

Time-dependent Reliability Analysis of Timber-Concrete Composite Beams

Nikola Velimirović^{1,2*}, Dragoslav Stojić¹, Miodrag Djordjević²,
Gordana Topličić-Ćurčić¹

RESEARCH ARTICLE

Received 16 November 2016; Revised 06 January 2017; Accepted 13 February 2017

Abstract

A timber-concrete composite system is increasing the use of timber in construction. Some structures cannot be built by timber alone, but this type of composite system makes it a possibility. The deterioration of structures in service is generally uncertain over time. The appropriate approach therefore for a structural analysis is a time-dependent reliability analysis which considers the randomness and uncertainties of the deterioration process over a specific time period. This paper describes two methods for time-dependent reliability analysis, a random variable deterioration rate model and a gamma process deterioration model. It also, compares the deterioration prediction as well as service life estimation of timber-concrete composite beams under normal use, based on proposed models.

Keywords

probabilistic deterioration models, timber-concrete composite, reliability analysis, deterioration prediction, service life estimation

1 Introduction

All structures are required to be safe for use during their projected service life. They should withstand particular environmental influences along with a service load which has an effect on their deterioration. Application of reliability analysis is becoming increasingly important in civil engineering. Based on time dependency, reliability analysis could be generally classified into two categories, time-invariant and time-dependent reliability analysis [1]. Time-invariant reliability analysis considers the state of the structure's condition at a certain moment and does not change over time. Structural reliability, however, is a function of time for the majority of problems within engineering. Considering that deterioration of structures in service is generally uncertain over time, it should be represented ideally using a probabilistic deterioration model. Therefore, a more appropriate approach for a structural analysis is a time-dependent reliability analysis which considers the randomness and uncertainties of deterioration process over a time period.

The limit state function can be generally defined as follows:

$$g(X(t)) = 0 \quad (1)$$

where $X(t)$ is a probabilistic deterioration model.

The time-dependent reliability is given by:

$$R(T) = P[g(X(t)) > 0], \forall t \in [0, T] \quad (2)$$

The time-dependent reliability tells us the likelihood that no failure will occur over a certain time period.

The corresponding probability of failure in the considered time interval is then:

$$p_f(T) = 1 - P[g(X(t)) > 0], \forall t \in [0, T] \quad (3)$$

The timber-concrete composite (TCC) structure is a structural system in which a timber beam is connected to an upper concrete flange using different types of connectors. This structural system has successfully been used in bridges all around the world, in strengthening existing timber floors as well as in constructing new floors in residential and office buildings [2].

¹University of Niš, Faculty of Civil Engineering and Architecture, Aleksandra Medvedeva 14, 18000 Niš, Serbia

²University of Niš, Faculty of Science and Mathematics, Višegradska 33, 18000 Niš, Serbia

*Corresponding author, email: velimirovic.nikola@gmail.com

Generally, this composite system increases the use of timber in construction because some structures cannot be built by timber alone. This becomes a possibility with a TCC system. Yeoh et al. [3] presented a survey on the current state of art in TCC research during recent years. Because of its lower weight compared to the reinforced concrete section, TCC has reduced seismic effects. The stiffness of the timber section is significantly increased by connecting to concrete flange. Many properties of a timber floor are improved in that way: for example, load carrying capacity, thermal and sound insulation and fire resistance. The long-term behaviour of the TCC composite system is a very complex problem and it depends on the creep, mechano-sorptive creep, swelling, shrinkage and hydrothermal changes within the constituent elements [4]. The fluctuation in deflection of TCC beams over time under the constant load is caused by variations of environmental temperature and relative humidity. A reduction in temperature causes an increase in deflection because of the different thermal expansion coefficients and conductivities of timber and concrete. Additionally, a reduction in timber moisture content produces a decrease in deflection. Explanation of this type of behaviour is based on the shrinkage/swelling produced by temperature and moisture content variations. The differential shrinkage/swelling of the concrete flange and timber beam cannot freely occur due to the connection system which restrains the possibility of either part to move relative to the other. Consequently, additional deformations will be induced in the composite beam [5].

2 Probabilistic Deterioration Models

Time-dependent processes have been modelled traditionally without consideration of the variations in the process over time using deterministic models [6]. In order to find the relation between the average deterioration path and time, a regression model is appropriate to use. In this model, the increase in deterioration is modelled using a deterministic regression function which could be chosen based on past experiences or expert judgment. In most of engineering problems, expected average deterioration in time t can be represented using a power function:

$$E[X(t)] = at^b \propto t^b \quad (4)$$

where a is the rate parameter and b is the time order parameter. Estimation of the regression model parameters is usually performed using the least-squares fitting method.

In past decades, there has been significant research done in the field of time-dependent performance prediction of deteriorating structures. Most of them had the aim to improve the accuracy of models which can predict structural performance under time-dependent deterioration. The performance prediction process can be seen as the main part of the life-cycle management of the structures and infrastructures. Frangopol and Soliman [7] provided a brief overview of the recent achievements in the field of the life-cycle management of the deteriorating

