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RESEARCH ARTICLE

Localization of Simulated Damage on a Steel Beam from Random Vibrations

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Abstract
The performance of a few localization schemes using measured mode shapes was tested in an experimental case study with respect to the Ultimate Limit State (ULS). The first question to be posed was: Is it possible to indicate and locate damage under laboratory conditions before the ULS is reached? Relatively simple localization criteria were chosen, which do not require extensive FE analysis. A new combination of them, designated here as Combined Localization Criterion (CLI), was proposed that performed well in the presented case. A simple supported beam with the damage progressing in three consecutive stages was used for the experiments. Mode shapes in the range of up to 100 Hz were extracted from the response of the structure to the air stream.

Keywords
damage localization, change of natural modes, flexibility matrix, flexibility curvatures, case study, damage detection, vibration monitoring

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1 Introduction

Monitoring of building structures is a continuously developing discipline of applied sciences. Ageing building structures (e.g. 66% of railway bridges in Europe are more than 50 years old [1]) raise the need for techniques capable of detecting damage at an early stage of progress, preventing fatal collapses or just to help to determine the remaining service life.

Measurement of vibrations represents an opportunity to solve these problems. Monitoring can help to detect, localize, quantify the damage or even help to determine the remaining bearing capacity [2, 3]. This article is narrowly focused on damage localization from vibration response measurements.

Localization of damage became a subject of intensive research in connection with trusses for space stations during the eighties [4]. In this article, it is stated that natural frequencies only will probably not be able to localize damage in real applications. Other possibilities of damage localization are also discussed using changes to the stiffness matrix [e.g. 5], to the kinetic energy or just to the imbalance of structural parts.

Later, a promising damage localization was reported using mode shape curvatures [6, 7]. The principle was successfully applied on the real structure of the Z24 bridge [8]. It was also proved that higher derivatives of mode shapes are sensitive damage indicators in general [9].

Changes of flexibility constructed from measured mode shapes represent another possibility to localize damage [2, 10]. The topic is discussed from a theoretical and practical point of view in [11]. The successful combination of flexibility and curvatures can be found in [12].

The damage index based on strain energy capable of localizing and quantifying damage was presented in [13, 14]. It was applied and enhanced by many other authors [15, 16, 17, 18]. An interesting application tailored especially for trusses can be found in [19].

A common feature of the preceding titles was the application of measured mode shapes. There were also attempts to use the transfer function directly for damage localization [20, 21, 22, 23]. Power spectral density was suggested for damage localization in [24] and a similar approach is adopted in [25].

The direct application of dynamic stiffness from an impact test is another approach that was applied on masonry arches [26]. Applications of nonlinear phenomena for damage localization were not considered here.

Another point of view is the choice of reference data which can be taken either from analysis or from measurement of the intact structure. Extensive literature reviews also exist on the topic and can be found in [1, 4, 27, 28].

The broader practical use of localization schemes in framework of monitoring of building structures in everyday engineering applications (like experimentally supported assessment of existing structures) would need a well-defined relationship with the design standards. Because various kinds of experimental experiences, including uncertainties and local technical and environmental conditions, are difficult to transfer from literature quotations, initially a "safe" indication of the ULS (as the ULS is in fact a matter of standards) was posed as a target here, at first on an elementary example of a simple supported beam under laboratory conditions.

As a main contribution of this presentation is considered the fact, that the experimental verification showed that the tested criteria could locate the damage before the ULS was reached which implies that further research, and search for suitable applications, is possible. It was also demonstrated that a better localization results can be reached through the combination of different criteria (*CLI* criterion).

2 Localization criteria

From the literature sources only those localization criteria were chosen and applied that don´t require an extensive FE-analysis (like stiffness matrix computation) and utilize just the measured information or the measured information plus the lumped mass matrix which can be assembled quite easily.

The equation (1) describes the free motion of a structure when neglecting the damping:

$$\left(\mathbf{M}\cdot\omega^2-\mathbf{K}\right)\cdot\mathbf{y}=0. \quad (1)$$

The eigenvalue solution provides us with the matrix of natural mass normalized mode shapes $\mathbf{\Phi}$ which fulfills the following condition for the mass matrix M and the stiffness matrix K:

$$\mathbf{\Phi}^{\mathrm{T}}\cdot\mathbf{M}\cdot\mathbf{\Phi}\cdot\mathbf{\Omega}=\mathbf{\Phi}^{\mathrm{T}}\cdot\mathbf{K}\cdot\mathbf{\Phi}\,, \quad (2)$$

where $\mathbf{\Omega}$ is the matrix of eigenvalues (ω_i^2 on the main diagonal). The inversion of (2) can be written as:

$$(\mathbf{\Omega})^{-1}=(\mathbf{\Phi}^{\mathrm{T}}\cdot\mathbf{K}\cdot\mathbf{\Phi})^{-1}\,. \quad (3)$$

