic parameters of the EA were chosen after a few numerical experiments:

- Population size – 100 individuals;
- Number of generations – 150;
- Stopping criterion – the first met, of: maximum number of generations – 200; number of stall generations – 100; objective function tolerance – 1e-6;
- Crossover probability – 0.8;
- Crossover type – Laplace crossover [11];
- Mutation type – power mutation [12]

Every optimization problem is solved 50 times; more runs do not change significantly the median results. Evidently, it is not known whether the EA solution is precisely a global or even a local minima point. The exact minima point in the neighbourhood of the obtained solution can be reached using other heuristics, e.g., pattern search (PS) algorithm. After the best optimization results are defined, we start the PS from the three best solutions allowing 16000 additional objective function evaluations with initial mesh size 1, mesh expansion factor 2, and function tolerance 1e-6. In almost all optimization problems the best result after the PS is achieved when starting the algorithm from the best result of EA. However, the gains in total mass of structure do not exceed 1.5%; i.e., the EA locates the solution points with a sufficient precision. Hereafter, only the best results after PS will be refereed.

Solution time depends on the number of FEM mesh nodes that, in turn, is determined by the number of typical sections n. 50 independent runs of the program package usually take from 4 to 24 hours on a typical PC.

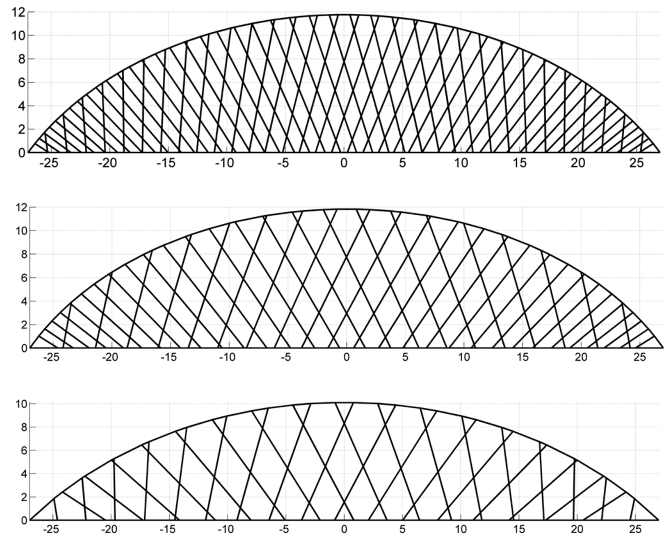


Fig. 3 The best three solutions for the bridge of 54 m span and load case L2: 6773.2, 6827.7, and 6850.8 kg

The results of optimization clearly indicate the multimodal character of objective function. For example, in Fig. 3 the topologies of the three best solutions for the bridge structure of 54 m span and traffic load case L2 are shown. At close values of objective function – the total masses of 6773.2 kg (n = 42),

Table 2 Optimization results at different initial conditions and taking discrete cross-sections palette