infrastructure systems. They have discussed various probabilistic performance prediction and evaluation approaches. In general, probabilistic deterioration models appear to be more flexible in modelling the deterioration process of structures in comparison to deterministic deterioration models. Unlike the widely used deterministic deterioration models, such as the regression model that has limited validity, because they cannot capture temporal effects in deterioration process, probabilistic deterioration models are trying to imitate uncertainty of the deterioration in real conditions [8]. For the proper life-cycle maintenance management of structures, it is necessary to include uncertainties associated with the performance prediction, damage initiation and propagation and the effect of maintenance actions on structural [7]. Deterioration is usually assumed to be a Markov process, stochastic process with independent increments. Types of Markov processes which are used for modelling stochastic deterioration are discrete time Markov process (Markov chains) and continuous time Markov process [9]. The Markov chain model is a widely applied technique in the performance assessment for deteriorating structures [10]. In this model, the structural deterioration process is regarded as structural condition state changes over discrete time intervals. It is represented by the transition probabilities from one condition state to another. Regrettably, in reality the transition probabilities are usually unknown. They should be determined from a reverse calculation based on accumulated inspection data. In the traditional Markov chain model all the transition probabilities are considered to be deterministic. Considering that the structural deterioration process is uncertain over the life-cycle of the structure, Zhang et al. [11] proposed refined Markov chain deterioration model where all the transition probabilities are modelled as random variables. This novel approach brings the randomness of the transition probabilities into consideration, instead of utilizing only the mean as in the previous approaches [12]. As we mention earlier, modelling of deterioration processes could also be done using continuous time Markov processes, such as the Brownian motion with drift and the gamma process. The gamma process has only independent increments which makes it more adequate for modelling deterioration that is monotone process, compared to Brownian motion with drift which has the independent increments and decrements.

This paper considers two different probabilistic deterioration models, random variable deterioration rate model and gamma process deterioration model, and compares the deterioration prediction as well as service life estimation based on the proposed models.

2.1 Random variable deterioration rate model

Random variable (RV) deterioration rate model presents a parametric process that could be generally presented as a deterministic function of time and random parameter [13]:

$$X(t) = f(R, t) = R \cdot t^b \quad (5)$$

The main idea of this approach is to model different rates of deterioration within the population as random variable R with the certain probability density function.

$$R = \frac{X(t)}{t^b} \quad (6)$$

This model does not consider temporal uncertainties, but only a sample uncertainty of the deterioration process (Figure 1). Deterioration of each sample within the population is a deterministic function with a known form and has a specific deterioration rate which is constant over time.

$$r_j = \frac{x_j(t)}{t^b} \quad (7)$$

where j is number of samples in population, $x_j(t)$ is the realization of the stochastic process and r_j is specific sample of deterioration rate.

The gamma distribution is chosen for modelling deterioration rate and its probability density function is given by:

$$f_R(r) = Ga(r|\eta, \delta) = \frac{r^{\eta-1}}{\Gamma(\eta) \cdot \delta^\eta} \cdot e^{-\frac{r}{\delta}} \quad (8)$$

with shape parameter $\eta > 0$ and scale parameter $\delta > 0$, where

$$\Gamma(a) = \int_{z=0}^{\infty} z^{a-1} e^{-z} dz \quad (9)$$

is the gamma function for $a > 0$.

We can have $X(t)$ denote the deterioration at time t , $t \geq 0$, and have the probability density function of $X(t)$, in conformity with the definition of the random variable deterioration rate model, be given by:

$$f_{X(t)}(x) = Ga\left(x|\eta, \delta t^b\right) = \frac{x^{\eta-1}}{\Gamma(\eta) \cdot (\delta t^b)^\eta} \cdot e^{-\frac{x}{\delta t^b}} \quad (10)$$

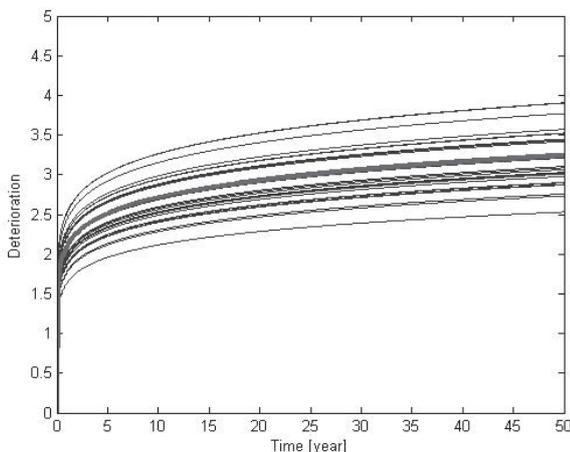


Fig. 1 Simulated sample paths of RV deterioration rate model and average deterioration path

Mean, variance and coefficient of variance of $X(t)$ are given respectively:

$$\begin{aligned} E[X(t)] &= E[R] \cdot t^b = \eta \cdot \delta \cdot t^b \\ Var[X(t)] &= Var[R] \cdot t^{2b} = \eta \cdot \delta^2 \cdot t^{2b} \\ CoV[X(t)] &= \frac{\sqrt{Var[X(t)]}}{E[X(t)]} = \frac{1}{\sqrt{\eta}} = const. \end{aligned} \quad (11)$$

The presented probabilistic deterioration model is more appropriate for modelling the processes with relatively constant conditions, such as corrosion and wear [13,14]. Therefore, variations in the deterioration rate over population appear as a result of individual differences among samples.

2.2 Gamma Process Deterioration Model

Gamma process (GP) deterioration model was first proposed by Abdel-Hameed as an appropriate approach for modelling the deterioration randomly occurring in time [9]. Gamma process is a stochastic process with independent non-negative increments that are gamma distributed with an identical scale parameter (Figure 2). We observe the gamma process with shape function $k(t) > 0$ and scale parameter $\theta > 0$, as continuous-time stochastic process $\{X(t); t \geq 0\}$ with the following properties:

- $X(0) = 0$ with probability one;
- $\Delta X(t) = X(t + \Delta t) - X(t) \sim Ga(\Delta k(t), \theta)$; $\Delta k(t) = k(t + \Delta t) - k(t)$
- $\Delta X(t)$ are independent.

where $k(t)$ is supposed to be a non-decreasing, right-continuous, real-valued function for $t \geq 0$, with $k(0) \equiv 0$.

