Periodica Polytechnica Civil Engineering

61(1), pp. 39–50, 2017 DOI: 10.3311/PPci.10095 Creative Commons Attribution ①

RESEARCH ARTICLE

Empirical methods of calculating the mechanical parameters of the rock mass

Balázs Vásárhelyi1*, Dorottya Kovács1

Received 03 October 2016; accepted 30 October 2016

Abstract

The knowledge of the main mechanical constants of a rock mass (such as strength, deformability and the Poisson's ratio) is one of the most important for rock engineering design on or in rock mass. Until now, several empirical relationships were determined for calculating these material constants based on both the quality of the studied rock mass (ie. RMR or GSI values) and the mechanical parameters of an intact rock.

The goal of this paper is to review the empirical relationships between the mechanical properties of rock masses and the rock mass classification systems. The engineering properties involve not only the uniaxial compressive strength and deformation modulus of rock masses, but also Poisson's ratio and tensile strength, among the others, which are of crucial for designing rock engineering structures.

These different methods are compared and a general equation is determined in this paper.

The presented expressions are yet to be tested with experimental data and empirical relationships should not replace in situ tests for final design.

Keywords

rock mass strength, deformability, Poisson's rate, empirical rock mass classification

¹Department of Engineering Geology and Geotechnics Faculty of Civil Engineering, Budapest University of Techology and Economics H-1521 Budapest, P.O.B. 91, Hungary *Corresponding author email: vasarhelyi.balazs@epito.bme.hu

1 Introduction

Large scale rock mass characterization introduces material parameters related to mechanical properties. The most important properties are the deformation modulus; the unconfined strength and the Poisson's ratio value of the rock mass in interest. These material parameters are frequently related to laboratory data characteristics of intact rock samples and to the classical rock mass classification systems (e.g. *RQD*, *Q*, *RMR* or *GSI*). These rock mass quality measures quantify the relation between the rock mass and the intact rock.

The established empirical relations between the mechanical parameters of rock masses (unconfined compressive strength, deformation modulus) and the rock mass classification systems (*RMR* or *GSI* values) show exponential increasing deformation modulus and compressive strength with the increasing quality of the rock mass.

The paper summarizes the observed correlations published between the mechanical properties of rock masses and one of the rock mass classification systems. It was found, that the deformation modulus and the strength of a rock mass data may reflect a simple exponential relationship of the observed quantities. According to analysis of different proposed equations a new formula is suggested and the modification ratio (i.e. ratio of the deformation modulus and the strength of rock mass) is also determined.

However, it is important to note, that there are huge differences between the published date and the empirical formulas. The reason for this variance is the difference in situ testing methods, that may give different values of mechanical parameters even for the same rock mass. According to Bieniawski [1], even a single testing method, such as flat jack test, can lead to a widely scattering results even where the rock mass is very uniform. The other reason for the discrepancy in the different calculation methods is the directional effect. Most rock masses are anisotropic and do not have single deformation modulus [2].

There is no mechanical (physical) interpretation of the above empirical equations but these were analysed by e.g. [3]. Recently, Ván and Vásárhelyi [4] suggested a damage mechanical approach to analyse them. In this paper their method will be also presented.

2 Nomenclature

- c_i: cohesion of intact rock
- c_m : cohesion of rock mass
- D: Disturbance factor (0: undisturbed to 1: very disturbed

[16])

- E_m : deformation modulus of rock mass
- E_r: deformation modulus of intact rock

GSI: Geological Strength Index (values between 0 and 100) MR Modification Ratio (ratio of the deformation modulus and the strength of the rock [3])

- Q Rock Quality according to Barton [24]
- RMR: Rock Mass Rate (values between 0 and 100) [11]
- RQD: Rock Quality Designation (values between 0 and 100) [5]

WD weathering degree (values between 1 and 4 according to [42])

- ϕ_i : internal friction angle of intact rock
- $\phi_{\rm m}$: internal friction angle of rock mass
- $v_{\rm m}$: Poisson's ratio value of rock mass
- $v_{\rm r}$: Poisson's ratio value of intact rock
- $\sigma_{\rm c}$: unconfined compressive strength of intact rock
- $\sigma_{\rm cm}$: unconfined compressive strength of rock mass
- σ_t : tensile strength of intact rock
- $\sigma_{\rm tm}$: tensile strength of rock mass

3 Deformation modulus of rock masses

In the literature several relationships have been suggested of the deformation modulus and the rock mass quality measures. These commonly used equations are the deformation modulus and *RQD*, *RMR*, *GSI* and *Q* systems

3.1 RQD based methods

The Rock Quality Designation (*RQD*) method developed by Deere [5] and couple of years later [6] it was suggested, that this method can be also used for determining the deformation modulus of rock masses, as well. Collecting several in situ measurements Coon and Merritt [7] developed the first relationship between the modulus ratio (E_m/E_r , i.e. the ratio deformation modulus of rock mass and intact rock, respectively) and the *RQD* value. In Fig. 1 their published results is shown. Later, Gardner [8] improved their approach and suggested the following form:

 $E_m = \alpha_E E_r$

where

$$\alpha_E = 0.0231 (RQD) - 1.32 \ge 0.15$$
(1b)

For RQD > 57 %, Eq. (1) is the same as the relation of Coon and Merritt [7], while for RQD < 57 % the Eq. (1) gives $E_m/E_r =$ 0.15. Note that, this method is adopted by the American Association of State Highway and Transportation Officials in the Standard Specification for Highway Bridges [9]. This equation has the following limitations (according to Zhang and Einstein [10]):

- The range of RQD < 60 % is not covered only an arbitrary value of E_m/E_r can be selected in this range
- For RQD = 100 %, E_m is assumed to be equal to E_r . This is obviously unsafe in design practice because RQD =100 % does not mean that the rock is intact. There may be discontinuities in rock masses with RQD=100 % and thus E_m may be smaller, then E_r even when RQD = 100% [10].



Fig. 1 Variation of E_m/E_r with RQD [7]

Zhang and Einstein [10] added further collected from published literature to cover the entire range $0 \le RQD \le 100$ % (see Fig. 2).