Span m	Load case	f_{min} kg	f_{median} kg	f_{max} kg	h m	n	α_i deg.	α_f deg.	w_i	k	$EI_{y\,arch} / EI_{y\,tie}$
		482.2	489.1	550.5	3.64	15	94.1	34.4	0.80	1.19	3.58
	L1	**474.1**	489.0	506.1	4.11	16	103.2	30.0	0.66	1	4.34
		491.5	494.5	519.0	3.56	16	87.1	42.7	1	1	2.38
		602.7	636.1	663.0	3.86	18	110.0	27.5	0.84	1.35	2.17
18	L2	628.3	637.4	662.9	3.89	23	106.6	37.5	0.39	1	7.50
		633.8	639.3	664.3	3.98	23	101.1	44.6	1	1	5.48
		786.5	811.8	950.4	4.20	23	106.3	34.1	0.69	1.10	2.17
	L3	788.7	885.1	929.2	4.25	28	110.0	50.0	0.59	1	2.16
		796.9	887.2	933.2	4.27	27	100.9	47.0	1	1	2.16
		1677.4	1726.1	1819.1	5.80	23	96.8	27.4	0.87	1.17	3.24
	L1	1680.9	1726.6	1796.6	5.95	25	99.3	39.7	0.54	1	3.24
		1728.9	1738.8	1860.4	5.24	25	83.7	41.4	1	1	2.17
		1982.9	2025.5	2164.7	7.00	26	102.3	24.5	0.80	1.17	3.90
30	L2	1984.7	2017.8	2101.4	7.07	35	109.9	35.2	0.33	1	3.90
		2016.7	2035.7	2174.0	6.12	29	95.9	43.0	1	1	2.60
		2355.6	2578.7	2747.5	6.79	28	95.0	35.3	1.39	1.33	2.23
	L3	2540.7	2608.5	2727.3	6.64	32	109.2	37.4	0.57	1	3.38
		2563.8	2610.7	2735.3	6.78	32	104.2	48.4	1	1	3.38
		3233.0	3278.2	3405.6	9.62	28	101.0	36.4	0.51	0.96	4.92
	L1	**3210.7**	3275.8	3398.3	9.27	36	100.8	28.6	0.74	1	7.37
		3255.5	3308.7	3419.1	9.54	34	95.1	28.9	1	1	7.37
		4105.1	4185.8	4319.4	9.41	44	106.0	25. 5	0.56	1.00	5.70
42	L2	4105.1	4185.8	4319.4	9.41	44	106.0	25.5	0.56	1	5.70
		4186.1	4219.7	4334.8	9.97	30	97.9	30.9	1	1	5.70
		4683.1	5348.1	5486.6	8.16	25	110.0	27.9	0.96	1.45	1.84
	L3	5248.7	5365.2	5526.5	9.30	34	105.9	34.7	0.53	1	3.98
		5308.0	5394.2	5533.7	9.62	30	100.3	41.0	1	1	3.98
		5102.5	5244.4	6225.4	10.55	41	103.4	23.4	0.67	1.12	5.70
	L1	5215.1	5252.5	6100.7	9.48	29	96.2	36.1	0.54	1	4.01
		5261.2	5414.3	6299.4	9.77	32	86.7	39.0	1	1	4.01
		6773.2	7541.2	7876.1	11.76	42	97.4	32.7	1.31	1.37	3.98
54	L2	7480.0	7611.0	7849.3	11.44	39	104.0	27.9	0.60	1	6.12
		7573.8	7662.3	7811.7	11.94	33	97.6	35.2	1	1	6.12
		8234.3	8347.5	9637.4	11.22	28	105.0	25.6	0.75	1.21	3.60
	L3	8575.7	9563.4	9926.3	10.90	15	109.1	49.8	0.64	1	2.07
		9038.6	9634.7	9841.6	10.89	42	97.5	50.0	1	1	1.48

6827.7 kg (27), and 6850.8 kg (19) – besides different geometrical parameters, the obtained arrangements of hangers distinctly point to the discrepant topologies. In this case, also the difference in objective function values for three best solutions is highest, reaching 1.1%. Here we should note one advantage of using stochastic optimization algorithms that generally produce several solutions with close objective function values but distant topologies: the designer may choose the third solution with only 34 hangers and lower arch rise instead of slightly lighter solution with 80 hangers.

Since the objective function values of few best solutions are very close, in the Table 2 only the results on the best solutions for each span and each load case are provided along with leading design parameters and the vertical stiffness ratios between the tie and arch. In the Table 2 also the median and maximum objective function values of all 50 independent numerical experiments are shown in order to characterize the optimization quality. Evidently, the discrete geometrical properties of cross-section assortments burden the optimization process. This is clearly seen, e.g., from the results on 18 m span and different load cases: the EA chose the same profiles for various loadings. Since the design parameters are interdependent, it results in rather far-off values of remaining parameters.