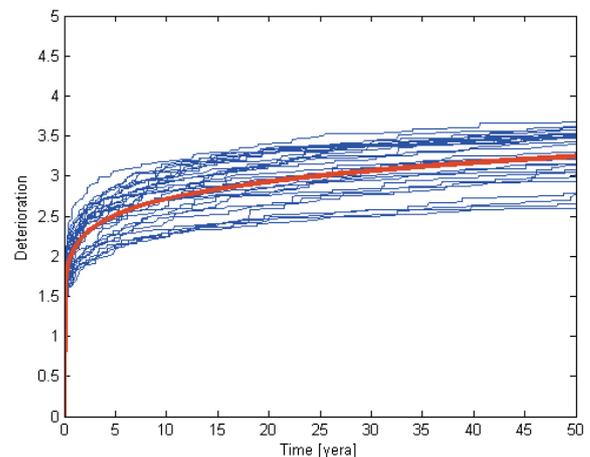


Fig. 2 Simulated sample paths of Gamma process deterioration model and average deterioration path

Probability density function of $X(t)$, according to the definition of the gamma process deterioration model could be given as:

$$f_{X(t)}(x) = Ga\left(x \mid ct^b, \theta\right) = \frac{x^{ct^b-1}}{\Gamma(ct^b) \cdot \theta^{ct^b}} \cdot e^{-\frac{x}{\theta}} \quad (12)$$

with mean, variance and coefficient of variation

$$\begin{aligned} E[X(t)] &= ct^b \cdot \theta \\ Var[X(t)] &= ct^b \cdot \theta^2 \\ CoV[X(t)] &= \frac{1}{\sqrt{ct^b}} \end{aligned} \quad (13)$$

The gamma process deterioration model presents the proper model for gradual damage accumulating monotonically over time, such as wear, fatigue, creep, crack growth, erosion, corrosion and swell [9]. Beside this, the gamma process deterioration model has been considered as an appropriate approach for building element deterioration prediction [15] and for bridge deterioration prediction [16].

3 Estimation of deterioration model parameters

Being that the deterioration models present a basis for predicting the end of service life of structural elements and planning for maintenance actions, an accurate estimation of the parameters of the deterioration process models from available inspection data is of critical importance. We assume that a typical inspection data set consists of time of conducted inspections $t_i, i = 1, \dots, n$, where $0 = t_0 < t_1 < \dots < t_n$, and corresponding observations of the cumulative amounts of deterioration for each inspected sample $x_{ij}, 0 = x_{0j} \leq x_{1j} \leq \dots \leq x_{nj}$, where $j = 1, \dots, m$ and m is number of samples in population. The method of maximum likelihood, well-known estimation method in statistics, has been applied in this paper.

3.1 Maximum likelihood estimation of RV deterioration rate model parameters

Estimation of RV deterioration rate model parameters, η and δ , can be obtained based on the available inspection data set. Maximum likelihood estimation can be done by maximizing the logarithm of the likelihood function for the considered population of deterioration rates $r_j, j = 1, \dots, m$. Likelihood function is given as follows:

$$\begin{aligned} L(\eta, \delta) &= \prod_{j=1}^m f_R(r_j) = \prod_{j=1}^m Ga(r_j \mid \eta, \delta) = \\ &= \prod_{j=1}^m \frac{r_j^{\eta-1}}{\Gamma(\eta) \cdot \delta^\eta} \cdot e^{-\frac{r_j}{\delta}} \end{aligned} \quad (14)$$

Based on the given likelihood function, log-likelihood function is:

$$\begin{aligned} l(\eta, \delta) &= \log L(\eta, \delta) \\ &= (\eta - 1) \sum_{j=1}^m \ln r_j - \frac{1}{\delta} \sum_{j=1}^m r_j - m\eta \cdot \ln \delta - m \ln \Gamma(\eta) \end{aligned} \quad (15)$$

According to the first partial derivatives of log-likelihood function with respect to η and δ , the maximum-likelihood estimates η and δ could be obtained based on following system of equations:

$$\begin{aligned} \frac{\partial l(\eta, \delta)}{\partial \eta} &= \sum_{j=1}^m \ln r_j - m \cdot \ln \delta - m \cdot \psi(\eta) = 0 \\ \frac{\partial l(\eta, \delta)}{\partial \delta} &= \frac{1}{\delta^2} \sum_{j=1}^m r_j - \frac{m \cdot \eta}{\delta} = 0 \end{aligned} \quad (16)$$

3.2 Maximum likelihood estimation of GP deterioration model parameters

In a similar way, already described above, we are able to estimate GP deterioration model parameters c and θ . Statistical data required for this approach is deterioration increments of each observed sample over time interval between two inspections, $\Delta x_{ji} = x_{ji} - x_{j(i-1)}, j = 1, \dots, m; i = 1, \dots, n$. Likelihood function is represented as a product of independent probability density functions:

$$\begin{aligned} L(c, \theta) &= \prod_{j=1}^m \prod_{i=1}^n f_{X(t_i)-X(t_{i-1})}(\Delta x_{ji}) \\ &= \prod_{j=1}^m \prod_{i=1}^n Ga(\Delta x_{ji} \mid c[t_i^b - t_{i-1}^b], \theta) \\ &= \prod_{j=1}^m \prod_{i=1}^n \frac{\Delta x_{ji}^{c[t_i^b - t_{i-1}^b]-1}}{\Gamma(c[t_i^b - t_{i-1}^b]) \cdot \theta^{c[t_i^b - t_{i-1}^b]}} \exp\left(\frac{-\Delta x_{ji}}{\theta}\right) \end{aligned} \quad (17)$$

