

The Effect of Particle Damper's Position on the Dynamic Response of Classical Columns

Angeliki Papalou^{1*}

RESEARCH ARTICLE

Received 17 November 2016; Revised 12 March 2017; Accepted 04 May 2017

Abstract

Preservation of the original architectural features of historic structures during their restoration and strengthening is challenging. A simple way to preserve the appearance of ancient classical columns and simultaneously protect them from earthquakes is to replace damaged or missing parts by a system that absorbs the incoming seismic energy. This paper examines the effectiveness of particle dampers in reducing the dynamic response of multi-drum and monolithic columns with capital. The properties of the damper that influence the column's dynamic response were also examined with emphasis on the position of the damper. The measurements suggest that the replacement of the capital by a particle damper has small effect on the dynamic response of multi-drum and monolithic columns. The damper can enhance the column's seismic safety if it replaces one of its top drums.

Keywords

monuments, classical columns, ancient columns, particle dampers

1 Introduction

The preservation of historical monuments requires extensive experience and knowledge of the structural techniques and the materials used to build them. During the last few decades there is a particular interest in preserving historical heritage [1]. Ancient temples around the Mediterranean Sea are representatives of classical architecture, built with precision and care. A good understanding of their behavior, especially in high seismicity areas, is needed in order to preserve and strengthen them enough to resist future earthquakes. The ancient temples' massive multi-drum columns, used to support their roof, are their most recognizable element. The proportions and profiles of ancient columns were characteristics to their architectural order. Small columns were often made from a single piece of stone. These monolithic columns can be found in several ancient temples such as the temple of Apollo in Corinth, the temple of Aphaia in Aegina, etc. Taller columns though consisted of pieces of stone (drums) placed precisely one above the other without any mortar but were instead connected with wooden parts placed in their center. In high seismicity areas these tall ancient columns could withstand earthquake loadings by absorbing part of the incoming energy through rocking and sliding of their drums. Several monuments though have been damaged due to earthquakes (e.g., the collapse of two Greek temples in Selinunte).

Simple re-erection of monuments with the existing damaged remains increases the chances of a future collapse of the re-erected structure. Restoration processes that include the replacement of missing or damaged material with similar one are preferable. However, this replacement does not usually increase the monument's strength to earthquake loadings. In areas of high seismicity, the enhancement of seismic safety will increase the monuments' life expectancy and avoid further restoration due to earthquake damage. Another drawback of conventional restoration techniques is that they usually affect the appearance of the monument. A simple way to increase the seismic safety of a monument without altering its appearance is to add a particle damper that looks the same as the regular restoration material but is hollow containing particles. There have

¹Department of Civil Engineering T.E., Technological Educational Institute of Western Greece, Megalou Alexandrou 1, 26334 Partras, Greece

*Corresponding author email: email: papalou@teiwest.gr

been several studies of the dynamic behavior of monuments consisting of classical columns [2–13] but there have been only limited efforts to investigate their seismic enhancement [14–18]. Previous research [14–17] suggested that particle dampers can be used to reduce the motion of a vibrating column. Particle dampers are passive control systems that absorb the incoming energy through impact of particles with each other and with the walls of the container resulting in exchange of momentum between the particles and the primary system. Such devices have been used for years to reduce the vibrations of machines and structures [19–37]. Papalou et al. [14–17] reported that a properly designed particle damper replacing one of the top drums of a classical multi-drum column without capital can achieve substantial reduction of the motion. The particles inside the damper have to be placed in one layer, occupying 40–60% of the empty space and to have mass equal to 1–3% of the mass of the column. Previous research on the effectiveness of particle dampers has focused only on multi-drum columns without capital [14–17]. There has been no published work, to the best of our knowledge, on the use of particle dampers on monolithic columns and on multi-drum columns with capital. Monolithic columns though are often encountered in relatively short ancient temples. In addition, the capital has been found to play an important role on the dynamic response of classical columns [13]. Many times restoration of classical columns includes either the addition of a new capital, if the original one is missing, or extensive repair with new material if the original one is damaged (Fig. 1). This paper, building upon the results obtained from previous research [14–17], examines the effect of particle damper's position on the dynamic response of multi-drum and monolithic columns with a capital on top. Two small marble column-models are used, a monolithic and a multi-drum one consisting of four drums.



Fig. 1 Remains of ancient temple after restoration

2 Experimental set-up

The experimental investigation was conducted using a four-drum marble column-model and a monolithic one. Their dimensions were selected to be 1:5 of the dimensions of an ancient short column from the temple of Athena in Alifeira. The total height of the columns was 0.665 m and the masses of the monolithic and the four-drum column were 20.4 and 20.6 kg respectively (Fig. 2). The height of each drum was 0.15 m while their diameter varied depending on their position. The diameter of the bottom of the lower drum was 0.134 m and the diameter of the top part of the top drum was 0.1 m. The height of the capital was 0.065 m and its width 0.175 m. The drums and the capital were not connected with each other but they were smoothly polished and placed carefully one above the other in full contact through a peripheral ring that was formed on the top of each drum. Each column was placed on the top of a marble plate with dimensions 0.165 x 0.165 x 0.019 m and the system was attached onto a shaker introducing motion in one direction. Small rods were inserted at the cylindrical surface of the drums and capital and fishing lines connected the rods loosely with an external metal frame in order to avoid damage of the column during shaking.

