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RESEARCH ARTICLE

Locating Emergency Facilities Using the Weighted *k*-median Problem: A Graph-metaheuristic Approach

Ali Beitollahi[2], Ali Kaveh[1*], Vahid Reza Mahdavi[3]

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Abstract

An efficient approach is presented for addressing the problem of finding the optimal facilities location in conjunction with the k-median method. First the region to be investigated is meshed and an incidence graph is constructed to obtain connectivity properties of meshes. Then shortest route trees (SRTs) are rooted from nodes of the generated graph. Subsequently, in order to divide the nodes of graph or the studied region into optimal k subregions, k-median approach is utilized. The weights of the nodes are considered as the risk factors such as population, seismic and topographic conditions for locating facilities in the high-risk zones to better facilitation. For finding the optimal facility locations, a recently developed meta-heuristic algorithm that is called Colliding Bodies Optimization (CBO) is used. The performance of the proposed method is investigated through different alternatives for minimizing the cost of the weighted k-median problem. As a case study, the Mazandaran province in Iran is considered and the above graph-metaheuristic approach is utilized for locating the facilities.

Keywords

optimal locating, colliding bodies optimization, graph methods, weighted k-median method, risk

1 Centre of Excellence for Fundamental Studies in Structural Engineering, Iran University of Science and Technology, Tehran-16, Iran.
2 Building and Housing Research Center (BHRC), Tehran-14, Iran
3 Postdoctoral Researcher, School of Civil Engineering, Iran Univ. of Science and Technology, P.O. Box 16846-13114, Iran.
* Corresponding author, email: alikaveh@iust.ac.ir

1 Introduction

In recent years, finding the location of facilities has received considerable attention due to its societal importance. A location problem deals with the choice of a set of points for establishing certain facilities in such a way that, taking into account different criteria and verifying a given set of constraints, they optimally fulfill the needs of the users [1–4]. These problems have historically been primarily modelled as a minimax facility location problem. The objective in the minimax format is to locate a new facility such that the maximum distance to the areas requiring emergency services is minimized. The objective function is based on the idea that even the farthest and/or the least important client should get a reasonable level of service by attempting to ensure that any emergency is addressed in a reasonable time. There have been a number of different objective functions applied to the problem. Research has also addressed the location of an emergency service facility on networks, on the plane, location on the restricted plane and others [5]. Another variation is the case where demand points have different weights reflecting different degrees of risks [6].

One of the most famous problem for dealing with locating facilities is the k-median problem. In this problem, the sum of the distances of the customers (demand points) from the nearest facility is an essential factor. This problem is also known as the minimax location problem. Considering that n demand points being located in the space and they are in fixed positions in static facility locations, the main objective is to locate k facilities among these demand points in such a way that the sum of distances between demand points and their nearest facility becomes minimum [7–9].

Some algorithms have been developed for the problem for finding optimal domain of finite elements methods [10–13], and some review papers on this topic are also available [13]. Finding the medians of a graph is an NP-hard combinatorial optimization problem, and the exact solution of the problem is complex and highly time consuming for graphs with a large number of nodes. Such algorithms can be found in the work of [11]. Therefore, many approximate algorithms are developed for finding the medians of a graph. The simplest approach to deal with domain decomposition is referred to as the k-median method [10]. In this method, a graph is associated to the connectivity property of model. Then, the optimal medians of the graph are selected through the sum of the distances of the nodes from medians. Recently, metaheuristics such as Genetic algorithms [14, 15], bionomic approaches [16] and ant colony algorithm [17, 18] have been developed in order to obtain solutions for the *k*-median problem.

As a newly developed type of meta-heuristic algorithms, the colliding bodies optimization (CBO) was introduced for design of structural problems [19]. This algorithm can be considered as a multi-agent method, where each agent is a Colliding Body (CB). Each CB is considered as an object with a specified mass and velocity before the collision. After collision occurs, each CB moves to a new position according to the new velocity. This algorithm utilizes simple formulation; and it requires no parameter tuning.

In this paper, an algorithm based on the k-median concept is presented for finding optimal domains for locating the emergency facilities considering the risk factors using the CBO algorithm. Computer programs have been developed to perform this method and five numerical examples are presented to illustrate the application and efficiency of the proposed method.