Considering the logarithm of the likelihood function, we get corresponding log-likelihood function:

$$\begin{aligned} l(c, \theta) &= \log L(c, \theta) \\ &= \sum_{j=1}^m \sum_{i=1}^n (c[t_i^b - t_{i-1}^b] - 1) \log \Delta x_{ji} \\ &\quad - c \sum_{j=1}^m \sum_{i=1}^n [t_i^b - t_{i-1}^b] \log \theta \\ &\quad - \sum_{j=1}^m \sum_{i=1}^n \log \Gamma(c[t_i^b - t_{i-1}^b]) \\ &\quad - \frac{1}{\theta} \sum_{j=1}^m \sum_{i=1}^n \Delta x_{ji} \end{aligned} \quad (18)$$

First partial derivatives of obtained log-likelihood function with respect to c and θ are presented as follows:

$$\begin{aligned} \frac{\partial l(c, \theta)}{\partial c} &= \sum_{j=1}^m \sum_{i=1}^n [t_i^b - t_{i-1}^b] \left\{ \log \Delta x_{ji} - \psi(c[t_i^b - t_{i-1}^b]) - \ln \theta \right\} = 0 \\ \frac{\partial l(c, \theta)}{\partial \theta} &= \frac{1}{\theta^2} \sum_{j=1}^m \sum_{i=1}^n \Delta x_{ji} - \frac{c}{\theta} \sum_{j=1}^m \sum_{i=1}^n [t_i^b - t_{i-1}^b] = 0 \end{aligned} \quad (19)$$

where the function $\psi(\cdot)$ is digamma function. The maximum-likelihood estimates c and θ could be obtained by numerically solving system of equations.

4 Time-dependent reliability of TCC beams in the serviceability limit state

The main cause for the deterioration and the reduction in safety and reliability of the existing structures and infrastructures are environmental conditions. In order to estimate the probability of failure, it should be recognized that both applied loads and the resistance of a structure may fluctuate in time during the lifetime of the structure. When considering long-term deflection of timber-concrete composite beams, as the most serious criterion in the serviceability limit state, time-dependent reliability analysis is appropriate because of diverse effects which develop in a different manner over time in the life cycle.

The aim of this paper is to demonstrate the application of the two different reliability-based deterioration prediction models for assessing the service life of TCC beams using available inspection data. We observe a simply supported TCC beam (Figure 3) of span 4.5m. The shear connectors are glued-in steel rods $\phi 20/150$ mm, which are embedded in the pre-drilled holes perpendicular to the grain and coated with epoxy resin, at constant intervals of 240 mm. The concrete flange is made of concrete strength class C25/30, the timber beam of sawn softwood timber structural strength class C27 and the steel rods are made of the steel grade S235.

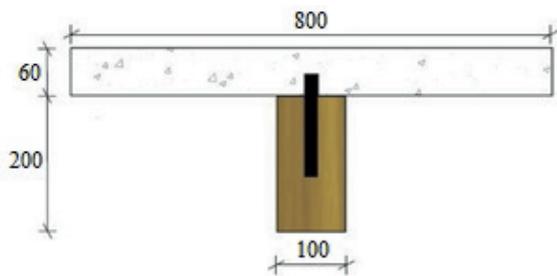


Fig. 3 Cross section of the timber-concrete composite beam

It is supposed that these beams are in indoor conditions, where temperature (T) is 22 ± 4 °C and the relative humidity (RH) is $50 \pm 5\%$. The relative mid-span deflection of the beam over time under normal use conditions is defined as follows:

$$X(t) = \frac{u(t) - u_{el}}{u_{el}} \quad (20)$$

where $u(t)$ is mid-span deflection at time t and u_{el} is the elastic deflection measured immediately after applying the service load that is assumed to be an initial deflection, deflection at time t_0 . Based on the relative mid-span deflection definition, we could establish maximum permissible value of $X(t)$ as ρ :

$$\rho = \frac{u_L - u_{el}}{u_{el}} \quad (21)$$

In agreement with the Eurocode 5 [17], suggested serviceability limit value for long-term deflection u_L of simply supported beam is $l/200$, where l is the span length. It is assumed that observed TCC beam will reach serviceability limit state when the maximum mid-span deflection exceeds the proposed serviceability limit value of relative mid-span deflection. In our

case, the serviceability limit value of relative mid-span deflection is 4.9518 and it is presumed to be constant over time.

For a comprehensive time-dependent reliability analysis of TCC beams under normal use conditions, we need to define the limit state function. In this paper we consider serviceability limit state of maximum deflection that could be expressed as:

$$g(\rho, X, t) = \rho - X(t) \quad (22)$$

According to this definition, probability of failure for considered TCC beams is given as follows:

$$P_f(t) = P[g(\rho, X, t) \leq 0] = P[X(t) \geq \rho] \quad (23)$$

In the absence of deterioration measurement data under normal use conditions and because time-consuming experiments under normal operating conditions are costly, accelerated aging test was applied. Accelerated deterioration data has been generated using an available deterministic model. This model is based on conducted experimental tests, presented by Fragiacomo [18]. This approach serves to simulate the condition of the set of 30 identical beams exposed to indoor conditions. The observed beams are monitored through periodic inspections at year 10 and year 20. In that way inspections reveal the progress of the deterioration of each inspected beam. Deterioration increase in each time interval is not constant within the considered population, rather it is random due to the uncertain influence of the environment over the life-cycle. The uncertainties of the deterioration process in a large population of structural elements are simulated by Monte Carlo simulation technique. Variation in environmental conditions will cause dispersion of mid-span deflection within the population over time (Figure 4), and therefore variation in lifetime among the samples.