Fig. 2 E_m/E_r in function with *RQD* according to [10] (see Eq. (2)

Zhang and Einstein [10] analysed the followings: (1) testing method; (2) directional effects; (3) discontinuity conditions; (4) intensity of *RQD* to discontinuity frequency and they proposed the following relationships between *RQD* and E_m/E_r (see Fig. 2)

(1a)

Lower bound:

$$E_m / E_r = 0.2x 10^{0.0186 RQD - 1.91}$$
(2a)

Upper bound:

$$E_m / E_r = 1.8 \times 10^{0.0186 RQD - 1.91}$$
 (2b)

Mean:

$$E_m / E_r = 10^{0.0186 RQD - 1.91}$$
(2c)

Eq. (2c) in exponential form [4] the mean value can be recalculated in the following form:

$$E_m / E_r = e^{\frac{RQD-100}{22.52}}$$
(3)

RQD does not consider the discontinuity conditions, however they have a great effect on the rock mass deformation modulus [2]. Kayabasy et al. [12] derived the following relation form a database of 57 tests showing the influences of weathering degree of the discontinuities on the rock mass deformation modulus:

$$E_{m} = 0.1423 \left[\frac{E_{r} \left(1 + 0.01 R Q D \right)}{W D} \right]^{1.1747}$$
(4a)

where *WD* is the weathering degree (1: fresh, 2: slightly weathered, 3: moderately weathered, 4: highly weathered – according to [42]. Applying multiple regression analysis, and considering the same independent variables, the following equation was obtained [12]:

$$E_m = 4.32 - 3.42WD \Big[0.19E_r (1 + 0.01RQD) \Big] \quad (4b)$$

By assigned 58 new test values to the database of Kayabasy et al. [12], Gokceoglu et al [18] derived the following correlation based on regression analysis:

$$E_{m} = 0.001 \left[\frac{\left(E_{r} / \sigma_{c} \right) \left(1 + 0.01 R Q D \right)}{W D} \right]^{1.5528}$$
(5)

The prediction graph for the deformation modulus of rock mass in function of *RQD* and the weathering degree is presented in Fig. 3, according to Gokceoglu et al. [18].

3.2 RMR or GSI based methods

The proposed correlations between the deformation modulus of rock mass and the *RMR/GSI* values can be divided into two parts:

- the deformation modulus of the rock mass calculate independently the deformation modulus of the intact rock
- the deformation modulus of the intact rock is also used for determining the deformation modulus of the rock mass.

In this chapter there are not differences between the RMR and the *GSI* values, they used parallel.



Fig. 3 Prediction graph for the deformation modulus of rock mass, in the function of RQD and the weathering degree [18]

3.2.1 Independent equations

Firstly Bianiawski [13] suggested a linear relationship between deformation modulus of the rock mass and the RMR value. He studied seven projects and assumed the deformation modulus of the rock mass is independent of the deformation modulus of intact rock:

$$E_m = 2RMR - 100 \quad (GPa) \tag{6}$$

This equation does not give modulus values for RMR < 50% (see Fig. 3), thus it cannot be used for poorer rock masses.

Later, Serafim and Pereira [14] proposed the more known expression, which can be used from poor to very good rock mass quality:

$$E_m = 10^{\frac{RMR-10}{40}}$$
(7)

According to the suggestion of Serafim and Pereira [14], the deformation modulus of rock mass is independent of the deformation modulus of intact rock. Fig. 4 shows graphically both expressions and their comparison.



Fig. 4 Correlation between the in situ deformation modulus of deformation and RMR

Comparing Eq. (7) with in situ measurements it was found, that it can be used for good quality rocks, however, for poor quality rocks it appears to predict too high values [15]. Based upon practical observations and back analysis of excavation behaviour in poor quality rock masses, the following modification to Serafim and Pereira's equation [14] is proposed for $\sigma_c < 100$ MPa:

$$E_m = \sqrt{\frac{\sigma_c}{100}} 10^{\frac{GSI-10}{40}} (GPa)$$
(8)

Note that, *GSI* has been substituted for *RMR* in this equation and that the modulus $E_{\rm m}$ is reduced progressively as the value of $\sigma_{\rm c}$ falls below 100 MPa. This reduction is based upon the explanation that the deformation of better quality rock masses are controlled by the discontinuities while, for poorer quality rock masses, the deformation of the intact rock parts contributes to the overall deformation process [15]. Eq. (8) is plotted in Fig. 5.



Fig. 5 Deformation modulus versus GSI [15], see Eq. (8)

Later, Hoek et al. [16] empirically estimated E_m based on *GSI* and *D* (Disturbance factor) in the following form:

$$E_m = \left(1 - \frac{D}{2}\right) \sqrt{\frac{\sigma_c}{100}} 10^{\frac{GSI-10}{40}} \quad (GPa) \tag{9a}$$

 $\sigma_{c} < 100 \text{ MPa}$

$$E_m = \left(1 - \frac{D}{2}\right) 10^{\frac{GSI-10}{40}} \quad (GPa) \tag{9b}$$

 $\sigma_a > 100 \text{ MPa}$

Similarly to Eq. (1), Galera et al. [17] found linear connection in case of RMR > 50 % values:

$$E_m = 0.0876 RMR \quad (GPa) \tag{10a}$$

In case of poorer rock masses, i.e. RMR < 50 %, they proposed the following form [17]:

$$E_{m} = 0.0876RMR + 1.056(RMR - 50) + 0.015(RMR - 50)^{2}$$

(GPa) (10b)

Some researchers suggest that there is an exponential correlation between the deformation modulus of rock mass and the *RMR/GSI* values:

$$E_m = a e^{bRMR} \quad (GPa) \tag{11}$$

where *a* and *b* constants. Table 1 shows the published data.

Table 1 Pubslished constants of Eq. (11)				
a	b	Ref.		
0.33	0.064	[2]		
0.0736	0.0755	[18]		
0.1451	0.0654	[18]		

Finally, Hoek and Diederichs [19] suggested the following equation (see Fig. 6):

$$E_m = 100000 \left(\frac{1 - D/2}{1 + e^{((75 + 25D - GSI)/11)}} \right) (GPa)$$
(12)



Fig. 6 Plot od Simplified Hoek and Diederichs equation (9) according to [19]

Note that the constant $a = 100\ 000$ in Eq. (12) is not directly related to the physical properties of rock masses [19]. The sensitivity of Eq. (12) was analyzed by Ván and Vásárhelyi [52, 53]. According to their results, the deformation modulus of the rock mass highly sensitive for the *D* and *GSI* values.