Knowing that the flexibility matrix $\mathbf{F}$ is the inverse of the stiffness matrix $\mathbf{K}$, (3) can be rewritten as

$$\mathbf{F}=(\mathbf{K})^{-1}=\mathbf{\Phi}\cdot(\mathbf{\Omega})^{-1}\cdot\mathbf{\Phi}^{\mathrm{T}}=\mathbf{\Lambda}\cdot\mathbf{\Phi}\cdot\mathbf{\Phi}^{\mathrm{T}}\,, \quad (4)$$

where $\mathbf{\Lambda}$ is the diagonal matrix of reciprocal eigenvalues $1/\omega_i^2$. The flexibility matrix already converges well with a few of the first natural mode shapes [10], which is convenient from an experimental point of view.

For the localization of the damage the following criterion [12] can be applied

$$\boldsymbol{fc}=\mathrm{diag}(\mathbf{F}_0-\mathbf{F}_\mathrm{d}), \quad (5)$$

where $\boldsymbol{fc}$ is a vector of the same dimension as there is the number of measured DOFs. $\mathbf{F}_0$ is the flexibility matrix of the intact (or reference) structure and $\mathbf{F}_\mathrm{d}$ is the flexibility matrix of the damaged structure.

In [2 and 10] the maximum element in each column of F is used instead of the main diagonal in (5), but the results are very similar. The elements of the main diagonal could be physically interpreted more easily because they in fact correspond to elementary displacements on the particular mode shapes loaded in the same place with a unitary force. As the measurement quality of each of the mode shapes can be rather different, it may be beneficial to use only selected modes for the damage localization:

$$\boldsymbol{fc}_\mathrm{i}=1/\omega_{\mathrm{i},0}^2\cdot\boldsymbol{\varphi}_{\mathrm{i},0}^2-1/\omega_{\mathrm{i,d}}^2\cdot\boldsymbol{\varphi}_{\mathrm{i,d}}^2, \quad (6)$$

which is in fact the equation (5) written for the i^{th} natural mode. It should be noted that the natural mode shapes in the equation (6) are mass normalized. The above criteria can be physically interpreted and therefore can also be used for the damage quantification theoretically.

The criterion for modified flexibility was defined in [29]:

$$\boldsymbol{fd}=\sum\mathbf{\Psi}_{\mathrm{i},0}^{\,2}-\mathbf{\Psi}_{\mathrm{i,d}}^{\,2}, \quad (7)$$

where the mode shapes are unity normalized $\mathbf{\Psi}_\mathrm{i}^{\mathrm{T}}\cdot\mathbf{\Psi}_\mathrm{i}=1$ and the summation is carried out for the n measured mode shapes. The indices 0 and d are used in the same sense as in equations (5) and (6). Although this criterion in the analytical case study using 3 mode shapes on a clamped beam provided erroneous results for some damage locations [in 29] it can provide us with reasonable results in individual damage cases (like the one presented below), or when using a higher number of mode shapes.

Based on inspiration from [7, 9, 12, 15, 17 and 29], the flexibility curvature can be defined for linear structures as (8)

$$\boldsymbol{fc}_i''(xj)=(2\cdot fc_i(xj)-fc_i(x_{j-1})-fc_i(x_{j+1}))\,/\,((x_{j+1})/2-(x_{j-1})/2)^2$$

and $\boldsymbol{fc}_i''(xj)$ can be further normalized

$$\boldsymbol{nfc}_i''=\boldsymbol{fc}_i''\,/\,max(abs(\boldsymbol{fc}_i'')). \quad (9)$$

The combined localization criterion (*CLI*) can now be defined as the product of equations (7) and (9)

$$CLI=\boldsymbol{fd}\cdot\sum\boldsymbol{nfc}_i''. \quad (10)$$

The computation of the *CLI* is not complicated. It requires only the lumped mass matrix and the measured mode shapes on the intact and damaged structure. It is strictly oriented at the localization of dominant damage and can´t be used for damage quantification because both of its components are based on relative values. Only the positive values of the *CLI* can indicate the damage position theoretically.

The *CLI* performed well using the models of simple supported and cantilever beams in preliminary analytical simulations with 6 modes and the considered damage, however it was not tested on more complicated frame structures and for other damage scenarios.

Each criterion can be considered as a "point of view on the condition of the structure". Its performance should be verified for a given type of structure and damage, before launching a monitoring project. A suitable combination of the criteria may be beneficial, such as the *CLI* in the presented case.