To obtain the relation between the average deterioration level and time, available inspection data was fitted by the power function. This is performed based on the regression analysis using a least-squares fitting method, also suggested by Nicolai *et al.* [19]. As the regression of available data fits a power function very well with high coefficient of determination ($R^2 = 0.9175$), the expected deterioration of TCC beams under normal use conditions can be modelled as follows:

$$E[X(t)] = 3.238 \cdot t^{0.1104} \quad (24)$$

where $E[X(t)]$ presents the expected deterioration of relative mid-span deflection under indoor conditions at time t . Having parameter estimate of the power b , we are able to estimate the other two parameters of the proposed RV deterioration rate model and GP deterioration model, the shape and scale parameters.

Maximum likelihood estimation of shape and scale parameters of RV deterioration rate model is obtained according to procedure previously described in chapter 3.1. As we have available data from two conducted inspections at year 10 and year 20, random variable deterioration rate is given as follows:

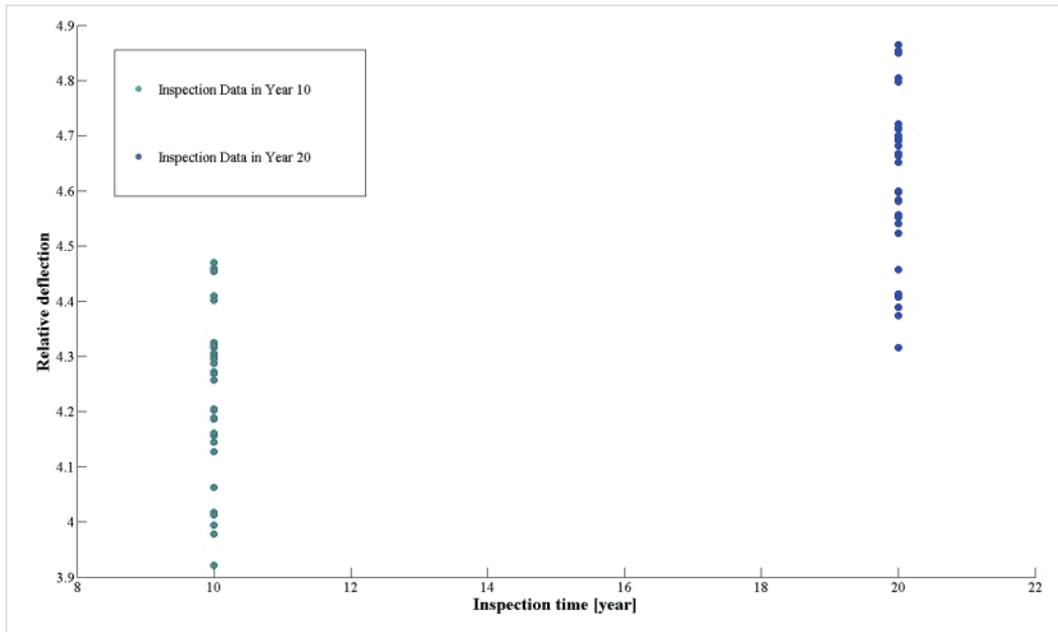


Fig. 4 Inspection data for observed beams

$$R = \frac{X(t_2) - X(t_1)}{t_2^b - t_1^b} \quad (25)$$

Respectively, the deterioration rate of specific sample is:

$$r_j = \frac{X_j(t_2) - X_j(t_1)}{t_2^b - t_1^b} \quad (26)$$

Parameter estimation of the suggested gamma process model using method of maximum likelihood is conducted in the manner described in chapter 3.2.

Table 1 Estimated parameters of time-dependent reliability analysis methods at year 20

RV deterioration rate model		Gamma process model	
Shape (η)	Scale (δ)	Shape (c)	Scale (θ)
834.069	0.0039	699.912	0.0047

4.1 Deterioration prediction

The continuous challenge in optimizing the maintenance of structures is manifested in the accurate assessment and modeling of the structural lifecycle performance over time. In the available literature we can find various deterministic deterioration models for predicting long-term behavior of TCC beams presented by different authors [18], [20], [21]. These deterministic prediction models have inherent limitations in real conditions, because they do not include any variations and uncertainty in model variables. Hence they provide point estimation of the future condition of the structure. A more appropriate approach for deterioration prediction of existing structures is based on the analysis of the physical deterioration in accordance with engineering knowledge and experience. It is also based on the statistical data obtained from the condition inspections which reveal the current state of the deterioration.