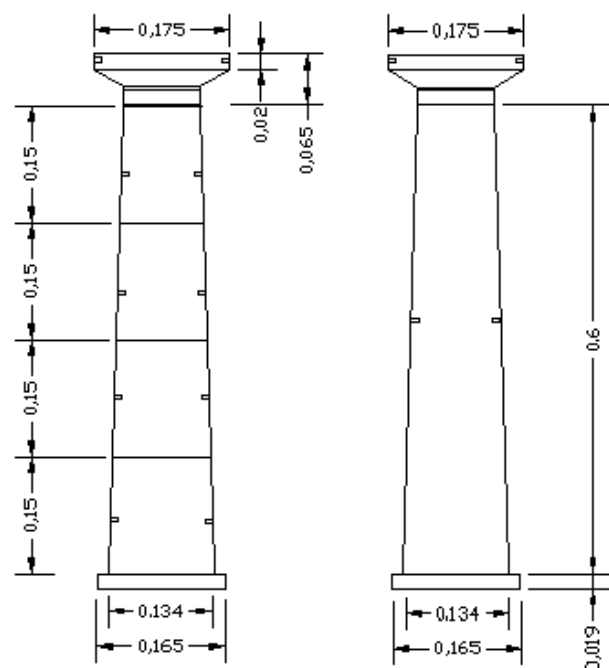


Fig. 2 Dimensions of column models

The particle dampers used for these experiments were also made of marble. The damper in the form of the capital had a cavity of diameter equal to 7.6 cm and height equal to 2.5 cm. Two dampers in the form of drums were also used. One had the form of the top drum and the other the form of the third one (the one below the top drum) with cavity's diameter and depth 7.6 cm and 6 cm respectively. Spherical particles of 2.54 cm diameter made of tungsten were placed inside the damper. The dimensions of the damper's cavity and the number of particles were selected in order to satisfy the criteria suggested by Papalou et al. [14–17]. Tungsten particles were selected because of their

high density. This way the criteria were satisfied with a small number of particles. Dampers with smaller cavity diameter and a smaller number of particles were also used but the results were similar or slightly worse to the results presented.

The motion of the drums and capital were monitored using laser transducers and accelerometers. The motion of the shaker was monitored using a draw-wire sensor and an accelerometer. The experiments were recorded with a video-camera. The motion of the column is presented with the relative displacement of the capital with respect to the base in the direction of motion (since the capital was moving more than the rest of the column).

3 Dynamic response of column-models

3.1 Harmonic excitation

The natural frequencies of the column models were identified using harmonic excitations in the form of sine waves. The duration of the excitation signals was 10 sec and their frequency range from 1–10 Hz. The frequency was increased in increments of 0.5 to 1 Hz depending on the observed motion; the highest the motion of the column the lower the frequency increment.

Multi-drum column

Initially the four-drum column was excited by sinusoidal signals. The relative motion of the column with respect to the base was very small for frequencies below 2 Hz. At 2 Hz even though the relative motion was not visible, noise could be heard from slight rocking of the drums. At 3 Hz the top part of the column was moving more and slight rocking was observed between the 2nd and 3rd drum. At 4–4.5 Hz the motion of the top part of the column substantially increased with slight rocking of the 2nd and 4th drum and intense rocking of the 3rd drum. Residual displacement and rotation was observed mostly at the top drums. At 5 Hz the rocking of the drums and residual displacement and rotation decreased for most of the drums. At 6 Hz and above the motion was smaller. It was apparent that there was a mode introducing substantial displacement and rotation of the drums close to 4 Hz.

Monolithic column

Next the monolithic column was excited with the same sinusoidal signals. Some noticeable motion of the top part was observed at 4.5 Hz. At 5 Hz rocking of the base and substantial relative motion of the top part of the column occurred. At the end of the experiment residual rotation and displacement of the base were observed. At 6 Hz and above the rocking and motion of the column were smaller. It was apparent that there was a mode introducing substantial displacement and rotation of the drums close to 5–5.5 Hz.

3.2 Random excitation

The column-models without damper were excited by a random signal of 20 sec duration (Fig. 3) containing frequencies from 0 to 20 Hz. The amplitude of this random time history was higher towards its end.

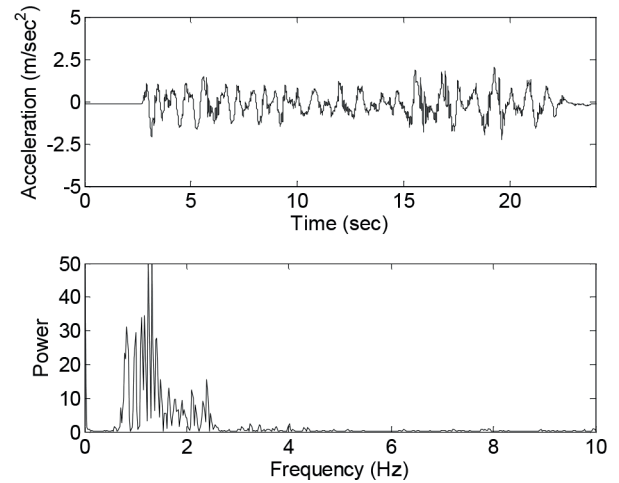


Fig. 3 Time history of the input random signal

Multi-drum column

Initially, the multi-drum column was excited by the random signal. At the first few seconds of excitation, the capital and the top drum were sliding slightly. Towards the end of the excitation rocking of the 2nd, 3rd and 4th drum occurred (Fig. 4).