3.2.2 Normalized equations

Several researchers normalized the deformation modulus of rock mass by the deformation modulus of intact rock. Using these results usually the regression coefficient is better than the previously presented method, mostly for the poorer quality rock masses.

In this chapter, the published E_m/E_r relationships are presented and they are recalculated in the following form, if it is possible [4]:

$$\frac{E_{rm}}{E_r} = e^{\frac{GSI-100}{A}}$$
(13)

where A is a general constant.

Firstly, Nicholson & Bieniawski [20] suggested a power equation for calculating the ratio of the deformation modulus of rock mass and the elastic modulus of intact rock:

$$E_{rm} / E_i = 1 / 100(0.0028RMR^2 + 0.9 exp(RMR / 22.82))$$
(14)

This equation can be transformed to exponential form with slight differences:

$$E_m / E_r = e^{\frac{RMR - 100}{22.94}}$$
(15)

Mirti et al [21] has developed the following empirical correlation:

$$\frac{E_{rm}}{E_r} = \frac{1 - \cos(\pi x RMR / 100)}{2}$$
(16)

Using the version of 2002 Hoek-Brown criteria [16], Sonmez et al. [22] determined the following equation:

$$E_{rm} / E_r = \left(s^a\right)^{0.4},$$
 (17)

Where *s* and *a* are Hoek-Brown parameters, which are depend on the *GSI* value in case of undisturbed rock mass:

$$s = e^{\frac{GSI-100}{9}} \tag{18}$$

and

$$a = 0.5 + \frac{1}{6} \left(e^{-\frac{GSI}{15}} - e^{-\frac{20}{3}} \right)$$
(19)

Recalculating Eq. (16) to exponential form:

$$\frac{E_{rm}}{E_r} = e^{\frac{GSI-100}{38.11}}$$
(20)

Note that, *s* value can be also calculated in case of very disturbed rock mass, i.e. the disturb factor (D) is equal to 1 [16]:

$$s = e^{\frac{RMR-100}{6}} \tag{21}$$

In this case the simple form is:

$$\frac{E_{rm}}{E_r} = e^{\frac{GSI-100}{25.04}}$$
(22)

Carvalcho [23] suggested similar equation as Sonmez et al [22], assuming, that the ratio of the deformation modulus of rock mass and the elastic modulus of intact rock depend only on the Hoek-Brown constant (s):

$$E_{mn} / E_n = s^{0.25}, \tag{23}$$

Where according to Eq. (17)

$$=e^{\frac{RMR-100}{9}}$$
 (24)

i.e.

$$\frac{E_{rm}}{E_r} = e^{\frac{RMR-100}{36}}$$
(25)

This equation corresponds to another suggestion of Galera et al. [17], which is based on pure empirical calculation. According to Eq. (22) the calculation of disturbed *s* value is:

$$\frac{E_{rm}}{E_r} = e^{\frac{RMR-100}{24}}$$
(26)

Hoek and Diederichs [18] recalculated several Chinese and Taiwanese in situ measured data thus they suggested the following equation (see also Fig. 7):

$$E_m = E_r \left(0.02 + \frac{1 - D/2}{1 + e^{((60 + 15D - GSI)/11)}} \right) (GPa)$$
(27)

where D is the disturbance factor [16].



Fig. 7 Plot of normalized in situ rock mass deformation modulus against Hoek and Diederichs equation (26). Each data point represents the average of multiple tests at the same site in the same rock mass [19]

The sensitivity of Eq. (27) was calculated by Ván and Vásárhelyi [52, 53]. Their results show that the calculated value are highly depend on the exact calculation of both GSI and D factor.

Based on the developed and applied proposed correlations, Ván and Vásárhelyi [4] collected the pure exponential relationships, according to Eq. (13).

$$\frac{E_{rm}}{E_r} = e^{\frac{GSI-100}{A}}$$
(28)

In Table 2 the selected values are summarized in the cases of disturbed and undisturbed rock mass. In Table 2 the equation of Zhang and Einstein [10] is also used.

Table 2 Collecting the different relationships	s' constants, using Eq. (13)
--	------------------------------

Ref.	Α		Note
	Undisturbed (D = 0)	Disturbed (D = 1)	
[20]	·	22.94	Not defined
[22]	38.11	25.04	
[23]	36.00	24.00	
[10]		22.52	Not defined
Average	37.06	24.38	

S

3.3 Q based methods

Until this point, the developed empirical formulas are independent of the deformation modulus of the intact rock.

The first relationship between rock mass quality (Q-value) and the deformation modulus of the rock mass was published by Barton et al. [24]. The following equation was proposed:

$$E_m = clogQ(GPa) \tag{29}$$

where *c* is a general constant: minimum 10, maximum 40 and the mean value is 25 (see Fig. 8). Eq. (27) is only applicable to Q > 1 and generally hard rocks.

Couple years later, Barton [25] modified Eq. (29) (see Fig. 8):

$$E_m = 10 \ Q^{1/3} (GPa) \tag{30}$$

According to the last modification of Barton [26] the deformation modulus of the rock mass depends on the unconfined strength of the rock, as well:

$$E_{rm} = 10 \left(Q \frac{\sigma_c}{100} \right)^{1/3} \tag{31}$$



RMR Classification

Fig. 8 Estimation the rock mass deformation modulus using the Q value [25]

4 Strength of rock mass

4.1 RQD based methods

First, Kuhawy and Goodman [27] suggested the calculation method from the *RQD* value:

- unconfined compressive strength (σ_{cm}) of rock masses is 0.33 σ_c if the RQD < 70 %.
- in case of RQD = 100 %, the $\sigma_{\rm cm} = 0.8 \sigma_{\rm c}$.
- there is a linear relationship between the two values, i.e.:

$$\frac{\sigma_{cm}}{\sigma_c} = 0.0157 RQD - 0.7667 \quad (70 \le RQD \le 100\%) \quad (32)$$

The suggestion of the AASHTO [9] is similar:

$$\frac{\sigma_{cm}}{\sigma_c} = 0.0231RQD - 1.32 \ge 0.15 \tag{33}$$

According to the AASHTO [9], the strength of rock mass is equal to the strength of intact rock, if RQD = 100 %.

Zhang [28] analysed the equations above and recognised, that in case of a very poor rock mass quality (RQD < 25 %) and a fair quality rock mass (RQD = 50-75 %), different $\sigma_{\rm cm}/\sigma_{\rm c}$ values should be expected.