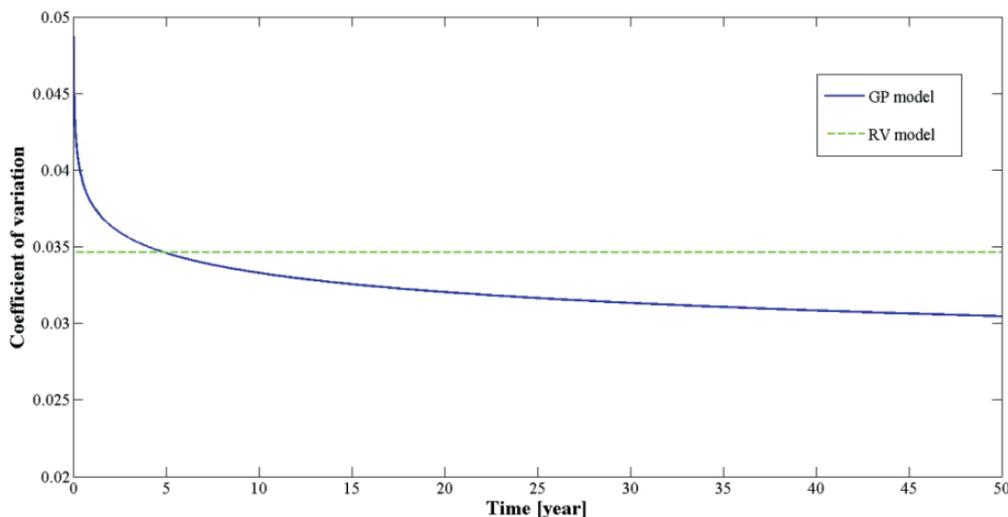


Fig. 5 Coefficient of variation of expected deterioration $X(t)$

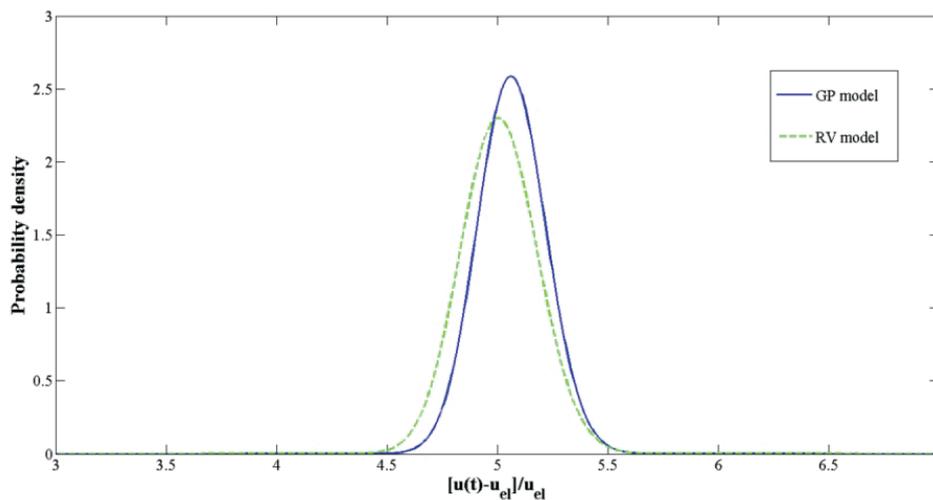


Fig. 6 Comparison of expected deterioration $X(t)$ at year 50, using prediction based on GP deterioration model and RV deterioration rate model

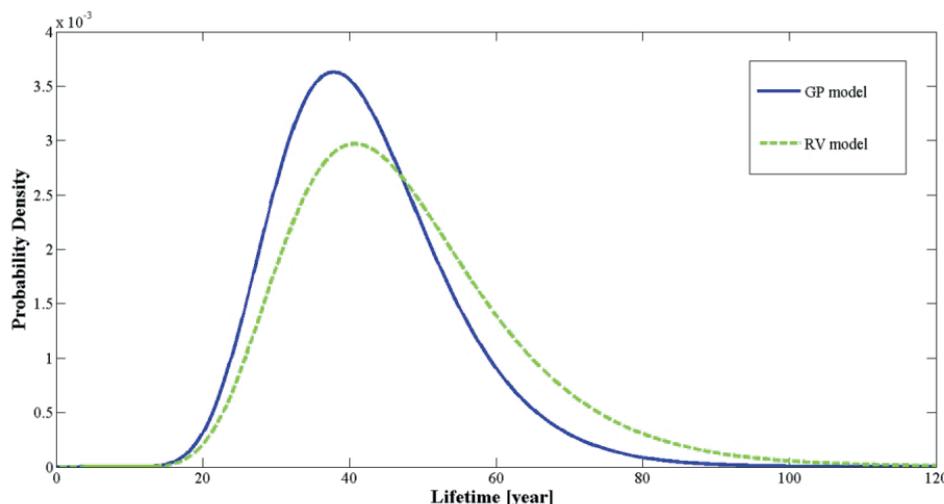


Fig. 7 Comparison of probability density functions of service life, obtained from GP deterioration model and RV deterioration rate model

Considering that the RV deterioration rate model depends on only one random variable whose mean and standard deviation are constant over time, the coefficient of variation of this probabilistic deterioration model is constant over time as well. Furthermore, coefficient of variation of GP deterioration model is variable over time and presents the function of time with the negative exponent, $t^{-(b/2)}$. According to this fact, coefficient of variation of GP deterioration model has higher value within the first couple of years. Its value however is rapidly decreasing and quickly becomes lower than the coefficient of variation of the RV deterioration rate model, which is an indicator of quality and stability for long term predictions (Figure 5).

Based on figure 6, we can conclude that gamma process model gives a more stable prediction of expected deterioration than the RV deterioration rate model.