2 Description of the proposed methodology

2.1 The k-median problem

The aim of the *k*-median problem is to cut a node set, N, into k nodes, $N_k \in N$, such that the sum of the distances of nodes to the median nodes becomes minimum. The problem of *k*-median can be stated as optimizing a function which decomposes the domain G into k subdomains G_1, G_2, … G_k where k is the number of subdomains. The objective function which must be minimized is formulated as:

$$\sigma_0(N_k) = \sum_{j \in N} v_j d(N_k, j) \tag{1}$$

where $\sigma_0(N_k)$ is called the out-transmission of nodes N_k, N_k is the median node number, v_j is the weight of node j and $d(N_k, j)$ is defined as:

$$d(N_k, j) = \min[d(i', j)] : (i' \in N_k) \tag{2}$$

let i' be the node of N_k which produces the minimum in Eq. (2), then we say the node j is allocated to i'. A SRT has been rooted from each node to obtain the shortest distance between nodes [12]. In order to find the node numbers of the medians of a graph, we consider the coordinates of medians as the variables of the optimization process. Then, the nearest nodes from this coordinate are selected as the medians of the graph.

2.2 Proposed method

The presented method for locating the facilities to minimize the accessibility and risk is described through a case study, as outlined in the following:

Step 1. Description of the region: A region is first selected and its geographical properties are then defined. In this paper, the study is performed for the Mazandaran province in northern of Iran, which is one of the most densely populated area in this country and located in the southern coast of the Caspian Sea. Mazandaran has a population of 3,073,943 and has a surface area of 23,842 km², which corresponds to 1.46% of the total surface area of Iran.

Step 2. Meshing the region: For the purpose of planning and design, the region area is divided into some square meshes. In this study, the selected region is divided into a mesh of 4096 squares (the sizes of square meshes are 5000 m×5000m).

Step 3. Construct the incidence graph: Once the region is meshed (in previous step), we associate an incidence graph with the meshed region. The nodes of the graph are in a one-to-one correspondence with the mesh of the considered region, and two nodes of graph are connected by an edge if the corresponding meshes have at least one common node.

Step 4. Defining the regional scale: In this step, the weight of each node of the graph is calculated based on the regional scale. Two groups of factors can be considered at this scale: (1) natural factors, which include seismicity, geological features, land instability and topography; and (2) man-made factors such as regional land uses, transportation networks, lifelines, population density, and disaster management organizations and capabilities. These factors are the weight of nodes in Eq. (1) to minimize the risk for the specific neighborhood.

Step 5. Minimizing the cost function: Performing optimization process on the objective function (Eq. (2)) for finding the facilities to minimize the accessibility and risk.

3 Formulation of the CBO algorithm

As stated previously, the CBO is a recently developed metaheuristic algorithm with its formulation driven from the one-dimension collision laws between two bodies; in which one object collides with another object and after collision, objects move in concordance with the principle of conservation of energy. In this algorithm the mass of each object is related to the inverse of its corresponding objective function. According to the conservation of energy, after collision, the heavier object moves less than the lighter one and the change of its mass is smaller [19].

In the CBO algorithm each solution candidate X_i containing a number of variables (i.e. $X_i = \{X_{i,j}\}$) is considered as a colliding body (CB). The massed objects are composed of two main equal groups; i.e. stationary and moving objects, where the moving objects move to follow stationary objects and a collision occurs between pairs of objects. This is done for two purposes: (i) to improve the positions of moving objects; (ii) to push stationary objects towards better positions. After the collision, the new positions of the colliding bodies are updated based on the new velocity by using the collision laws.

The CBO procedure can briefly be outlined as follows:

1) The initial positions of CBs are determined with random initialization of a population of individuals in the search space:

$$x_i^0 = x_{\min} + rand(x_{\max} - x_{\min}), \quad i = 1, 2, ..., n, \tag{3}$$

where, x_i^0 determines the initial value of the ith CB. $x_{\min}$ and $x_{\max}$ are the minimum and the maximum allowable values vectors of variables; *rand* is a random number in the interval [0,1]; and n is the number of CBs.