Until this point, several researchers in rock mechanics and rock engineering have studied the relation between the unconfined compressive strength ratio $\sigma_{\rm cm}/\sigma_{\rm c}$ and the deformation modulus ratio $E_{\rm rm}/E_{\rm r}$ and found that these can be related approximately by the following equation (Ramamurthy [38]; Singh et al. [40]; Singh and Rao [41], Galera et al. [17]):

$$\frac{\sigma_{cm}}{\sigma_c} = \left(\frac{E_{rm}}{E_r}\right)^q = \alpha_E^q \tag{34}$$

in which the power q varies from 0.5 to 1.0 and is most likely in the range of 0.61 to 0.74 with an average of 0.7. However, using e.g. the AASHTO [9] the upper bound value is q =1. The power q value in Eq. (33) may vary significantly for different rock types and discontinuity conditions [28]. Using the average value of q (= 0.7) and the E_m/E_r versus *RQD* relation in Eq. (2c) [36], the new formula is the following:

$$\frac{\sigma_{cm}}{\sigma_c} = 10^{0.012RQD-1.34} \tag{35}$$

Fig. 9. shows a comparison of the σ_{cm}/σ_c versus *RQD* relation, according to Zhang [28]. It was mentioned [28], that the Eq. (34) covers the entire range ($0 \le RQD \le 100$ %) continuously. For *RQD* > 70 %, Eq. (31) corresponds with suggestions of [27] and [9].



Fig. 9 Comparison of σ_{cm}/σ_c versus *RQD* relations by different methods (according to [28])

4.2 RMR or GSI based methods 4.2.1 Independent equations

There was only one equation in which the strength of the rock mass is independent of the strength of the intact rock. According to Asef et al [29], the strength of rock mass exponentially increases with the rock mass rate (*RMR*), independently of other parameters:

$$\sigma_{cm} = 0.5e^{0.06RMR} \tag{36}$$

4.2.2 Normalized equations

To estimate the unconfined compressive strength (σ_{cm}) of rock masses, there are various suggested correlations considering the discontinuity characteristics. Their functional form is exponential but with different parameters. Excepts one, all the equations were recalculated to the following form, according to [4],

$$\frac{\sigma_{cm}}{\sigma_c} = e^{\frac{RMR-100}{B}}$$
(37)

where B is a general constant.

Yudhbir et al. was the first at 1983 [30] who suggested the following equation:

$$\frac{\sigma_{cm}}{\sigma_{c}} = e^{7.65 \frac{RMRI-100}{100}}$$
 (38a)

This equation can be transformed:

$$\frac{\sigma_{cm}}{\sigma_c} = e^{\frac{RMR-100}{13.07}}$$
(38b)

• Two years later Ramamurthy et al. [31] found the following empirical correlation:

$$\frac{\sigma_{cm}}{\sigma_{c}} = e^{\frac{GSI-100}{18.5}}$$
(39)

• Kalamaras and Bieniawski [32] published this relationship:

$$\frac{\sigma_{cm}}{\sigma_c} = e^{\frac{RMR-100}{24}} \tag{40}$$

• Using high number of measured data, Sheorey [33] determined the following empirical relationship:

$$\frac{\sigma_{cm}}{\sigma_c} = e^{\frac{RMR-100}{20}}$$
(41)

These equations are based on empirical results, except the equation of Hoek et al. [34]. With the Hoek-Brown strength criterion for rock masses, the unconfined compressive strength can be expressed as

$$\sigma_{cm} = \sqrt{s}\sigma_c \tag{42}$$

where *s* is the Hoek-Brown constant [35] can be calculated for:

• undisturbed (or interlocking) rock masses:

$$s = e^{\frac{RMR-100}{9}} \tag{43}$$

i.e. using Eqs. (42) and (43):

$$\frac{\sigma_{cm}}{\sigma_c} = e^{\frac{RMR-100}{18}} \tag{44}$$

This equation corresponds to the Hoek's suggested one, which is published in the paper of Zhang [36]:

$$\frac{\sigma_{cm}}{\sigma_c} = 0.036e^{\frac{GSI}{30}} \tag{45}$$

• disturbed rock masses:

$$s = e^{\frac{RMR-100}{6}} \tag{46}$$

i.e. using Eqs. (42) and (46):)

$$\frac{\sigma_{cm}}{\sigma_c} = e^{\frac{RMR-100}{12}} \tag{47}$$

Using the version 2002 of the Hoek-Brown equation [16], the ratio of the strength of the rock mass and the intact rock is:

$$\sigma_{cm} = s^a \sigma_c \tag{48}$$

Where

$$a = 0.5 + \frac{1}{6} \left(e^{-\frac{GSI}{15}} - e^{-\frac{20}{3}} \right)$$
(49)

and in case of undisturbed rock mass:

$$s = e^{\frac{RMR-100}{9}}$$
(50)

in case of disturbed rock mass:

$$s = e^{\frac{GSI-100}{6}} \tag{51}$$

According to Eqs. (42) and (51), in case of undisturbed rock mass:

$$\frac{\sigma_{cm}}{\sigma_c} = e^{\frac{GSI-100}{15.24}} \tag{52}$$

And in case of disturbed rock mass:

$$\frac{\sigma_{cm}}{\sigma_c} = e^{\frac{RMR-100}{10.16}} \tag{53}$$

In Fig. 10. the above presented relationships are shown by the publication of Zhang [2].

According to the calculation of Ván and Vásárhelyi [4], the general constants of Eq. (37) are the following (Table 3).



Fig. 10 Variation of ratio of rock mass unconfined compressive strength (σ_{cm}) to intact rock unconfined compressive strength (σ_c) with RMR or GSI ratings [2]

 Table 3 Collecting the different relationships' constants, using Eq. (37)

Ref.	В		Note
	Undisturbed (D = 0)	Disturbed (D = 1)	
[27]	-	13.07	Not defined
[28]	18.5	-	Not defined
[29]	24.00	-	Not defined
[30]	20.00	-	Not defined
[31])	18	12	-
[33]	15.24	10.16	-
Average	19.15	11.74	-

4.3 Q methods

Similarly to the deformation modulus of rock mass, the strength of rock mass can be calculated from the Q value. According to Bhasin and Grimstad [37] and Singh and Goel [38] the following equation is proposed:

$$\sigma_{cm} = 7\gamma f_c Q^{1/3} \quad (MPa) \tag{54}$$

where $f_c = \sigma_c/100$ for Q > 10 otherwise $f_c = 1$; and γ is the unit weight of the rock mass in [g/cm³]. It means, the strength of intact rock does not influence the strength of rock mass for poor rock mass quality.