4.2 Service life prediction

Service life analysis considers the maximum time at which the expected deterioration will exceed the service life threshold. The first time when the sample path of deterioration $X(t)$ exceeds the proposed serviceability limit value of relative mid-span deflection ρ could be assigned as the service life T .

$$F_T(t) = P[T \leq t | X(t) \geq \rho] \quad (27)$$

In case that the deterioration is modelled using a random variable deterioration rate model, the cumulative distribution function of service life could be presented as:

$$F_T(t) = 1 - GA(\rho; \eta, \delta t^b) = 1 - \frac{\Gamma\left(\eta, \frac{\rho}{\delta t^b}\right)}{\Gamma(\eta)} \quad (28)$$

where

$$\Gamma(a, x) = \int_{t=x}^{\infty} t^{a-1} e^{-t} dt \quad (29)$$

is the incomplete gamma function for $x \geq 0$ and $a > 0$.

Taking into account that expected deterioration is presented by gamma process deterioration model, the cumulative distribution function of service life can be given as:

$$F_T(t) = 1 - GA(\rho; ct^b, \theta) = 1 - \frac{\Gamma\left(ct^b, \frac{\rho}{\theta}\right)}{\Gamma(ct^b)} \quad (30)$$

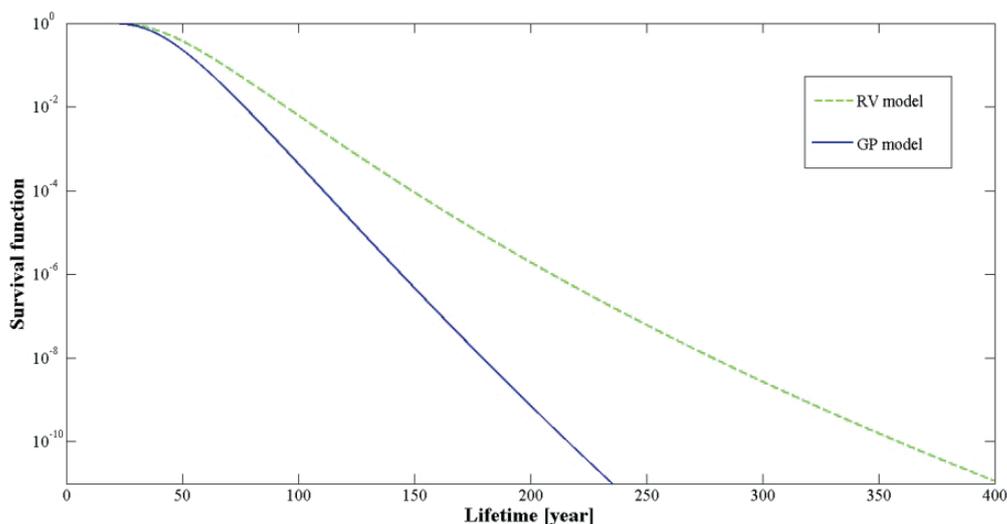


Fig. 8 Comparison of probability density functions of service life, obtained from GP deterioration model and RV deterioration rate model

The survival function offers the answer on question what is the probability that the considered structure will survive past a certain time (Figure 8). This function is defined as:

$$S(t) = 1 - F_T(t) \quad (31)$$

5 Conclusions

The continuous challenge in optimizing life-cycle maintenance of structures is manifested in the accurate assessment and modelling of the structural life-cycle performance over time. The widely used deterministic deterioration models have inherent limitations, because they cannot capture temporal effects in the deterioration process. Unlike them, probabilistic deterioration models are trying to imitate uncertainty of deterioration in real conditions. When considering long-term deflection of TCC beams, as the most serious criterion in the serviceability limit state, time-dependent reliability analysis is appropriate because of the diverse effects that develop in a different manner over the life-cycle.

This work has presented the application of two methods for time-dependent reliability analysis on TCC beams under normal use. Compared to the random variable deterioration rate model, the gamma process deterioration model is better for deterioration analysis. RV deterioration rate model does not consider temporal uncertainties, only sample uncertainty of the deterioration process. The main idea of this approach is to model different rates of deterioration for each sample within the population and therefore this model incorporates variability in deterioration process across the population. However, the gamma process deterioration model considers deterioration increase as the sum of the series of non-negative random increments. In this model, deterioration rate in each time interval is not constant, rather it is random due to the uncertain influence of the environment over the life-cycle. Considering that the RV deterioration rate model depends on only one random variable which mean and standard

deviation are constant over time, the coefficient of variation of this probabilistic deterioration model is constant over time as well. Furthermore, coefficient of variation of the GP model is variable over time and presents the function of time with the negative exponent, $t^{-(b/2)}$. According to this fact, coefficient of variation of GP deterioration model has higher value within the first couple of years. However, its value rapidly decreases and quickly becomes lower than the coefficient of variation of the RV deterioration rate model, which is an indicator of quality and stability for long term predictions.

In summary, a careful consideration of the temporal uncertainty associated with the evolution of deterioration is very significant for a time-dependent reliability analysis of timber-concrete composite structures.