2) The magnitude of the body mass for each CB is defined as:

$$m_k = \frac{1}{fit(k)}, \quad k = 1, 2, ..., n \tag{4}$$

where *fit*(i) represents the objective function value of the agent i; n is the population size. Obviously, a CB with good values exerts a larger mass than the bad ones.

3) The arrangement of the CBs objective function values is performed in ascending order. The sorted CBs are equally divided into two groups:

- The lower half of CBs (stationary CBs); These CBs are good agents which are stationary and the velocity of these bodies before collision is zero. Thus:

$$v_i = 0, \quad i = 1, ..., \frac{n}{2} \tag{5}$$

- The upper half of CBs (moving CBs): These CBs move toward the lower half. Then the better and worse CBs, i.e. agents with upper fitness value of each group will collide together. The change of the body position represents the velocity of these bodies before collision as:

$$v_i = x_{i-\frac{n}{2}} - x_i, \quad i = \frac{n}{2} + 1, ..., n \tag{6}$$

where, v_i and x_i are the velocity and position vector of the ith CB in this group, respectively; $x_{i-\frac{n}{2}}$ is the ith CB pair position of x_i in the previous group.

4) After the collision, the velocity of bodies in each group is evaluated using the collision laws and the velocities before collision. The velocity of each moving CB after the collision is:

$$v_i^{'} = \frac{(m_i - \varepsilon m_{i-\frac{n}{2}})v_i}{m_i + m_{i-\frac{n}{2}}}, \quad i = \frac{n}{2} + 1, ..., n \tag{7}$$

where, v_i and v_i are the velocity of the ith moving CB before and after the collision, respectively; m_i is the mass of the ith CB; $m_{i-\frac{n}{2}}$ is mass of the ith CB pair. Also, the velocity of each stationary CB after the collision is:

$$v_i^{'} = \frac{(m_{i+\frac{n}{2}} + \varepsilon m_{i+\frac{n}{2}})v_{i+\frac{n}{2}}}{m_i + m_{i+\frac{n}{2}}}, \quad i = 1, ..., \frac{n}{2} \tag{8}$$

where $v_{i+\frac{n}{2}}$ and v_i are the velocity of the ith moving CB pair before and the ith stationary CB after the collision, respectively; m_i is mass of the ith CB; $m_{i+\frac{n}{2}}$ is mass of the i th moving CB pair. ε is the coefficient of restitution (COR) and for most of the real objects, its value is between 0 and 1. It is defined as the ratio of the separation velocity of two agents after collision to the approached velocity of two agents before collision. In the present algorithm, this index is used to control of the exploration and exploitation rates. For this goal, the COR decreases linearly from unit to zero. Thus, is defined as:

$$\varepsilon = 1 - \frac{iter}{iter_{\max}} \tag{9}$$

where iter is the actual iteration number and $iter_{max}$ is the maximum number of iterations, with COR being equal to unit and zero representing the global search and local search, respectively.

5) New positions of the CBs are obtained using the generated velocities after the collision in the position of stationary CBs.

The new positions of each moving CB is:

$$x_i^{new} = x_{i-\frac{n}{2}} + rand \circ v_i^{'}, \quad i = \frac{n}{2} + 1, ..., n \tag{10}$$

where, x_i^{new} and $v_i^{'}$ are the new position and the velocity after the collision of the ith moving CB, respectively; $x_{i-\frac{n}{2}}$ is the old position of the ith stationary CB pair. Also, the new positions of stationary CBs are obtained by:

$$x_i^{new} = x_i + rand \circ v_i^{'}, \quad i = 1, ..., \frac{n}{2} \tag{11}$$

where, x_i^{new}, x_i and v_i are the new position, old position and the velocity after the collision of the ith stationary CB, respectively. *rand* is a random vector uniformly distributed in the range (−1,1) and the sign "" denotes an element-by-element multiplication.

6) The optimization is repeated from Step 2 until a termination criterion, specified as the maximum number of iteration, is satisfied. It should be noted that, a body's status (stationary or moving body) and its numbering are changed in two subsequent iterations.