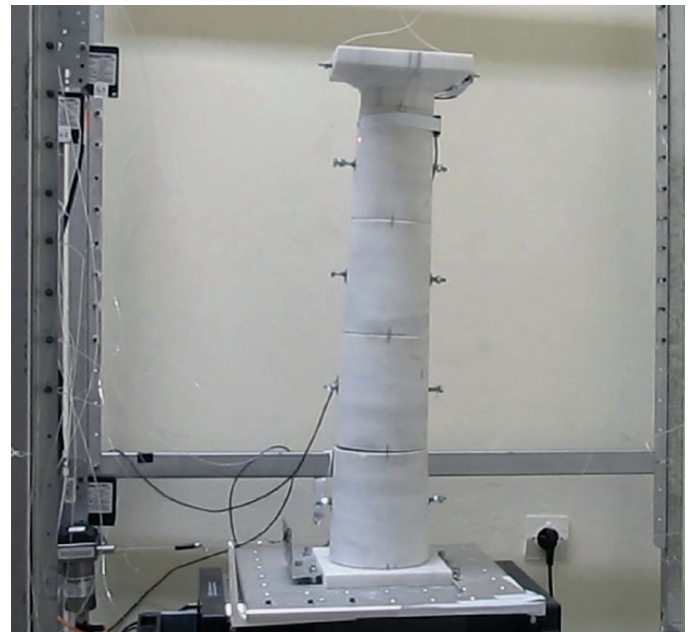


Fig. 4 Photograph showing the multi-drum column's response under random excitation

The relative displacement of the capital was small in the first few seconds but increased substantially towards the end of the excitation (Fig. 5). The effectiveness of the particle damper in reducing the dynamic response of the multi-drum column was examined by replacing the capital with a particle damper having the same shape and containing particles. Four tungsten spherical particles were placed inside the cavity corresponding to 3% mass ratio (mass of particles to the mass of the column m_p/M) and occupying 45% of the area of the cavity (cross section area of the particles divided by the area of the cavity A_p/A). As the column was excited by the random signal (when there was substantial motion to introduce enough momentum to the particles) the particles were moving in the opposite direction

from the direction of motion of the column hitting the walls of the damper, exchanging momentum with the column and reducing its motion (Fig. 6). The response of the column was reduced by more than 25% when there was substantial movement of the particles.

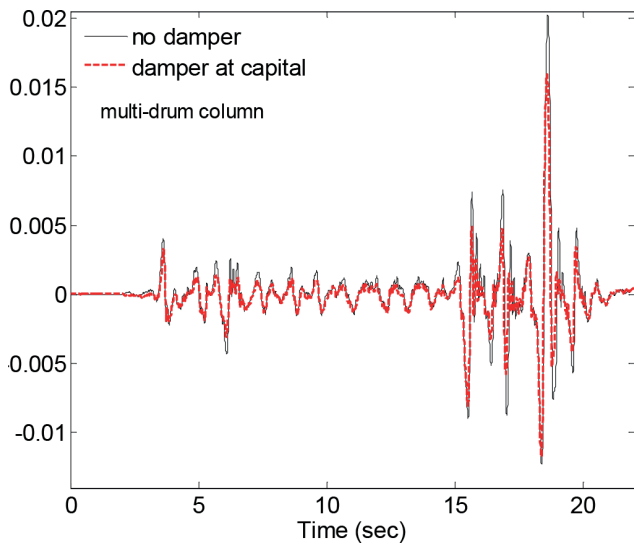


Fig. 5 Relative displacement of the of the multi-drum column's capital without and with a particle damper replacing the capital

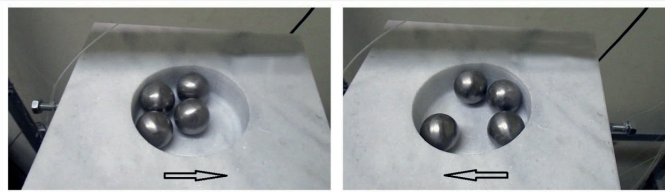


Fig. 6 Photograph showing the motion of the particles opposing the motion of the column

Next the top drum was replaced by a damper while the original capital was placed at the top. Different numbers of particles were used inside the cavity to find the best mass ratio and corresponding area. Fig. 7 presents the response of the capital without damper and with damper replacing the top drum with mass ratios of the particles equal to 1.5% and 3% with corresponding area ratios 22% and 45% respectively. The maximum reduction (27%) occurred in 3% mass ratio and corresponding 45% area ratio (Fig. 8). This reduction was similar with the reduction obtained when the particle damper replaced the capital. The results are in agreement with the results of previous research [14–17] as far as the mass ratio and area occupied by the particles. Similar results were obtained when, instead of the top drum, the third drum was replaced by a damper with the same size cavity and number of particles.

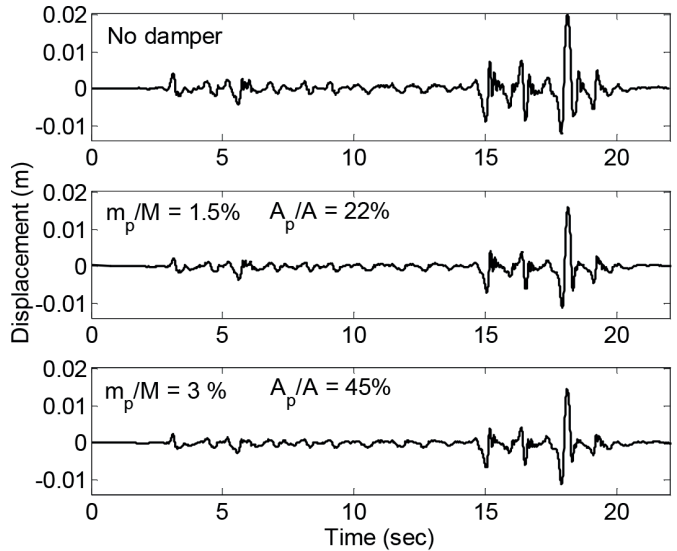


Fig. 7 Relative displacement of capital without and with damper with different mass and area ratios