References

- [1] Hu, Z., Li, H., Du, X., Chandrashekhara, K. "Simulation-based time-dependent reliability analysis for composite hydrokinetic turbine blades." *Structural and Multidisciplinary Optimization*, 47(5), pp. 765–781. 2013. <https://doi.org/10.1007/s00158-012-0839-8>
- [2] Dias, A., Skinner, J., Crews, K., Tannert T. "Timber-concrete-composites increasing the use of timber in construction." *European Journal of Wood and Wood Products*, 74(3), pp. 443–451. 2016. <https://doi.org/10.1007/s00107-015-0975-0>
- [3] Yeoh, D., Fragiaco, M., Francheschi, M. D., Boon, K. H. "The state-of-the-art on timber-concrete composite structures - a literature review." *Journal of Structural Engineering*, 137(10), pp. 1085–1095. 2011.
- [4] Stepinac, M., Rajčić, V., Brabalić, J. "Influence of long term load on timber-concrete composite systems." *Grđevinar*, 67(3), pp. 235–246. 2015. <https://doi.org/10.14256/JCE.1170.2014>
- [5] Fragiaco, M., Schänzlin J. "The effect of moisture and temperature variations on timber-concrete composite beams." In: *11th World Conference on Timber Engineering 2010, (WCTE 2010)*, Trentino, Italy, Jun. 20–24. 2010. pp. 113–120.
- [6] Buijs F. A., Hall J. W., Sayers P. B., Van Gelder P. H. "Time-dependent reliability analysis of flood defences." *Reliability Engineering & System Safety*, 94(12), pp. 1942–1953. 2009. <https://doi.org/10.1016/j.res.2009.06.012>

- [7] Frangopol, D. M., Soliman, M. "Life-cycle of structural systems: recent achievements and future directions." *Structure and Infrastructure Engineering*, 12(1), pp. 1–20. 2016. <https://doi.org/10.1080/15732479.2014.999794>
- [8] Pandey, M. D., Yuan, X. X., van Noortwijk, J. M. "The Influence of Temporal Uncertainty of Deterioration on Life-Cycle Management of Structures." *Structure and Infrastructure Engineering*, 5(2), pp. 145–156. 2009. <https://doi.org/10.1080/15732470601012154>
- [9] van Noortwijk, J. M. "A survey of the application of gamma processes in maintenance." *Journal of Reliability Engineering and System Safety*, 94(1), pp. 2–21. 2009. <https://doi.org/10.1016/j.ress.2007.03.019>
- [10] Straub, D., Faber, M. H. "Risk based inspection planning for structural systems." *Structural Safety*, 27(4), pp. 335–355. 2005. <https://doi.org/10.1016/j.strusafe.2005.04.001>
- [11] Zhang, Y., Kim, C.W., Tee, K.F. "Maintenance management of offshore structures using Markov process model with random transition probabilities." *Structure and Infrastructure Engineering*, pp. 1–13. 2016. <https://doi.org/10.1080/15732479.2016.1236393>
- [12] Zhang, Y., Kim, C. W., Tee, K. F., Lam, J. S. L. "Optimal sustainable life cycle maintenance strategies for port infrastructures." *Journal of Cleaner Production*, 142(Part 4), pp. 1693–1709. 2017. <https://doi.org/10.1016/j.jclepro.2016.11.120>
- [13] Pandey, M. D., Lu, D. "Estimation of parameters of degradation growth rate distribution from noisy measurement data." *Structural Safety*, 43, pp. 60–69. 2013. <https://doi.org/10.1016/j.strusafe.2013.02.002>
- [14] Huysse, L., van Roodselaar, A. "Effects of inline inspection sizing uncertainties on the accuracy of the largest features and corrosion rate statistics." In *2010 8th International Pipeline Conference (IPC2010)*, Calgary, Alberta, Canada, Sep. 27–Oct. 1. 2010. 4, pp. 403–413. <https://doi.org/10.1115/IPC2010-31037>
- [15] Edirisinghe, R., Setunge, S., Zhang, G. "Application of Gamma Process for Building Deterioration Prediction." *Journal of Performance of Constructed Facilities*, 27(6). 2013. [https://doi.org/10.1061/\(ASCE\)CF.1943-5509.0000358](https://doi.org/10.1061/(ASCE)CF.1943-5509.0000358)
- [16] Aboura, K., Samali, B., Crews, K., Li, J. "Stochastic Processes for Modeling Bridge Deterioration." In: *Futures in Mechanics of Structures and Materials*, (T. Aravinthan, W. Karunasena, H. Wang (Eds.)), pp. 533–538. CRC Press / Balkema, Taylor and Francis Group, London, UK. 2009.
- [17] CEN. "Eurocode 5 - design of timber structures - part 1-1: General – Common rules and rules for buildings. EN 1995-1-1". *European Committee for Standardization*, Brussels, Belgium. 2004. <https://law.resource.org/pub/eu/eurocode/en.1995.1.1.2004.pdf>
- [18] Fragiocomo, M. "Long-term behaviour of timber-concrete composite beams. II: Numerical analysis and simplified evaluation." *Journal of Structural Engineering*, 132(1), pp. 23–33. 2006. [https://doi.org/10.1061/\(ASCE\)0733-9445\(2006\)132:1\(23\)](https://doi.org/10.1061/(ASCE)0733-9445(2006)132:1(23))
- [19] Nicolai, R. P., Dekker, R., van Noortwijk, J. M. "A comparison of models for measurable deterioration: An application to coatings on steel structures." *Reliability Engineering and System Safety*, 92(12), pp. 1635–1650. 2007. <https://doi.org/10.1016/j.ress.2006.09.021>
- [20] Jorge, L. F., Schänzlin, J., Lopes, S. M. R., Cruz, H., Kuhlmann, U. "Time-dependent behaviour of timber lightweight concrete composite floors." *Engineering Structures*, 32(12), pp. 3966–73. 2010. <https://doi.org/10.1016/j.engstruct.2010.09.007>
- [21] Kanócz, J., Bajzecerová, V., Šteller, Š. "Timber-concrete composite elements with various composite connections. Part 1: Screwed connection." *Wood Research*, 58(4), pp. 555–570. 2013.