5 Connection between the deformation modulus and the strength of rock mass

According to Galera et al. [17], there is an expression involving both rock mass modulus ($E_{\rm m}$) and rock mass strength ($\sigma_{\rm rm}$):

$$\frac{E_m}{E_r} = \left(\frac{\sigma_{rm}}{\sigma_c}\right)^{2/3}$$
(55)

The expression above has the merit of a useful cross-check and it conforms to an old practice proposed by Deere and Miller [43]: a strength - deformation representation featuring the concept of the "modulus ratio (MR)", i.e. the deformation modulus may be estimated from the uniaxial compressive strength (see also [3, 19, 44]:

$$E_i = MR\sigma_c \tag{56}$$

According to Ván and Vásárhelyi [4], theoretically the following relationships were determined:

$$\frac{E_m}{E_r} = \left(\frac{\sigma_{rm}}{\sigma_c}\right)^{\frac{RMR-100}{22}}$$
(57a)

$$\frac{E_{rm}}{\sigma_{cm}} = MR * e^{\frac{2(RMR-100)}{100}}$$
(57b)

6 Tensile strength of rock mass 6.1 RMR or GSI methods

The failure criteria of Hoek-Brown can also be used to obtain the tensile strength of rock mass. It can be determined by:

$$\sigma_{tm} = 05\sigma_c \left[m_b - \left(m_b^2 + 4s \right)^{0.5} \right]$$
(58)

where m_{b} and s are the Hoek-Brown constants (material constant of rock mass and the characteristic of rock mass, respectively).

Aydan et al [45] presented an empirical calculation method from Tokashiki's PhD thesis:

$$\frac{\sigma_{tm}}{\sigma_t} = \frac{RMR}{RMR + 6(100 - RMR)}$$
(59)

6.2 Q methods

According to Singh and Goel [38], the tensile strength of rock mass can be obtained by the similar Eq. (54):

$$\sigma_{cm} = -0.029\gamma f_c Q^{1/3} \quad (MPa) \tag{60}$$

where $f_c = \sigma_c/100$ for Q > 10 otherwise $f_c = 1$; and γ is the unit weight of the rock mass in [g/cm³].

7 Poisson's ratio value

The experiments on the Poisson's ratio of rock masses are quite rare. Due to the lack of a huge number of in situ data, there are not many suggestions for the calculation of the Poisson's ratio value in the rock mass classification system.

Aydan et al [46] analysed several uniaxial compressive strength tests and found that the Poisson's ratio decreases with increasing uniaxial compressive strength. According to their laboratory observations, they proposed the following form for rock masses:

$$v_m = 0.25 \left(1 + e^{-\sigma_{cm}/4} \right) \tag{61}$$

Eq. (61) consists of only the unconfined compressive strength of rock mass, which can be calculated from one of the above mentioned equations.

Tokshiki and Aydan [47] proposed a direct method of determining the Poisson's ratio from the *RMR* value (the prediction is plotted in Fig. 11):

$$v_m = 0.5 - 0.2 \frac{RMR}{RMR + 0.2(100 - RMR)}$$
(62)

Using Eq. (62), the Poisson rate (v) is 0.3 and 0 in case of RMR = 100 (i.e. intact rock) and RMR = 0 (extremely poor rock mass), respectively.

Later, Aydan et al [48] modified Eq. (62). According to their publication, if the Poisson's ratio of the rock is known, the following relationship can be used to determine the Poisson's ratio in the function of the Rock Mass Rate (*RMR*):





Fig. 11 Poisson rate in the function of the Rock Mass Rate [47]

Vásárhelyi [49] estimated the Poisson's ratio value of the rock mass based on theoretical background. He found a linear relationship: as the quality of the rock mass decreases, the Poisson's ratio increases. Two correlations were determined:

if the Poisson's ratio of the intact rock is known:

$$v_{rm} = -0.002GSI + v_i + 0.2, \tag{64a}$$

• in case of the Hoek-Brown constant (m_i) is known:

$$v_{rm} = -0.002GSI - 0.003m_i + 0.457 \tag{64b}$$

In Fig. 12 the Poisson ratio values are plotted in the function of GSI value in case of different Hoek-Brown constants (m_i) .



Fig. 12 Estimated Poisson's rate values (v_{rm}) in the function of the geological strength index (*GSI*) in case of different Hoek-Brown (*m*_i) constants

In Fig. 13. the calculated Poisson ratios were calculated from the equation of Aydan et al. [48] (Eq. 63) and the suggestion of Vásárhelyi [49], using Eq. (64a), calculating with different Poisson ratios of intact rock (v = 0.1...0.4). One can see, the result of Aydan et al. [48] is equal to the result of Vásárhelyi [49] if the Poisson ratio of the intact rock (v_i) is equal to 0.15.



Fig. 13 Comparaison of the calculation methods: Aydan et [48] and Vásárhelyi [49]

According to Vásárhelyi [49] the Poisson ratio of rock mass linearly depend on both Geological Strength Index (*GSI*) and Hoek-Brown parameter (m_i). Recently, Vásárhelyi et al. [54] analyzed this constant and suggested new calculation method for Hoek-Brown failure criteria. Probably, using that modification, the Poisson ratio of rock mass can be determined more precisely.

8 Mohr-Coulomb parameters

The failure criteria of the studied rock masses are very important for rock engineering design. The most important empirical failure criteria were collected and published by Sheorey [33]. It his paper, the influence of the quality of rock masses for the Mohr-Coulomb parameters (cohesion and the internal friction angle) are summarized.

For using the Mohr-Coulomb failure criterion, it is necessary to estimate the cohesion and the friction angle parameters of the rock masses:

$$\tau_f = c_m + \sigma_n' tan\phi_m \tag{65}$$

where $\tau_{\rm f}$ is the shear strength of rock mass, $\sigma_{\rm n}$ ' is the effective normal stress on sliding plane, $c_{\rm m}$ and $\phi_{\rm m}$ are the cohesion and the internal friction angle of rock mass, respectively.