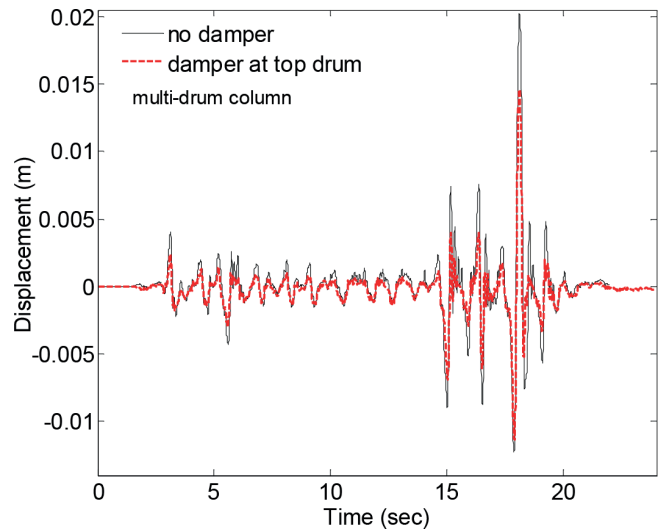


Fig. 8 Relative displacement of the multi-drum column's capital without and with a particle damper replacing the top drum

Monolithic column

The monolithic column was excited by the same random signal. The relative motion was small until the last few seconds where the column rocked a few times at its base (Fig. 9). The capital was moving together with the column without any slippage. Afterwards the capital was replaced by the damper containing four spherical tungsten particles ($m_p/M = 3\%$ and $A_p/A = 45\%$). At the beginning the motion was small similar to the motion of the column without damper, but towards the end when the excitation got stronger the particles started moving hitting the walls of the damper and reducing the motion of the column. Fig. 10 shows the relative motion of the capital with respect to the base. The peak response was reduced by 20% but the greatest reduction occurred afterwards reaching levels higher than 60%.

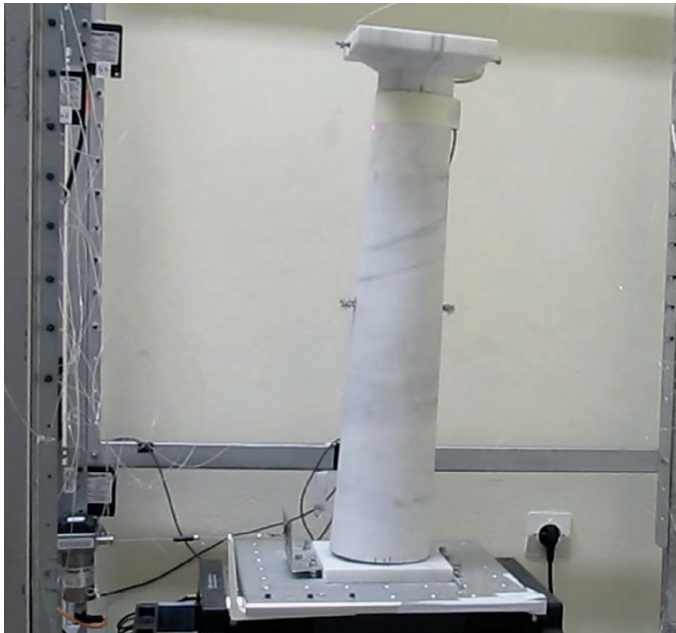


Fig. 9 Photograph showing the monolithic column's response under random excitation.

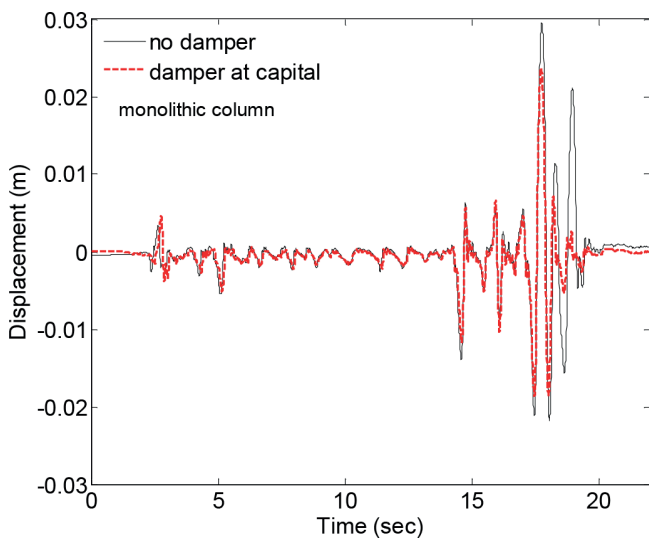


Fig. 10 Relative displacement of the monolithic column's capital without and with a particle damper replacing the capital

3.3 Earthquake excitation

The response of the column-models to earthquake excitations was quantified by selecting signals that could invoke substantial motion to the column and could cover different parts of the frequency spectrum. The signals were modified in amplitude or frequency content to account for the small size of the columns.

Multi-drum column

The multi-drum column was excited by two earthquake signals. The first earthquake signal had most of its energy built up to 3 Hz and its highest amplitude during the first few seconds. Fig. 11 presents the time history and frequency spectrum of this first earthquake signal (Earthquake I).