Fig. 1 Experimental setup

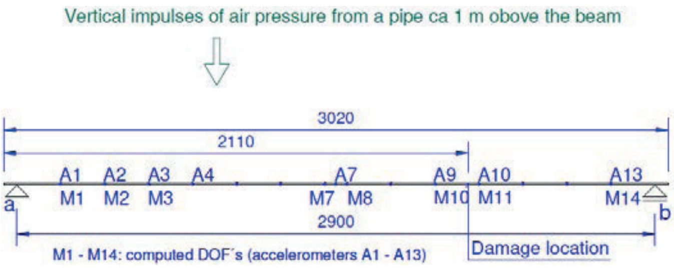

Fig. 2 Tested structure - geometry and position of transducers

3 Experimental verification

Without assuming any uncertainties, all the criteria worked well using FE-simulations. Therefore, a real measured data were considered as the crucial point and reason for the experimental verification.

The view of experimental site is on the figure 1. A simply supported hollow steel profile 40/10/2 mm was chosen for the purposes of this study. The geometry and damage location are shown in figure 2. There are 8 natural frequencies in the frequency range up to 100 Hz, but only five of them are related to bending modes. A uniform static load was assumed reaching 60% of the ULS in the middle of the beam. The damage was simulated by a saw cut into the upper flange at a location 600 mm away from the midspan of the beam. The saw cut was increased stepwise in three consecutive stages of 17mm, 28mm and 34mm, while with the last one (reduction of the moment of inertia about 60%) the ULS at the damage position under the uniform limit load would be exceeded. A standard static analysis was applied for determination of the ULS and the extent of the damage stages.

Vibrations were measured with Brüel&Kjaer (B&K) 4379 and 4370 accelerometers connected to B&K-NEXUS and B&K 2635 amplifiers (see also Fig. 2). Altogether, 13 simultaneously measured channels were used, but the channels were not phase calibrated, which increased the measurement uncertainty. The structure was loaded by vertical pulses of air generated from a valve with compressed air at irregular intervals (see Fig. 2) which should simulate extraction from ambient vibrations in situ. The sampling frequency was 1000 Hz and the mode shapes were extracted relative to a chosen reference transducer. The data from the second channel had to be discarded from the localization procedure because of electricity problems in the measurement chain (explains gap in Fig. 3–7).

