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RESEARCH ARTICLE

Modal Approximation Based Optimal Design of Dynamically Loaded Plastic Structures

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Abstract

The purpose of this study is to present an optimal design procedure for elasto-plastic structures subjected to impact loading. The proposed method is based on mode approximation of the displacement field and assumption of constant acceleration of impacted structure during whole time of deformation process until the plastic displacement limit is reached. Derivation of the method begins with the application of the principle of conservation of linear momentum, followed by determination of inertial forces. The final stage of the method utilizes an optimization technique in order to find a minimum weight structure. Eventually, effectiveness and usefulness of the proposed method is demonstrated on the example of a planar truss structure subjected to dynamic loading caused by a mass impacting the structure with a given initial velocity.

Keywords

structural dynamics, optimal design, elasto-plastic structures, short-time dynamic loading

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1 Introduction

Designing safe civil infrastructure is a critical issue in daily practice of many earthquake and structural engineers. On the other hand, there exists a problem related to economical design, which requires finding the cheapest structures fulfilling normative recommendations such as allowable stresses, accelerations or displacements. A reasonable trade-off between safety considerations and economic aspects requires continuous research towards new methods, procedures for optimal design of civil structures.

In the past six decades a great number of papers have been published dealing with the dynamic response of plastic structures. During the early period most of the developments in the field of analysis and optimal design of elasto-plastic structures were connected with the Italian school led by Maier and the Canadian school led by Cohn [1]

The classical theorems have been extended to problems concerning dynamic loading, large deformations, strain hardening and nonassociated constitutive models and also several computational methods have been developed see e.g. [2], and [3–5]. Systematic surveys can be found e.g. in [6]. As a consequence of the theorem of constant stresses of plasticity, rigid-plastic models have been used and introducing simple approaches like the modal and the kinematical approximations the overall response of beams and plates subjected to dynamic pressure and impact have been investigated e.g. [7–10]. For determination of the permanent deflections and response time a number of bounding theorems have been also developed.

Earlier, e.g. [11–13], and later several papers have studied the effects of strain rate sensitivity and large deformations e.g. [14–16]. In addition to the theoretical studies experimental investigations have been also conducted e.g. [14, 17]. A few papers proposed analytical and numerical methods to the optimal design of beams and plates subjected to dynamic pressure e.g. [18–21]. Following the theoretical and analytical investigations more and more numerical methods and very accurate computer programs have been developed which can be used for a complete time history analysis of any kind of dynamically loaded elasto-plastic structures taking effects of

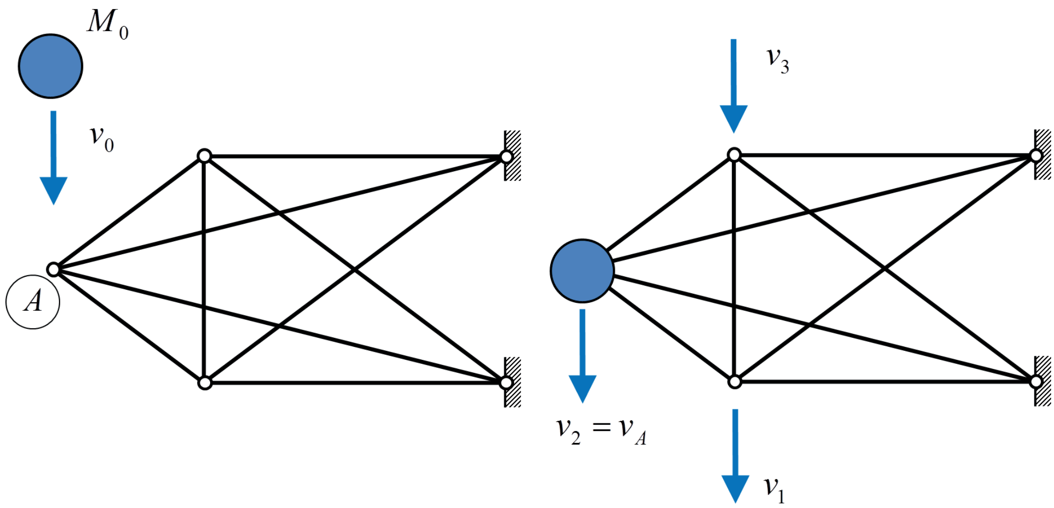

Fig. 1 Location of impact and velocity field just before and after impact occurred.

viscosity and large deformation into consideration. The survey of these methods is beyond the scope of this paper.