8.1 RMR based methods 8.1.1 Independent equation

According to Bieniawski [11] the cohesion and the friction angle of rock mass related to the *RMR* value. In the publication of Bieniawski [11] there is not exact calculation method between the *RMR* value and the Mohr-Coulomb parameters – he was suggested intervals for rock classes.

Sen and Sadagah [50] suggested a continuous system for the calculation of these constants:

$$c_m = 3.625 RMR \tag{66}$$

and

$$\phi_m = 25[1 + 0.01RMR] \text{ for } RMR \ge 20\%$$
 (67a)

$$\phi_m = 1.5 RMR \quad for \quad RMR < 20\% \tag{67b}$$

According to the publication of Aydan et al. [46] the internal friction angle of rock mass depends on the strength of rock mass, i.e.:

$$\phi_m = 20\sigma_{cm}^{0.25} \tag{68}$$

Later, Aydan and Kawamoto [51] found a linear connection between the internal friction angle and the Rock Mass Rate (RMR) value:

$$\phi_m = 20 + 0.5RMR \tag{69}$$

In this case the cohesion can be calculated from the friction angle and the strength of rock mass:

$$c_m = \frac{\sigma_{cm}}{2} \frac{1 - \sin \phi_m}{\cos \phi_m} \tag{70}$$

8.1.2 Normalized equations

Aydan et al. [48] suggested the following form for calculating the cohesion of rock mass, which can be applied when the Mohr-Coulomb parameters of intact rock is known:

$$c_m = \frac{RMR}{RMR + 6(100 - RMR)}c_i \tag{71}$$

8.2 Q method

The cohesion of rock mass can be calculated from the different parameters of *Q*-values [26]:

$$c_m = \left(\frac{RQD}{J_a} \frac{1}{SRF} \frac{\sigma_c}{100}\right) \tag{72}$$

9 Conclusions

The different empirical methods were summarized in this paper for calculating the mechanical parameters of rock masses, such as deformation modulus, compressive and tensile strength, Poisson ratio and the Mohr-Coulomb parameters.

Unfortunately, determination of the rock mass quality is not exact. Using the well-known rock mass classification systems (i.e. RMR, Q and GSI) in the radioactive waste repository at Bátaapáti (Hungary), the classification of the tunnel face was influenced by high subjectivity [55]. There is not exact relationship between the different classifications, as it is rather project dependent [56]. Recently, several authors published papers about the quantitative determination of GSI value but the differences among the results are extremely high [57]. It can be declared that the rock mass classification is not exact, it depends highly on the rock engineer, the applied measuring systems, the project, etc.

The sensitivity of some of the presented equations were calculated by Ván and Vásárhelyi [52, 53] and it was found that these relationships are highly dependent on the input parameters changing one parameter with 5 %, and the final results may change more than 50 %!

It would be useful to apply damage theory in rock mechanics – the first results in this were published by Ván and Vásárhelyi [4] and Kamera et al. [58].

The presented expressions are yet to be tested with experimental data and empirical relationships, so these should not replace in situ tests for final design.

References

- Bieniawski, Z. T. "Determining rock mass deformability: experience from case histories". *International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts*. 15 (5), pp. 237–247. 1978. DOI: 10.1016/0148-9062(78)90956-7
- Zhang, L. "Engineering Properties of Rocks". 290 p. Elsevier, 2005. https://app.knovel.com/web/toc.v/cid:kpEPR00001/viewerType:toc/root_ slug:engineering-properties
- [3] Palmström, A., Singh, R. "The deformation modulus of rock masses comparisons between in situ tests and indirect estimates". *Tunnelling and Underground Space Technology*. 16 (2), pp. 115-131. 2001. DOI: 10.1016/ S0886-7798(01)00038-4
- [4] Ván, P., Vásárhelyi, B. "Relation of rock mass characterization and damage". In: *Rock Engineering in Difficult Ground Conditions - Soft Rocks and Karst*. Vrkljan I. (Ed), pp. 399-404. Taylor & Francis Group, London. 2010. http://real.mtak.hu/14136/1/1322175.pdf
- [5] Deere, D. U. "Technical description of rock cores for engineering purposes". *Rock Mechanics and Engineering Geology*. 1 (1), pp. 16-22. 1963. http://www.oalib.com/references/7750673
- [6] Deere, D. U., Hendron, A. J., Patton, F. D., Cording, E. J. "Design of surface and near-surface construction in rock." In: Failure and Breakage of Rock. The 8th U.S. Symposium on Rock Mechanics (USRMS), Minneapolis, Minnesota Sep. 15-17, 1966. Fairhurst, C. (Ed), pp. 237-302. 1967. https:// www.onepetro.org/conference-paper/ARMA-66-0237