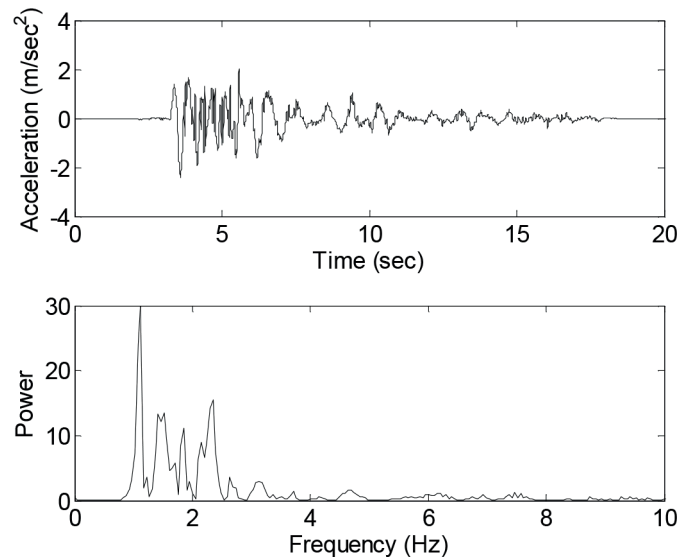


Fig. 11 Time history and frequency spectrum of Earthquake I

During the first few seconds the motion of the column was strong with slight rocking occurring at the bottom drum and more intense at the 2nd and 3rd one while the top drum and capital followed the motion of the 3rd drum. The relative displacement of the capital with respect to the base is presented in Fig. 12. In the next test the capital of the column was replaced by the damper containing four spherical tungsten particles. There was reduction of motion after the peak and towards the end of the excitation (Fig. 12).

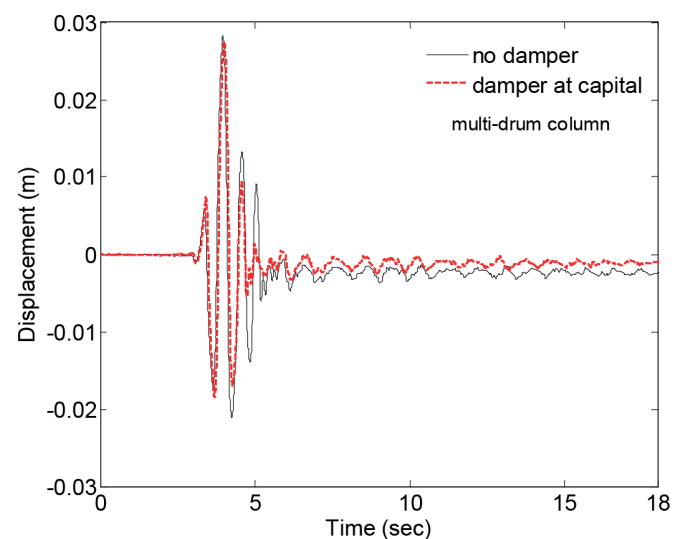


Fig. 12 Relative displacement of the of the multi-drum column's capital without and with a particle damper replacing the capital (Earthquake I)

Considerable reduction of motion was achieved when the damper replaced the top drum with the same particles (Fig. 13). The peak response was reduced by more than 35% and the motion was diminished afterwards. Similar results were obtained when the damper replaced the 3rd drum.

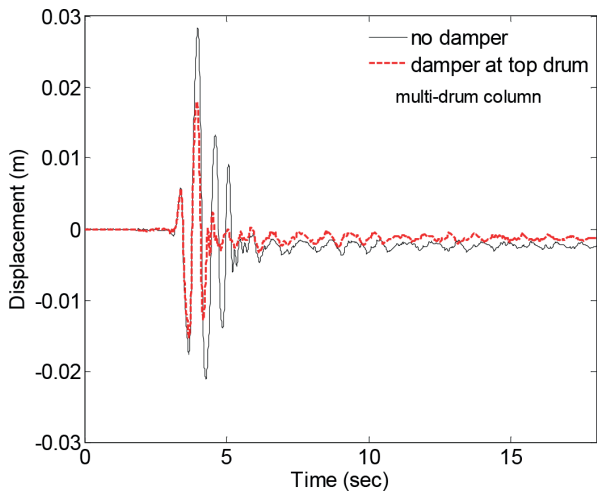


Fig. 13 Relative displacement of the multi-drum column's capital without and with a particle damper replacing the top-drum

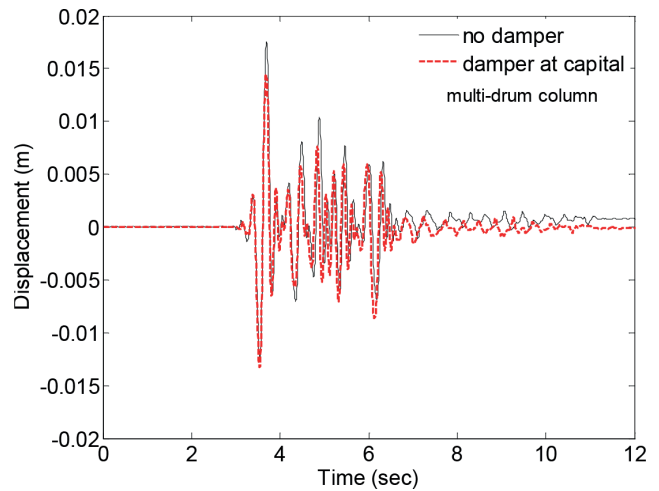


Fig. 15 Relative displacement of the of the multi-drum column's capital without and with a particle damper replacing the capital (Earthquake II)

Fig. 14 presents the time history and frequency spectrum of the second earthquake signal (Earthquake II). This is a short impulsive earthquake with its energy built in higher frequencies than the first one. During the excitation the 3rd drum experienced strong rocking while the 2nd and 4th drum only slight one. The dynamic response of the capital is depicted in Fig. 15. The peak response of the capital was smaller than the corresponding one on the previous earthquake.