Earlier attempts in optimal design took into account only structures made of elastic material [22–24], In such situations the structure was assumed to be optimally designed when its internal stresses did not exceed a certain limit. However, in the case of extreme loading such a design can lead to very stiff and expensive structures. For that reason researches started to look for optimal designs of structures made of elasto-plastic or rigid-plastic material which allowed to make use of a plastic reserve of a structure [25–27]. During the past two decades optimal plastic design of engineering structures subjected to extreme dynamic loading, such as earthquake or explosion, attracted a lot of attention among the engineering and scientific community [17, 28]. Many methods have been proposed for optimal plastic design of structures. Some of them based on gradient methods such as sequential quadratic programming and other on biologically inspired optimization such as genetic algorithms.

One has to remember that contrary to static optimization, dealing with optimal design of structures subjected to dynamic loading, one needs to either solve transient response of the structures under consideration or transform dynamic loading into equivalent quasi-static ones [29]. Fortunately, it turns out that the appropriate formulation of optimal design of dynamically loaded structures i.e. the formulation which utilizes modal approximation, allows to find optimal solution very effectively. Modal approach frequently used in linear structural dynamics proved its effectiveness, which was shown in [30] on the example of damage detection in steel structures. Moreover, researchers looked for a similar modal approach in structural dynamics of plastic structures. Examples of such attempts are papers by Kaliszky and Wierzbicki [13, 31]. In the present paper the modal approximation is used to find minimal mass of the structure subjected to impact loading.

The paper is organized as follows: in the next section the theoretical background required to formulate modal approximation based optimal design is recalled. The third section describes details of an iterative algorithm for solving a problem of optimal design of dynamically loaded plastic structures. Then, a numerical example of a planar truss structure showing the effectiveness of the proposed method is presented. Finally, the paper is concluded with an enumeration of the most important aspects of the work.

2 The problem under consideration

In the present paper we are looking for an optimal set of design variables, which minimize total weight of the structure. It is assumed that the structure is subjected to a short-time dynamic loading in the form of an impact caused by a mass M_0 with initial velocity v_0 (Fig. 1). The structure is made of elastic-plastic material, which elastic deformation is negligibly small. However, after exceeding yield limit deformation become significant and permanent. The primary objective of the designed structure is to cease the motion of the impacting mass within a given constraint on maximal allowable permanent plastic displacements.

Derivation of the method we start with recalling the modified [25] classical equations of motion of the structure

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{G}\mathbf{Q}(t) = \mathbf{f}(t) \tag{1}$$

where $\mathbf{M}$ is mass matrix of the system, $\ddot{\mathbf{q}}(t)$ is acceleration vector, $\mathbf{G}$ is kinematic matrix, $\mathbf{Q}(t)$ is internal force vector and finally $\mathbf{f}(t)$ is vector of external forces. In our method we assume that the structure's inertial properties can be described by diagonal mass matrix $\mathbf{M} = diag\{m_i\}$, $i = 1,2,...,n$, here n denotes the number of nodes. Additionally, there are no external forces applied to the system except those coming from impacting mass. However, these forces are included into system's dynamics by applying appropriate initial conditions i.e. displacements and velocities. Initial velocities are obtained by applying principle of conservation of linear momentum

$$M_0 v_0 = M_0 v_A + \sum_{i=1}^{n} m_i v_i \tag{2}$$

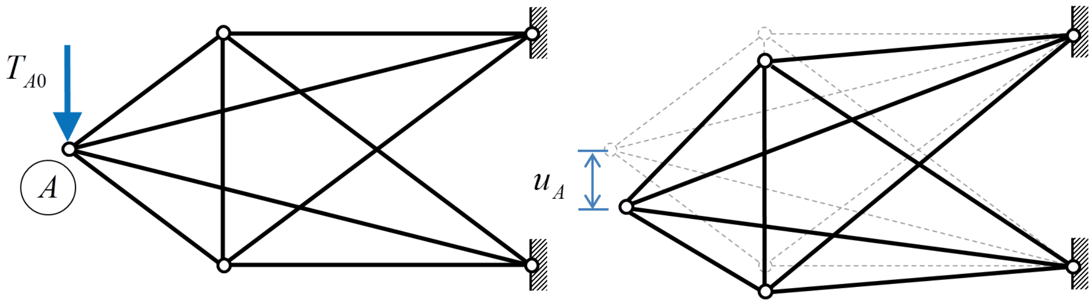

Fig. 2 Load capacity and static failure mechanism as an assumed mode of dynamic deformation