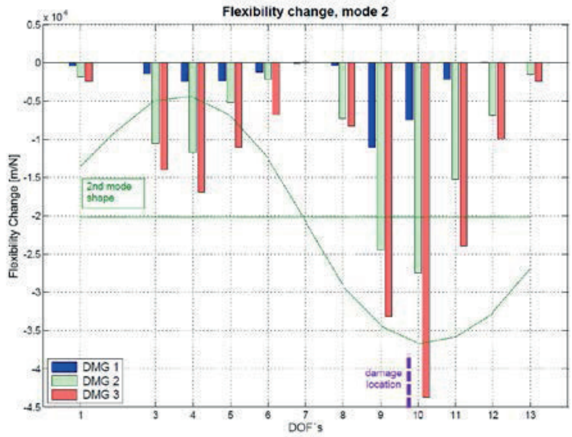

Fig. 3 criterion fc2 – localization of damage using the 2nd mode

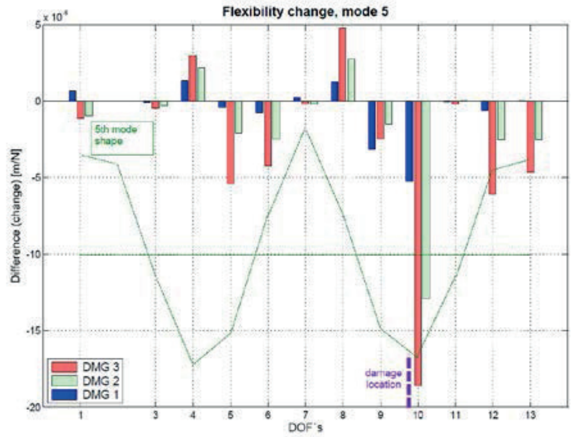

Fig. 4 criterion fc5 – localization of damage using the 5th mode

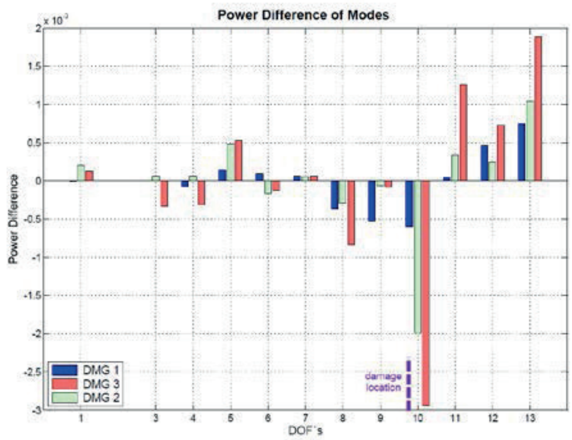

Fig. 5 criterion fd – localization of damage using 6 modes

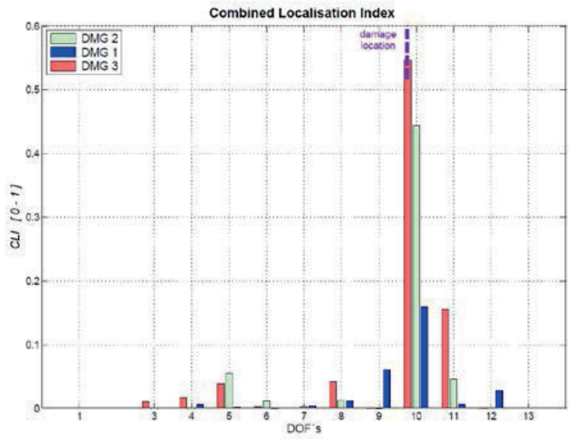

Fig. 6 criterion CLI – localization of damage using 6 modes

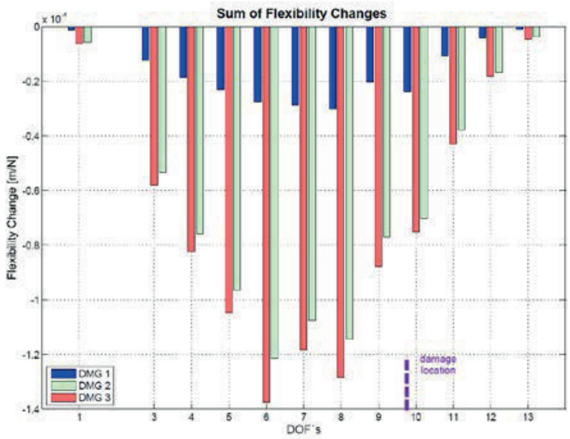

Fig. 7 criterion fc – localization of damage using 6 modes

Natural frequencies were identified as peaks in power spectral density functions of measured outputs and extracted from a curve fit. The applied identification of the modes was one of the simplest ways that could be used for evaluation of ambient vibrations on real structures. The analytical lumped mass matrix was used for the normalization of the mode shapes while evaluating the criteria $\boldsymbol{fc}$ and $\boldsymbol{fc}_j$.

The measured changes of natural frequencies are presented in Table 1. They distinctly mirror the stepwise deterioration of the structure which implies that they could serve as a "save" damage indication under the given laboratory conditions (more details in [31]). The MAC (e.g. [30]) criterion between the undamaged and damaged structure was higher than 99.89 in all cases and the changes, though very small, also consistently reflected the progress of the damage.

Table 1 Experimentally identified nat. frequencies at the stages of progressing damage (DMG 0,1–3)

nat. frequencies	DMG 0 [Hz]	DMG 1 [Hz]	DMG 2 [Hz]	DMG 3 [Hz]
1	2.51	2.46	2.34	2.32
2	11.62	11.58	11.45	11.37
3	22.48	22.47	22.4	22.37
4	44.39	44.4	44.35	44.28
5	65.41	65.29	64.9	64.55
6	94.12	94.03	93.73	93.41

The evaluation of the localization criteria showed that the criteria $\boldsymbol{fd}$, $\boldsymbol{fc}_2$, $\boldsymbol{fc}_5$ and *CLI* identified the position of the damage correctly (see figures 3–6). The criterion $\boldsymbol{fc}$ is influenced in a dominant way by the first mode, which was not determined reliably in this case, so that the criterion $\boldsymbol{fc}$ failed in localizing the damage position correctly (see Fig. 7). On the other hand, the sum of $\boldsymbol{fc}$ along the beam seems to reflect the extent of the progressing damage well. The results of the *CLI* criterion are promising, but it will be necessary to test it under more complicated conditions and in cases of multiple damage.

4 Conclusions

The presented study confirmed that the measured mode shapes could reliably reveal the position of the damage before the ULS was reached in the considered case under laboratory conditions. All the tested localization criteria required the experimental data of the vibration mode shapes on the intact and damaged structure and the theoretical mass distribution of the tested structure only.

The tested localization criterion fd performed well with this type of damage on the simple supported beam using 6 measured mode shapes. The criterion fc derived from the flexibility of the structure wasn´t able to localize the damage well, but its integral value along the length of the beam reflected well the progress of damage. The performance of localization schemes can be enhanced by combination of different localization criteria like in the case of the *CLI*.

The achieved results are promising from the point of view of further research and practical applications.

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