In common, in engineering practice the simpler network arch bridge design is preferable: the circular arch shape, and the equidistant placement of hangers along the tie. Therefore, next we optimise the same bridge schemes at different initial conditions: only allowing the circular shape of bridge and alternating distances between hangers, and circular shape plus equidistant placement of hangers. The obtained results are compared in the same Table 2.

In two cases the circular arch shape provides even better optimization results than the more general arch shape – the cases of 18 m L1 and 42 m L1. The gains do not exceed 1.6% and can be explained by a well suited pair of I-beam and hollow tube profiles for those particular span/load cases. In all remaining cases the results are as expected: more freedom in geometrical parameters produces better results. Going from elliptical arch to a circular arch, the losses are insignificant for smaller spans but reach 7.8% for 42 m L3 and 12.1% for 54 m L2. Further reducing the set of design parameters, placing the hangers at equal distances, the losses are even smaller, reaching only up to 5.4% for 54 m L3. The largest difference between objective functions for 1st and 3rd sets of design parameters comprises only 13.3% (54 m L3).

Changing the shape of the arch from elliptical to circular, the EA apparently tries reducing the initial distance between hangers. Only for the elliptical arch case of 42 m L1 – this was the only case with ellipse radii ratio <1, the initial distance is increased when the arch becomes circular. In all cases the optimal initial distance is around 0.5–0.7 of average distance.

Also, when going to a circular arch and then to a circular arch with an equidistant placement of hangers, the algorithm places the hangers at more upright angles: the α_i diminishes while the α_f increases. The number of hangers is rather stable parameter for smaller spans and lower loading, the dispersal increases for larger spans and loads.

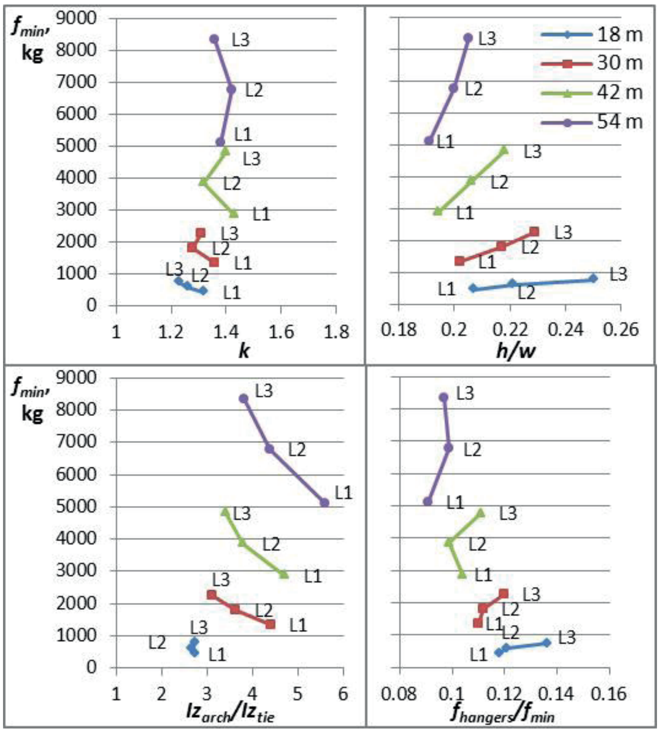


Fig 4 Dependencies of the main design parameters on the spans and load cases; continuous cross-section palette

Generally, the discrete nature of cross-section palette prevents drawing general conclusions about the optimal ranges of design parameters. In order to obtain deeper insights on the optimal values of parameters, the load-carrying structure of the bridge was re-optimized employing the welded hollow tube cross-sections for the arch and the tie. Now instead of design parameters No 8 and 9 (Table 1), four new continuous design parameters appear: the width and the height of the arch and the tie tubes; the range of parameters is 0.2 to 0.6 m. The thickness of the tube wall is chosen depending on the height of cross-section so that the local stability constraint is satisfied.