Next, we assume that the velocity field is proportional to the applied failure mechanism $v_i = \beta_i v_A$. This gives as the formula for velocity of structural nodes just after impact occurred

$$v_i = \frac{M_0 \beta_i}{M_0 + \sum_{j=1}^{n} \beta_j m_j} v_0 \tag{3}$$

Additionally, we assume that after impact all nodes of the structure are moving with constant acceleration $\ddot{q}_i = a_i = const$, which in turn allows to write the formula for time history of velocities in the following linear form

$$v_i(t) = a_i t + v_i \tag{4}$$

From the above equation we can determine time required for final velocity to be zero. Eventually, we can determine constant acceleration at structural nodes, which in turn allows us to represent the inertial forces acting on the structure in an explicit form

$$D_i = \frac{v_0^2}{2u_A} \left(\frac{M_0}{M_0 + \sum_{j=1}^{n} \beta_j m_j} \right)^2 \tilde{m}_i \tag{5}$$

where $\tilde{m}_i = M_0 + m_i$ for impacted point, and $\tilde{m}_i = m_i$ elsewhere.

In summary, the above optimal design problem can be formulated as follows

Minimize

$$M_s = \rho \sum_{e=1}^{ne} l_e A_e \tag{6}$$

subject to:

- Stationary equilibrium equations

$$\mathbf{GQ} + \mathbf{D}(A_e) = \mathbf{0} \tag{7}$$

- Constraints on allowable stress level

$$-A_e \sigma_0 \le Q_e \le A_e \sigma_0 \tag{8}$$

- Constraint on permanent plastic displacement in selected point A

$$T_{A0} u_A \le \frac{1}{2} \sum_{i=1}^{n} \tilde{m}_i v_i^2 \tag{9}$$

- Bound constraints

$$A_{\min} \le A_e \le A_{\max} \tag{10}$$

The inequality (9) is a consequence of principle of virtual velocity and it states the value of permanent plastic deformation is bounded by kinetic energy of the structure [32].

3 Iterative algorithm for optimal design of plastic structures based on modal approximation

In the previous section we have formulated the problem of optimal design of dynamically loaded elasto-plastic structures using mathematical programming. In order to determine optimal mass distribution for a given structure one can use two step optimization proposed by Logo [21]. However, in this paper we solve the problem in iterative way where instead of strict calculation of gradients of objective functions and constraints, we determine new cross section areas based on stress intensity [33]. It means that in structural members with smallest stress intensity there is an redundant material, which can be removed. In the next iteration of optimization process we check constraints imposed on displacement and allowable stress if these constraints are not violated we continue the process of removing redundant material until we encounter limiting value in either of these constraints. The whole process described above is presented in the form of flow chart in Figure 3.

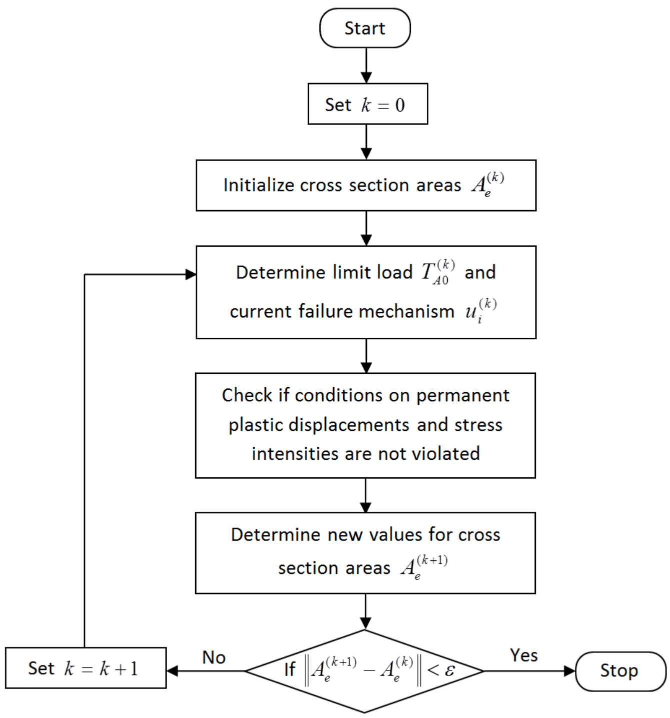

Fig. 3 Flow chart of the proposed algorithm.