- [7] Coon, R. F., Merritt, A. H. "Predicting in situ modulus of deformation using rock quality indexes". In: *Determination of the in Situ Modulus of Deformation of Rock*. ASTM International, Philadelphia. pp. 154-173. 1970.
- [8] Gardner, W. S. "Design of drilled piers in the Atlantic Piedmont". In: Foundations and Excavations in Decomposed Rock of the Piedmont Province. GSP 9, Smith, R. E. (Ed), American Society of Civil Engineers, New York, NY. pp. 62-86. 1987. http://cedb.asce.org/CEDBsearch/record. jsp?dockey=0051472
- [9] AASHTO. "Standard Specifications for Highway Bridges". 14th edition, American Association of State Highway and Transportation Officials, Washington DC. 1989.
- [10] Zhang, L., Einstein, H. H. "Using RQD to estimate the deformation modulus of rock masses". *International Journal of Rock Mechanics* and Mining Sciences. 41 (2), pp. 337–341. 2004. DOI: 10.1016/S1365-1609(03)00100-X
- Bieniawski, Z. T. "Engineering Rock Mass Classifications". 272 p. Wiley, Chichester. 1989. http://eu.wiley.com/WileyCDA/WileyTitle/ productCd-0471601721.html
- [12] Kayabasi, A., Gokceoglu, C., Ercanoglu, M. "Estimating the deformation modulus of rock masses: a comparative study". *International Journal* of Rock Mechanics and Mining Sciences. 40 (1), pp. 55–63. 2003. DOI: 10.1016/S1365-1609(02)00112-0
- [13] Bieniawski, Z. T. "Determining rock mass deformability: experience from case histories". *International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts.* 15 (5), pp. 237-247. 1978. DOI: 10.1016/0148-9062(78)90956-7
- [14] Serafim, J. L., Pereira, J. P. "Consideration of the geomechanics classification of Bieniawski". In: Proc. Int. Syrup. Eng. Geol. Underground Constr., Lisbon, 1. pp. II33-II42. 1983.
- [15] Hoek, E., Brown, E. T. "Practical estimates of rock mass strength". *International Journal of Rock Mechanics and Mining Sciences.* 34 (8), pp. 1165-1186. 1997. DOI: 10.1016/S1365-1609(97)80069-X
- [16] Hoek, E., Carranza Torres, C. T., Corkum, B., "Hoek-Brown failure criterion, 2002 edition". In: Proceedings of the fifth North American rock mechanics symposium, Toronto, Canada. 1. pp. 267-273. 2002. http:// citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.484.9671&rep=rep1 &type=pdf
- [17] Galera, J. M., Álvarez, M., Bieniawski, Z.T. "Evaluation of the deformation modulus of rock masses using RMR: Comparison with dilatometer tests". In: Proceedings of the ISRM Workshop W1, Madrid, Spain, Jul. 6-7, 2007. Taylor & Francis, Madrid. pp. 71-77. 2007. DOI: 10.1201/NOE0415450287.ch9
- [18] Gokceoglu, C., Sonmez, H., Kayabasi, A. "Predicting the deformation moduli of rock masses". *International Journal of Rock Mechanics and Mining Sciences*. 40 (5), pp. 701–710. 2003. DOI: 10.1016/S1365-1609(03)00062-5
- [19] Hoek, E., Diederichs, M. S. "Empirical estimation of rock mass modulus". *International Journal of Rock Mechanics and Mining Sciences*. 43 (2), pp. 203-215. 2006. DOI: 10.1016/j.ijrmms.2005.06.005
- [20] Nicholson, G.A., Bieniawski, Z.T. "A nonlinear deformation modulus based on rock mass classification". *International Journal of Mining* and Geological Engineering. 8 (3), pp. 181–202. 1990. DOI: 10.1007/ BF01554041
- [21] Mitri, H. S., Edrissi, R. Henning, J. G. "Finite element modeling of cablebolted stopes in hard-rock underground mines". In: SME Annual Meeting, Albuquerque, New Mexico. pp. 94-116. 1994.
- [22] Sonmez, H., Gokceoglu, C., Ulusay, R. "Indirect determination of the modulus of deformation of rock masses based on the GSI system". *International Journal of Rock Mechanics and Mining Sciences.* 41 (5), pp. 849–857. 2004. DOI: 10.1016/j.ijrmms.2003.01.006

- [23] Carvalho, J. "Estimation of rock mass modulus". Equation from the publication of [14], 2004.
- [24] Barton, N., Loser, F., Lien, R., ,Lunde, J. "Application of Q-System In Design Decisions Concerning Dimensions And Appropriate Support For Underground Installations". In: *Subsurface Space*. Bergman, M. (Ed), 2, pp. 553-561. Elsevier, 1981. DOI: 10.1016/B978-1-4832-8421-7.50080-6
- [25] Barton, N. "Permanent support for tunnels using NMT Special Lecture". In: Proc. Symp. of KRMS (Korea Rock Mechanics Society) and KSEG (Korea Society of Engineering Geology), pp. 1-26. 1995.
- [26] Barton, N. "Some new Q value correlations to assist in site characterization and tunnel design". *International Journal of Rock Mechanics and Mining Sciences.* 39 (2), pp. 185-216. 2002. DOI: 10.1016/S1365-1609(02)00011-4
- [27] Kulhawy, F. H., Goodman, R. E. "Foundations in rock". In: Ground Engineer's reference book. Chapter 15. Bell, F. G. (Ed). Butterworths, London, UK. 1987.
- [28] Zhang, L. "Determination and applications of rock quality designation (RQD)." *Journal of Rock Mechanics and Geotechnical Engineering*. 8 (3), pp. 389-397. 2016. DOI: 10.1016/j.jrmge.2015.11.008
- [29] Asef, M.R., Reddish, D.J., Lloyd, P.W. "Rock-support interaction analysis based on numerical modeling". *Geotechnical and Geological Engineering*. 18 (1), pp. 23- 37. 2000. DOI: 10.1023/A:1008968013995
- [30] Yudhbir, R. K., Lemanza, W., Prinzl, F. "An empirical failure criterion for rock masses". In: Proc. 5th ISRM Congress, Apr. 10-15, 1983, Melbourne, Australia, 1. pp. B1-B8. 1983. https://www.onepetro.org/conferencepaper/ISRM-5CONGRESS-1983-042
- [31] Ramamurthy, T., Rao, G. V., Rao, K. S. "A strength criterion for rocks". In: Proc. Indian Geotech. Conf., Roorkee, 1, pp. 59-64. 1985.
- [32] Kalamaras, G. S., Bieniawski, Z. T. "A rock mass strength concept for coal seams". In: Proc. 12th Conf. Ground Control in Mining. Morgantown, pp. 274-283. 1993.
- [33] Sheorey, P. R. "Empirical rock failure criteria". 200 p. Balkema, Rotterdam. 1997.
- [34] Hoek, E., Kaiser, P. K., Bawden, W. F. "Support of underground excavations in hard rock". 300 p. Balkema, Rotterdam. 1995. Hoek, E., Brown, E.T. "The Hoek-Brown criterion – a 1988 update". In: Proc. 15th Can. Rock Mech. Symp. University of Toronto, pp. 31-38. 1988.
- [35] Zhang, L. "Estimating the strength of jointed rock masses". Rock Mechanics and Rock Engineering. 43 (4), pp. 391-402. 2010. DOI: 10.1007/s00603-009-0065-x
- [36] Bhasin, R., Grimstad, E. "The use of stress-strength relationships in the assessment of tunnel stability". *Tunnelling and Undergound Space Technology*. 11 (1), pp. 93-98.1996. DOI: 10.1016/0886-7798(95)00047-X
- [37] Singh, B., Goel, R. K. "Rock mass classification a practical approach in civil engineering". 267 p. Elsevier Science, Amsterdam. 1999.
- [38] Ramamurthy, T. "Strength and modulus responses of anisotropic rocks". In: *Comprehensive Rock Engineering. Principle, Practice & Projects.* Hudson, J. A. (Ed), Pergamon, Oxford, UK. 1, pp. 313-329. 1993.
- [39] Singh, B., Goel, R. K., Mehrotra, V. K., Garg, S. K., Allu, M. R. "Effect of intermediate principal stress on strength of anisotropic rock mass". *Tunnelling and Undergound Space Technology.* 13 (1), pp. 71–79. 1998. DOI: 10.1016/S0886-7798(98)00023-6
- [40] Singh, M., Rao, K. S. "Empirical methods to estimate the strength of jointed rock masses". *Engineering Geology*. 77 (1–2), pp. 127–137. 2005. DOI: 10.1016/j.enggeo.2004.09.001
- [41] ISRM. "Suggested method for quantitative description of discontinuities in rock masses". *International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts.* 15 (6), pp. 319-368. 1978. DOI: 10.1016/0148-9062(78)91472-9