Apart from the efficiency of the CBO algorithm, which is illustrated in the subsequent section through numerical examples, the proposed algorithm does not contain internal parameters besides the coefficient of restitution (COR). The linear variation law adopted for COR, makes the proposed algorithm a parameter independent optimization technique. This is a definite strength of the CBO.

4 Numerical example based on a real case

In this section, the real data for locating emergency facilities in Mazandaran city are collected. Figure 1 shows the map of studied region and Mazandaran city. In this study the number of medians, k, is considered as 6. The number of agents is set to 20 individuals and the maximum number of iterations is also considered as 200. The algorithms are implemented in MATLAB. Capability and robustness of the proposed method is investigated for five different cases.

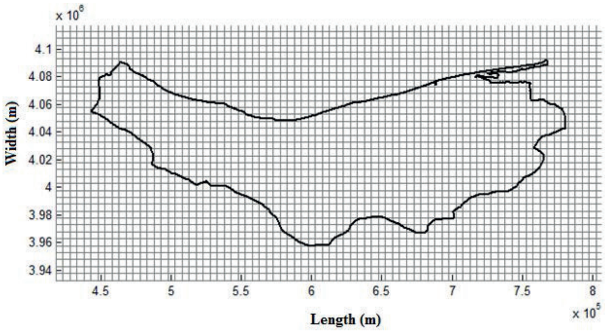

Fig. 1 Map of the meshed region and the Mazandaran province

Alternative 1: In the first case, suppose that decision maker intends to locate the facilities in the rectangular region displayed in Figure 1 such that sum distances of demands (graph's nodes) from the nearest facility be minimum. In this case, the weights of all nodes, v (as described in the Eq. (1)), is considered as unity. Figure 2 shows the optimal location of facilities and these facilitated regions obtained using the proposed method. As shown in this figure, the rectangular region is divided into k = 6 equal subregiones using the proposed method.

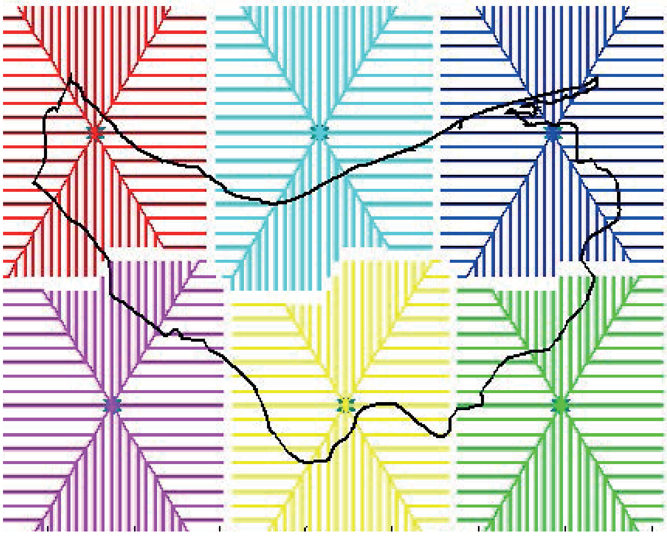

Fig. 2 The considered region divided into k = 6 subregions using the proposed method

Alternative 2: In this case, we have located the emergency facilities such that these facilate the demands that locate in the Mazandaran province. Hence, the weights of the internal and external nodes of the Mazandaran province boundary are considered as unity and zero, respectively. As shown in Figure 3, the Mazandaran province region is divided into k = 6 subregions using this method.

Alternative 3: In this case, the population of region is considered as the risk. The objective is locating the facilities with considering the population density of region and the accessibility of facilities, simultaneously. Hence, the weights of internal and external nodes of the Mazandaran province boundary are considered as the population density values and zero, respectively. Figure 4 shows the map of the normalized population density between zero and unity for the Mazandaran province. The proposed method is utilized for this case and as shown in Figure 5, the Mazandaran province region is divided into k = 6 segments using this method and the facilities are located close to the high population density zones.