Concerning parameters on which contradictory recommendations are proposed in the literature – for the number of hangers and their placement scheme, we find rather scattered optimal values: from 30 for both sets of hangers (18 m L1) to 60 (30 m L3). Plausible, including into the objective function the additional fictitious masses due to the price of hanger/arch and hanger/tie connection nodes, the EA would yield lower numbers of hangers. However, the ratio of the total mass of hangers to the total mass of the whole bridge is rather stable (Fig. 4), falling between 0.10 and 0.13. Studying the hanger inclination angles it is obvious that those parameters have less influence on the objective function. Optimal value of the last hanger angle is about 30°, while the first hanger should be at an angle 100° – 105°. Also is evident, that the hangers should be placed at alternating distances between them, starting from narrower distance of 0.8–0.9 of an average distance, and gradually increasing it.

The average optimal values of all main parameters in three best solutions (ratios of ellipse radii, of the rise of arch to span, of vertical stiffnesses of arch and tie, and of total mass of hangers to the total mass of bridge) are shown in Fig. 4. Thus, the optimal shape of arch is clearly elliptical with ratio of both ellipse radii from 1.20 to 1.40, and the ratio between rise and span falls into narrow interval from 0.19 to ~0.23. For lighter bridge, the vertical stiffness of an arch must be higher than stiffness of the tie; with increasing span the differences between stiffnesses should diminish.

Table 4 Comparison two bridge schemes: network arch and under-deck stayed bridges

Span m	Load case	Mass of network arch bridge kg	Mass of under-deck stayed bridge kg
18	L1	**482.2**	569.4
	L2	**602.7**	788.1
	L3	**786.5**	985.1
30	L1	1677.4	**1527.4**
	L2	**1982.9**	2164.8
	L3	**2355.6**	2746.6
42	L1	3233.0	**2943.4**
	L2	**4105.1**	4292.7
	L3	**4683.1**	5582.0
54	L1	5102.5	**5064.8**
	L2	**6773.2**	6966.8
	L3	**8234.3**	9002.5

Lastly, we provide the comparison of two alternative bridge schemes: the current results on network arch bridges and the optimization results of the novel bridge scheme, the under-deck stayed bridges for the same spans and loading cases under the adequate constraints according to the Eurocodes. [13, 14] claim this innovative unconventional bridge type is superior for medium spans. The optimized schemes of under-deck stayed bridges composed of steel profiles from the same assortment tables are provided in [15].

Generally, the Tveit's proposition that the network arch bridge is the most slender bridge scheme is true. Only for the least loadings the under-deck stayed bridges outperform the network arch scheme by 9.8–9.9% (30 m L1, 42 m L1). The network arch bridge is clearly more slender for the heaviest loads, being lighter even by 19.2–25.1% (42 m L3, 18 m L3).

4 Conclusions

A technique for the simultaneous topology/shape/sizing optimization of load-bearing structures of network arch bridges has been suggested. The main idea of the work is presenting the designer a simple and fast tool for obtaining the full initial project of a bridge by "one button click". Later, the obtained scheme can be validated by subsequent non-linear and dynamic analyses.

The technique covers four independent programs: the finite element linear static analysis program, the meshing pre-processor, the post-processor for evaluation of constraints and objective function, and the population-based optimization algorithm. If required, each part can be replaced by an appropriate program.

Based on the results obtained from the optimization of 12 moderate span network arch pedestrian bridge structures composed of industrial steel profiles, it was possible to conclude the following:

The optimal arch shape is elliptical. However, the losses in the total mass of the structure when simplifying the arch shape to the circular are more significant only for longer spans and heavier loadings and reach up to 12%.

The optimal rise of the arch is the most stable parameter, for all cases falling into narrow interval 0.2–0.23 of the span.

The hangers should be placed at inclined angles, starting from an angle >90° and ending with 25–50°.

The optimal values of remaining design parameters are more scattered and for the particular spans, loadings and cross-section assortments should be determined employing global optimization programs.