- [42] Deere, D. U., Miller, R. P. "Engineering Classification and Index Properties for Intact Rock". 327 p. Technical Report, University of Illinois, Urbana, Illinois. 1966. http://www.dot.ca.gov/hq/esc/geotech/references/Rock_ Cut_Slope_References/21%20_Deere_Engineering_Classification_of_ Index_Properties_of_Intact_Rx.pdf
- [43] Bieniawski, Z. T. "Misconceptions in the application of rock mass classifications and their corrections". In: ADIF Seminar on Advanced Geotechnical Characterization for Tunnel Design, Madrid, Spain, June 29, 2011. http://www.geocontrol.es/publicaciones/EB-189_adif_errores_en_ la_aplicacion_bieniawski_eng.pdf
- [44] Aydan, Ö., Ulusay, R., Tokashiki, N. "A New Rock Mass Quality Rating System: Rock Mass Quality Rating (RMQR) and Its Application to the Estimation of Geomechanical Characteristics of Rock Masses". *Rock Mechanics and Rock Engineering.* 47 (4), pp. 1255–1276. 2014. DOI: 10.1007/s00603-013-0462-z
- [45] Aydan, Ö., Akagi, T., Kawamoto, T. "The squeezing potential of rocks around tunnels; theory and prediction". *Rock Mechanics and Rock Engineering*. 26 (2), pp. 137–163. 1993. DOI: 10.1007/BF01023620
- [46] Tokashiki, N., Aydan, Ö. "The stability assessment of overhanging Ryukyu limestone cliffs with an emphasis on the evaluation of tensile strength of rock mass". *Doboku Gakkai Ronbunshuu C.* 66 (2), pp. 397–406. 2010. https://www.jstage.jst.go.jp/article/jscejc/66/2/66_2_397/_article
- [47] Aydan, Ö, Tokashiki, N., Genis, M. "Some considerations on yield (failure) criteria in rock mechanics". In: Proceedings of the 46th US rock mechanics/geomechanics symposium, Chicago, Illinois, June 24-27, 2012. ARMA 12-640 (on CD) 2012. https://www.onepetro.org/conferencepaper/ARMA-2012-640
- [48] Vásárhelyi B. "A possible method for estimating the Poisson's rate values of rock masses". Acta Geodaetica et Geophysica Hungarica. 44 (3), pp. 313-322. 2009. DOI: 10.1556/AGeod.44.2009.3.4
- [49] Sen, Z., Sadagah, B. H. "Modified rock mass classification system by continuous rating". *Engineering Geology*. 67 (3-4), pp. 269-280. 2003. DOI: 10.1016/S0013-7952(02)00185-0
- [50] Aydan, Ö., Kawamoto, T. "The stability assessment of a large underground opening at great depth". In: Proceedings of the 17th International Mining Congress., Turkey, Ankara, 1. pp. 277–288. 2001. http://www.maden.org. tr/resimler/ekler/eb2f1a06667bf99_ek.pdf

- [51] Ván, P., Vásárhelyi, B. "Sensitivity analysis of the Hoek-Diederichs rock mass deformation modulus estimating formula". In: The Second Half Century of Rock Mechanics: Proc. 11th ISRM Cong. Lisbon. Soussa, L. R., Grossmann, N. F., Ollala C. (eds), pp. 411-414. Taylor and Francis, 2007. http://real.mtak.hu/8273/1/1209074.pdf
- [52] Ván, P., Vásárhelyi, B. "Sensitivity analysis of GSI based mechanical parameters of the rock mass". *Periodica Polytechnica-Civil Engineering*. 58 (4), pp. 379-386. 2014. DOI: 10.3311/PPci.7507
- [53] Vásárhelyi, B., Kovács, L., Török, Á. "Analysing the modified Hoek– Brown failure criteria using Hungarian granitic rocks". *Geomechanics and Geophysics for Geo-Energy and Geo-Resources*. 2 (2), pp. 131-136. 2016. DOI: 10.1007/s40948-016-0021-7
- [54] Deák, F., Kovács, L., Vásárhelyi, B. "Comparison of different rock mass classification at Bátaapáti radioactive waste repository". In: BeFo - Rock Engineering & Technology for Sustainable Underground Construction: Eurock 2012, May 28-30, 2012, Stockholm, Sweden, pp. 247-259. https:// www.onepetro.org/conference-paper/ISRM-EUROCK-2012-027
- [55] Deák, F., Kovács, L., Vásárhelyi, B., "Geotechnical rock mass documentation in the Bátaapáti radioactive waste repository". *Central European Geology.* 57 (2), pp. 197-211. 2014. DOI: 10.1556/ CEuGeol.57.2014.2.5
- [56] Vásárhelyi, B., Somodi, G., Krupa, Á., Kovács, L.. "Determining the Geological Strength Index (GSI) using different methods". In: *Rock Mechanics and Rock Engineering: From the Past to the Future: Eurock* 2016. Ulusay, R., Aydan, O., Gerçek, H., Hindistan, M. A., Tuncay, E. (eds.), Taylor and Francis, Cappadocia. pp. 1049-1054. 2016.
- [57] Kamera, R., Vásárhelyi, B., Kovács, L., M. Tóth, T. "Relationship Between the Fractal Dimension and the Rock Mass Classification Parameters in the Bátaapáti Radioactive Waste Repository". *Engineering Geology for Society and Territory*. Vol. 6, Applied Geology for Major Engineering Projects. Lollino, G., et al (Eds), pp. 897-900. Springer International Publishing, Torino, 2015. DOI: 10.1007/978-3-319-09060-3_162