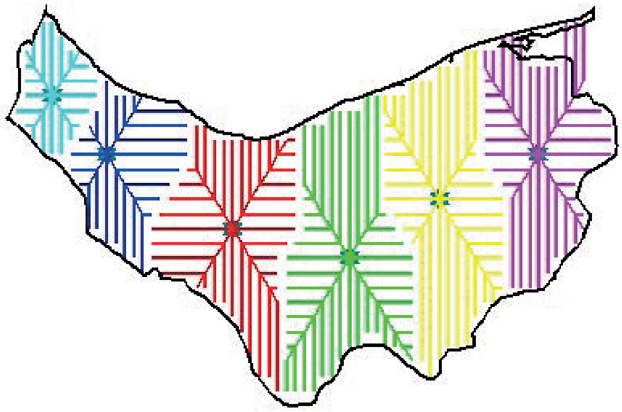

Fig. 3 The Mazandaran province divided into k = 6 subregions using the proposed method

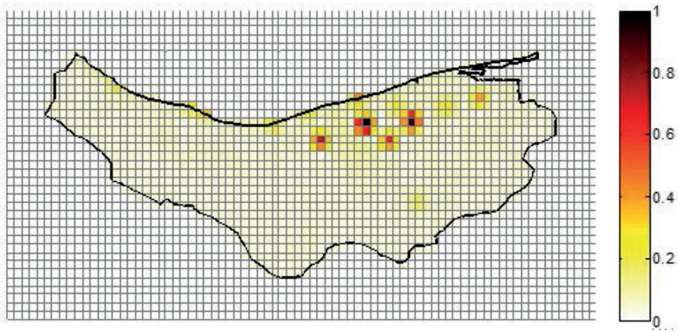

Fig. 4 The population density map for the Mazandaran province

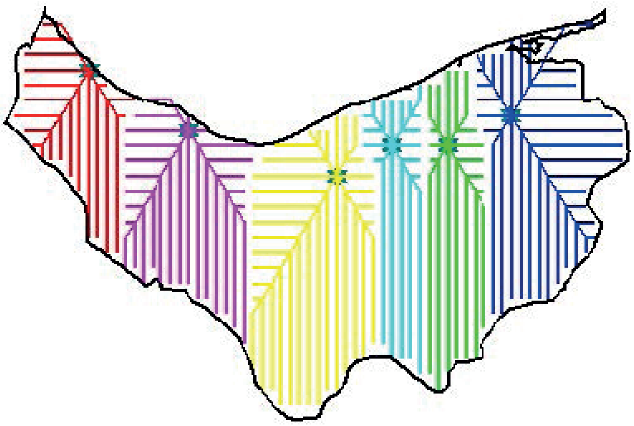

Fig. 5 The Mazandaran province divided into k = 6 subregions for case 3

Alternative 4: Similar to the previous case, the weight of demand points in the studied region is considered based on the earthquake risk for locating the emergency facilities in the high seismicity zones. For this purpose, distance of the demands to the nearest fault is considered as the risk factor and these values (in terms of meteric units) are shows in Figure 6. As illustrated in Figure 7, the Mazandaran province region is divided into k = 6 segments using this strategy and the emergency facilities are located close to the high potential seismicity zones.

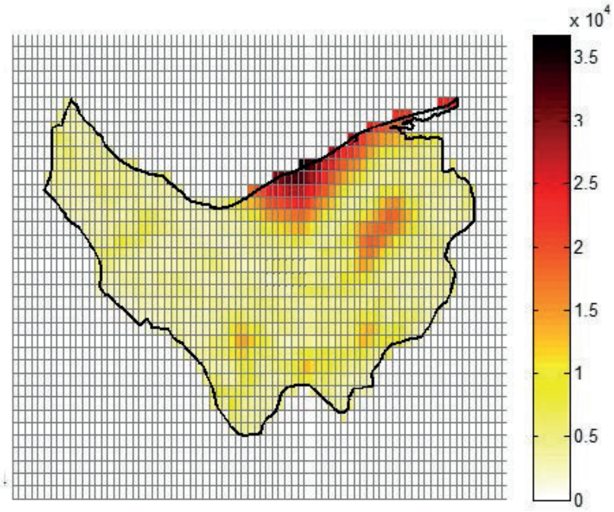

Fig. 6 The seismicity (distance to nearest fault based on meteric units) map for the Mazandaran province