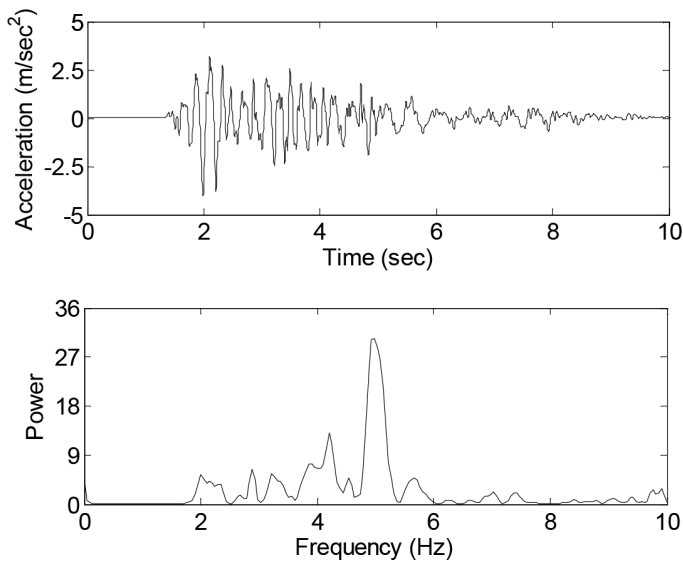


Fig. 14 Time history and frequency spectrum of Earthquake II

Next the capital of the column was replaced by the damper containing four spherical tungsten particles. Small reduction of peak motion occurred (about 15%) and there was no substantial reduction after the peak (Fig. 15). Similar results were obtained when the top or 3rd drum were substituted by the corresponding damper with reduction of the peak response about 10% (Fig. 16).

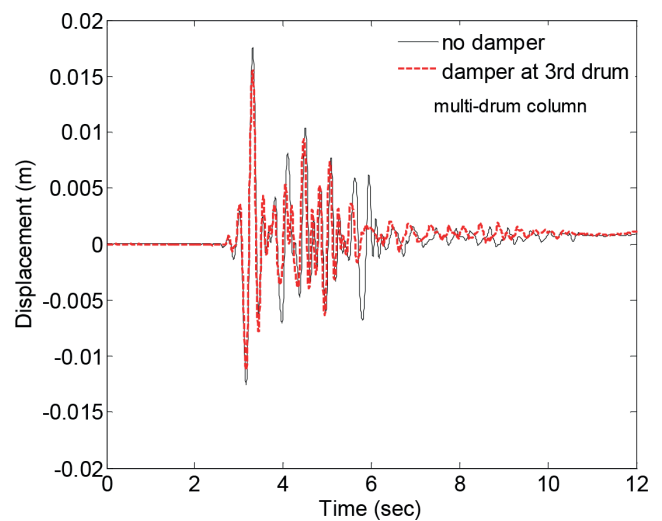


Fig. 16 Relative displacement of the of the multi-drum column's capital without and with a particle damper replacing the third drum (Earthquake II).

Monolithic column

The monolithic column was excited by the same earthquake signals. The first earthquake signal (Earthquake I) produced rocking of the base of the column at the first few seconds. Residual displacement and rotation were observed in the end of the experiment. Fig. 17 presents the relative displacement of the capital with respect to the base. The substitution of the capital with the damper containing four tungsten particles did not affect considerably the response. The reduction of the peak response was about 12% (Fig. 17) but the residual displacement and rotation increased.

The monolithic column was also excited by Earthquake II. The column experienced rocking at the base while the capital followed the motion of the rest of the column. Fig. 18 presents the relative displacement of the capital with respect to the base. The substitution of the capital with the damper containing four tungsten particles reduced the peak response about 10% (Fig. 18). In addition the response declined faster towards the end and small residual rotation and displacement were observed at the end of the excitation.

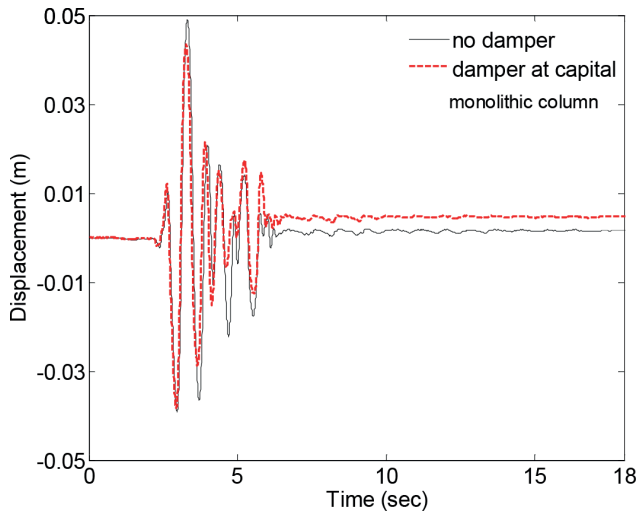


Fig. 17 Relative displacement of the of the monolithic column's capital with-out and with a particle damper replacing the capital (Earthquake I)

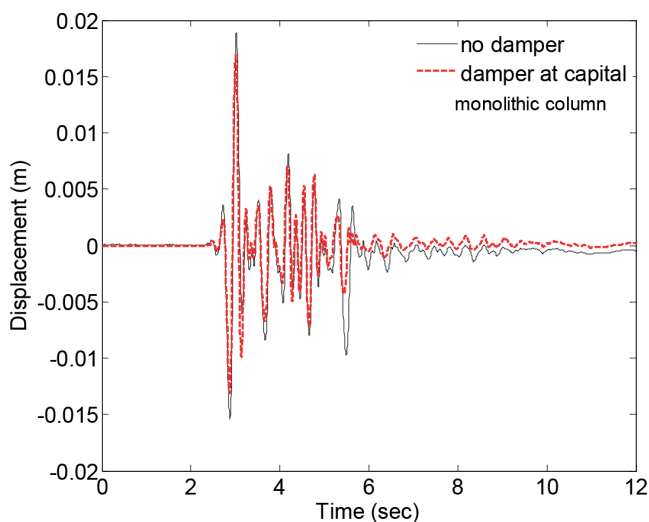


Fig. 18 Relative displacement of the of the monolithic column's capital with-out and with a particle damper replacing the capital (Earthquake II)

4 Conclusions

This paper quantifies the effectiveness of particle dampers in reducing the dynamic response of classical multi-drum and monolithic columns with capital with emphasis on the position of the damper. The damper can replace one of the top drums of a multi drum column or the capital. A small mass ratio of about 2–3% with the cross area of the particles 45–50% of the area of the cavity can reduce substantially the motion of the column when the damper replaces one of the top drums. The damper was more effective in reducing the motion of the multi-drum columns in long period earthquakes where there was substantial motion to invoke exchange of momentum between the particles and the primary system. The damper was not as effective in reducing the dynamic response of the column when it was replacing the capital in either the multi-drum or monolithic column. These results show that in restoration of monuments consisting of classical columns that are missing parts, the use of well designed particle dampers can enhance their seismic performance.