References

[1] Tveit, P. "Information on the Network Arch by Per Tveit". 2016. http://home.uia.no/pert/index.php/Home

[2] Teich, S. "Contribution to Optimizing Network Arch Bridges". (Beitrage zur Optimierung von Netzwerkbogenbrücken). PhD Dissertation. Technical University Dresden. 2012. http://www.qucosa.de/fileadmin/data/qucosa/documents/8604/Dissertation_Teich.pdf

[3] Brito, E. G. S. "Design of Network Arch Bridges". MSc Thesis, University of Porto. 2009.

[4] Rana, S., Islam, N., Ahsan, R., Ghani, S. N. "Application of evolutionary operation to the minimum cost design of continuous prestressed concrete bridge structure". *Engineering Structures*, 46, pp. 38–48. 2013. https://doi.org/10.1016/j.engstruct.2012.07.017

[5] Ahsan, R., Rana, S., Ghani, S. N. "Cost Optimum Design of Posttensioned I-Girder Bridge Using Global Optimization Algorithm". *Journal of Structural Engineering*, 138(2), pp. 273–284. 2012. https://doi.org/10.1061/(ASCE)ST.1943-541X.0000458

[6] Lute, V., Upahyay, A., Singh, K. K. "Genetic algorithms-based optimization of cable stayed bridges". *Journal of Software Engineering and Applications,* 4(10), pp. 571–578. 2011. https://doi.org/10.4236/jsea.2011.410066

[7] Hasançebi, O. "Optimization of truss bridges within a specified design domain using evolution strategies". *Engineering optimization*, 39(6), pp. 737–756. 2007. https://doi.org/10.1080/03052150701335071

[8] Smit, T. J. M. "Design and construction of a railway arch bridge with a network hanger arrangement". MSc Thesis, Delft University of Technology. 2013. http://homepage.tudelft.nl/p3r3s/MSc_projects/reportSmit.pdf

[9] Deep, K., Singh, K. P., Kansal, M. L., Mohan, C. "A real coded genetic algorithm for solving integer and mixed integer optimization problems". *Applied Mathematics and Computation,* 212(2), pp. 505–518. 2009. https://doi.org/10.1016/j.amc.2009.02.044

[10] Belevičius, R., Juozapaitis, A., Rusakevičius, D., Šešok, D. "Topology, Shape and Sizing Optimization of Under-Deck Stayed Bridges". In: *Proceedings of theFourth International Conference on Soft Computing Technology in Civil, Structural and Environmental Engineering.* (Tsompanakis, Y., Kruis, J., Topping, B. H. V. (Eds.)), Civil-Comp Press, Stirlingshire, United Kingdom, paper 12, 2015. https://doi.org/10.4203/ccp.109.12

[11] Deep, K., Thakur, M. "A new crossover operator for real coded genetic algorithms". *Applied Mathematics and Computation,* 188(1), pp. 895–911. 2007. https://doi.org/10.1016/j.amc.2006.10.047

[12] Deep, K., Thakur, M. "A new mutation operator for real coded genetic algorithms". *Applied Mathematics and Computation,* 193(1), pp. 211–230. 2007. https://doi.org/10.1016/j.amc.2007.03.046

[13] Ruiz-Teran, A. M., Aparicio, A. C. "Parameters governing the response of under-deck cable-stayed bridges". *Canadian Journal of Civil Engineering, 34*(8), pp. 1016–1024. 2011. https://doi.org/10.1139/l07-016

[14] Camara, A., Ruiz-Teran, A. M., Stafford, P. J. "Structural behaviour and design criteria of under-deck cable-stayed bridges subjected to seismic action". *Earthquake Engineering & Structural Dynamics*, 42(6), pp. 891–912, 2013. https://doi.org/10.1002/eqe.2251

[15] Belevičius, R., Juozapaitis, A., Rusakevičius, D. „Optimal schemes of under-deck stayed bridges for different spans and deck types". *Bauingenieur.* in press.