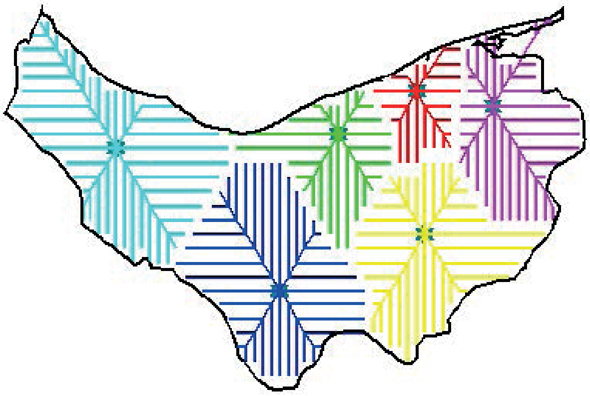

Fig. 7 The Mazandaran province divided into k = 6 subregions for case 4

Alternative 5: In the last case, the topological features is considered as the risk, and the purpose is to locate the emergency facilities in the high elevations. The Figure 8 shows the map of the elevation (in terms of meteric units) in the studied region. For this purpose, weights of the internal nodes of the Mazandaran province boundary are considered as the height elevation factors. As shown in Figure 9, the Mazandaran province region is divided into k = 6 segments using this strategy and the emergency centers are located in the high elevation zones.

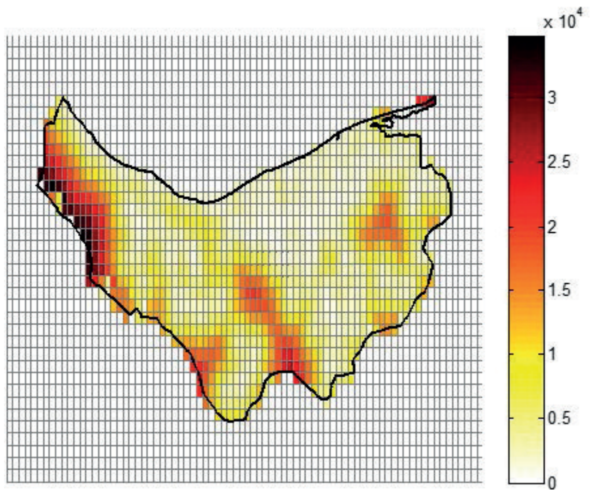

Fig. 8 The elevation map (based on meteric units) for the Mazandaran province

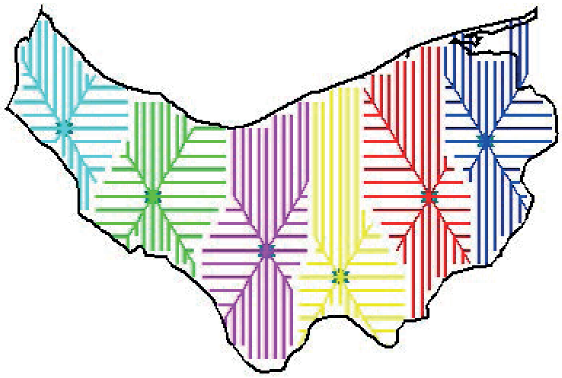

Fig. 9 The Mazandaran province divided into k = 6 subregions for case 5

5 Conclusions

An optimization method is proposed for emergency facilities locating problem for the Mazandaran province, based on Colliding Bodies Optimization (CBO) algorithm and weighted k-median method. The CBO mimics the laws of collision between objects. The very simple implementation and parameter independency are definite strength points of the CBO. In order to find the optimal facility locations, an incidence graph is used to transform the connectivity properties of the meshed region into that of the corresponding graphs. Then, the medians of the weighted graph are selected based on the utilized optimization algorithm to minimize the accessibility and risk. The validity and efficiency of the proposed method are illustrated using five alternatives for a case study based on real data. The outcome shows that the presented method is efficient for locating facilities by considering the accessibility and risk parameters. The proposed method can easily be utilized for different data sets of the other regions.

Acknowledgments

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