Acknowledgement

The author would like to thank the undergraduate student E. Haxhiraj for his assistance with the experimental work.

References

- [1] C., Papalou, A., Vozikis, K., Kamaterou, C., Lambropoulos, A., Shvets Y. "Architectural and Structural Analysis of Souli's Wells". *Journal of Architectural Conservation*, 20(1), pp. 16–27. 2014. <https://doi.org/10.1080/13556207.2014.886376>
- [2] Psycharis, I., Lemos, J., Papastamatiou, D., Zambas, C., Papantonopoulos, C. "Numerical study of the seismic behaviour of a part of the Parthenon Pronaos". *Earthquake Engineering and Structural Dynamics*, 32(13), pp. 2063–2084. 2003. <https://doi.org/10.1002/eqe.315>
- [3] Sarhosis, V., Asteris P., Wang, T., Hu W., Han Y. "On the stability of colonnade structural systems under static and dynamic loading conditions". *Bulletin of Earthquake Engineering*, 14(4), pp. 1131–1152. 2016. <https://doi.org/10.1007/s10518-016-9881-z>
- [4] Konstantinidis, D., Makris, N. "Seismic response analysis of multidrum classical columns". *Earthquake Engineering and Structural Dynamics*, 34(10), pp. 1243–1270. 2005. <https://doi.org/10.1002/eqe.478>
- [5] Papantonopoulos, C., Psycharis, I. N., Papastamatiou, D. Y., Lemos, J. V., Mouzakis, H. "Numerical prediction of the earthquake response of classical columns using the distinct element method". *Earthquake Engineering and Structural Dynamics*, 31(9), pp. 1699–1717, 2002, <https://doi.org/10.1002/eqe.185>
- [6] Papadopoulos, K., Vintzileou, E. "The seismic response of the columns of Epikouriori Apollo's". In: *3rd Greek Conference in Seismic Mechanics and Technical Seismology*, 5–7 November (in Greek), 2008.
- [7] Papaloizou, L., Komodromos, P. "Planar investigation of the seismic response of ancient columns and colonnades with epistyles using a custom-made software". *Soil-Dynamics and Earthquake Engineering*, 29(1), pp. 1437–1454. 2009. <https://doi.org/10.1016/j.soildyn.2009.06.001>
- [8] Ptilakis, K., Tavouktsi, E. "Seismic response of the columns of two ancient Greek temples in Rhodes and Lindos". In: *8th International Symposium on the Conservation of Monuments in the Mediterranean Basin*, Patra. 31 May–2 June, 2010.
- [9] Michaltsos, G. T., Raftoyiannis, I. G. "Rocking and sliding of ancient temple columns under earthquake excitations". *International Journal of Structural Stability and Dynamics*, 14(2), p. 28. 2014. <https://doi.org/10.1142/S0219455413500582>
- [10] Papaloizou, L., Polycarpou, P., Komodromos, P., Hatzigeorgiou, G. D., Beskos D. E. "Two-dimensional numerical investigation of the effects of multiple sequential earthquake excitations on ancient multi-drum columns". *Earthquake and Structures*, 10(3) pp. 495–521. 2016. <https://doi.org/10.12989/eas.2016.10.3.495>
- [11] Mouzakis, H. P., Psycharis, I. N., Papastamatiou, D. Y., Carydis, P. G., Papantonopoulos, C., Zambas, C. "Experimental investigation of the earthquake response of a model of a marble classical column". *Journal of Earthquake Engineering and Structural Dynamics*, 31(9), pp. 1681–1698. 2002. <https://doi.org/10.1002/eqe.184>
- [12] Drosos, V. A., Anastasopoulos, I. "Experimental investigation of the seismic response of classical temple columns". *Bulletin of Earthquake Engineering*, 13(1), pp. 299–310. <https://doi.org/10.1007/s10518-014-9608-y>
- [13] Papalou, A. "Examining the Dynamic Response of Classical Columns". *International Journal of Civil Engineering*, pp. 1–13. 2016. <https://doi.org/10.1007/s40999-016-0110-6>

- [14] Papalou, A., Strepelias, E. "Effectiveness of Particle Dampers in Reducing the Monuments' Response under Dynamic Loads". *Mechanics of Advanced Materials and Structures*, 23(2): pp. 128–135. 2015. <https://doi.org/10.1080/15376494.2014.943913>
- [15] Papalou, A., Strepelias, E., Roubien, D., Bousias, S., Triantafillou, T. "Seismic Protection of Monuments Using Particle Dampers in Multi-Drum Columns". *Soil Dynamics and Earthquake Engineering*, 77, pp. 360–368. 2015. <https://doi.org/10.1016/j.soildyn.2015.06.004>
- [16] Papalou A, Strepelias E. "Control of the Dynamic Response of Classical Columns with Defects". *Periodica Polytechnica Civil Engineering*, 59(3): pp. 303–308. 2015. <https://doi.org/10.3311/PPci.7870>
- [17] Papalou, A., Strepelias, E. "Structural Control of Monuments' Response under Sinusoidal Excitation using Particle Dampers". *Open Construction and Building Technology Journal*, 8, pp. 351–356. 2014. <https://doi.org/10.2174/1874836801408010351>
- [18] Vassiliou, M. F., Makris, N. "Dynamics of the vertically restrained rocking column". *Journal of Engineering Mechanics*, 141(12). 2015. [https://doi.org/10.1061/\(ASCE\)EM.1943-7889.0000953](https://doi.org/10.1061/(ASCE)EM.1943-7889.0000953)
- [19] Bapat, C.N., Sankar, S. "Multi unit impact damper—re-examined", *Journal of Sound and Vibration*, 103, pp. 457–469. 1985, [10.1016/S0022-460X\(85\)80015-8](https://doi.org/10.1016/S0022-460X(85)80015-8).
- [20] Popplewell, N., Semercigil, S. E. "Performance the bean bag impact damper for a sinusoidal external force". *Journal of Sound and Vibration*, 133(2), pp. 193–223. 1989. [https://doi.org/10.1016/0022-460X\(89\)90922-X](https://doi.org/10.1016/0022-460X(89)90922-X)
- [21] Papalou, A., Masri, S. F. "Performance of particle dampers under random excitation". *ASME Journal of Vibration and Acoustics*. Vol. 118(4), pp. 614–621. 1996. <https://doi.org/10.1115/1.2888343>
- [22] Papalou, A., Masri, S. F. "Response of impact dampers with granular materials under random excitation". *International Journal of Earthquake Engineering and Structural Dynamics*, 25(3), pp. 253–267. 1996. [https://doi.org/10.1002/\(SICI\)1096-9845\(199603\)25:3<253::AID-EQE553>3.0.CO;2-4](https://doi.org/10.1002/(SICI)1096-9845(199603)25:3<253::AID-EQE553>3.0.CO;2-4)
- [23] Papalou, A., Masri, S. F. "An experimental investigation of particle dampers under harmonic excitation". *Journal of Vibration and Control*, 4(4), pp. 361–379. 1998. <https://doi.org/10.1177/107754639800400402>
- [24] Papalou, A., Masri, S. F. "Experimental Studies of Particle Dampers under Random Excitation". In: *First World Conference on Structural Control*, August, Los Angeles, California. 1994. <https://doi.org/10.1002/eqe.4290241210>
- [25] Friend, R. D., Kinra, V. K. "Particle impact damping". *Journal of Sound and Vibration*, 233(1), pp. 93–118. 2000. <https://doi.org/10.1006/jsvi.1999.2795>
- [26] Olson, S. E. "An analytical particle damping model", *Journal of Sound and Vibration*, 264(5), pp. 1155–1166. 2003. [https://doi.org/10.1016/S0022-460X\(02\)01388-3](https://doi.org/10.1016/S0022-460X(02)01388-3)
- [27] Saeki, M. "Impact damping with granular materials in a horizontally vibrating system". *Journal of Sound and Vibration*, 251(1), pp. 153–161. 2002. [https://doi.org/10.1016/S0022-460X\(02\)01388-3](https://doi.org/10.1016/S0022-460X(02)01388-3)
- [28] Mao, K., Wang, M.Y., Xu Z., Chen, T. "DEM simulation of particle damping". *Powder Technology*, 142(2–3), pp. 154–165. 2004. <https://doi.org/10.1016/j.powtec.2004.04.031>
- [29] Xu, Z., Chen, M. Y., Chen, T. "Particle damping for passive vibration suppression: numerical modelling and experimental investigation". *Journal of Sound and Vibration*, 279(3–5), pp. 1097–1120. 2005. <https://doi.org/10.1016/j.jsv.2003.11.023>
- [30] Marhadi, K.S., Kinra, V.K. "Particle impact damping: effect of mass ratio ratio, material and shape". *Journal of Sound and Vibration*, 283(1–2), pp. 433–448. 2005. <https://doi.org/10.1016/j.jsv.2004.04.013>
- [31] Liu, W., Tomlinson, G.R., Rongong, J.A. "The dynamic characterisation of disk geometry particle dampers". *Journal of Sound and Vibration*, 280(3–5), pp. 849–861. 2005. <https://doi.org/10.1016/j.jsv.2003.12.047>
- [32] Yang, M. Y., Lesieutre, G. A., Hambric, S. A. and Koopmann, G. H. "Development of a design curve for particle impact dampers". In: *Proceedings of the SPIE*. 5386, p. 450–465. 2004. <https://doi.org/10.1117/12.540019>
- [33] Fang, X. and Tang, J. "Granular damping in forced vibration: qualitative and quantitative analyses". *Journal of Vibration and Acoustics*, 128(4), pp. 489–500. 2006. <https://doi.org/10.1115/1.2203339>
- [34] Li, K., Darby, A. P. "Experiments on the effect of an impact damper on a multiple-degree-of-freedom system". *Journal of Vibration and Control*, 12(5), pp. 445–464. 2006. <https://doi.org/10.1177/1077546306063504>
- [35] Hu, L., Huang, Q., Liu, Z. "A non-obstructive particle damping model of DEM". *International Journal of Mechanics and Materials in Design*, 4(1), pp. 45–51. 2008. <https://doi.org/10.1007/s10999-007-9053-z>
- [36] Wong, C., Daniel, M. C., Rongong, J. A. "Energy dissipation prediction of particle dampers". *Journal of Sound and Vibration*, 319(1–2), pp. 91–118. 2009. <https://doi.org/10.1016/j.jsv.2008.06.027>
- [37] Lu, Z., Masri, S. F., Lu, X. "Parametric studies of the performance of particle damper under harmonic excitation". *Structural Control and Health Monitoring*, 18(1), pp. 79–98. 2011. <https://doi.org/10.1002/stc.359>