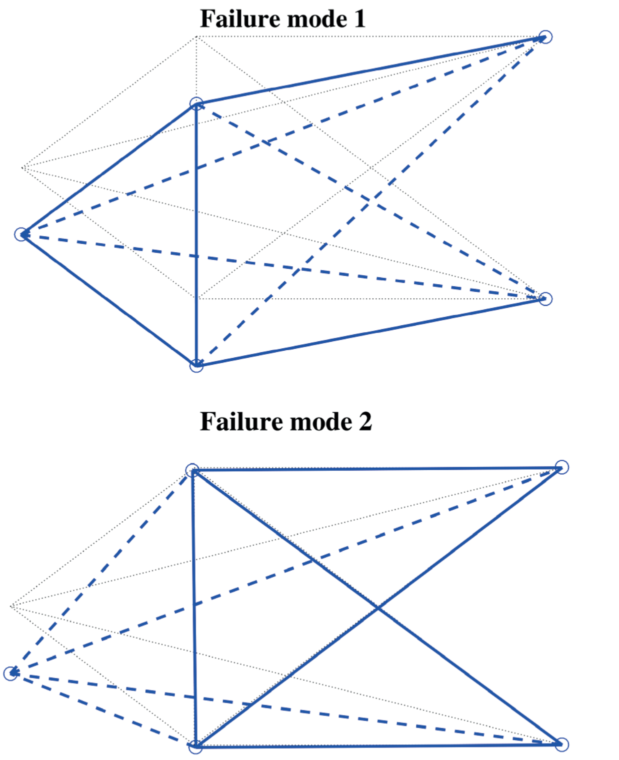

Fig. 4 Kinematically admissible modes of plastic deformation (dashed lines represent fully stressed members, solid lines indicate members within elastic range).

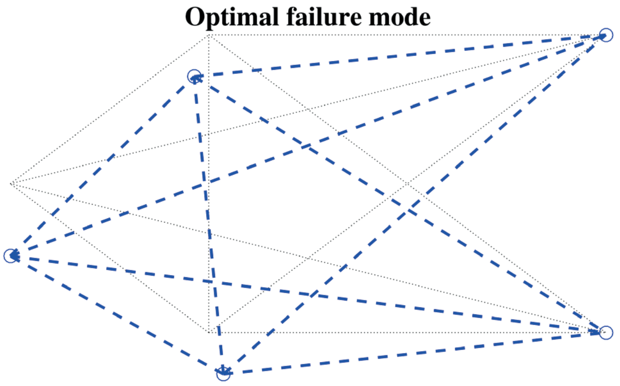

Fig. 5 Optimal mode of plastic deformation

In the Figure 4, two kinematically admissible modes of plastic deformation [34] are presented. Structural members plotted using dashed lines denotes fully stressed bars in which yield limit has been achieved. Solid lines indicate the structural members which operates below yield limit. The ultimate goal of the presented above optimization procedure is to determine cross section areas for which all members will undergo plastic deformation (Figure 5). This form of deformation will lead to fully stressed design treated here as an optimal one.

4 Numerical example - planar truss structure

As an illustrative example of the presented methodology a truss composed of 9 prismatic bars having rectangular cross section areas has been selected (Fig. 6). The truss is made of elastic-perfectly plastic material with Young modulus E = 210 GPa, yield limit σ_0 = 200 MPa and density ρ = 7850 kg/m^3. Mass M_0 = 1100 kg with initial velocity v_0 = 4 m/s hits the structure at node no.2 (Fig. 6).

Design variables in this example are cross section areas of all nine structural members.

Optimal solutions for three different values of allowable permanent plastic displacements have been shown in Fig. 8. The values of structural mass as a function of these displacement have been presented in Fig. 7 and Table 1. Stress intensities for optimal design have been shown in Table 2. Looking at Fig. 7 we can conclude that in presented examples the optimal mass distribution is inversely proportional to chosen permanent displacement, which can be summarized with the following statement: the smaller allowable plastic deformation the highest mass of the structure.

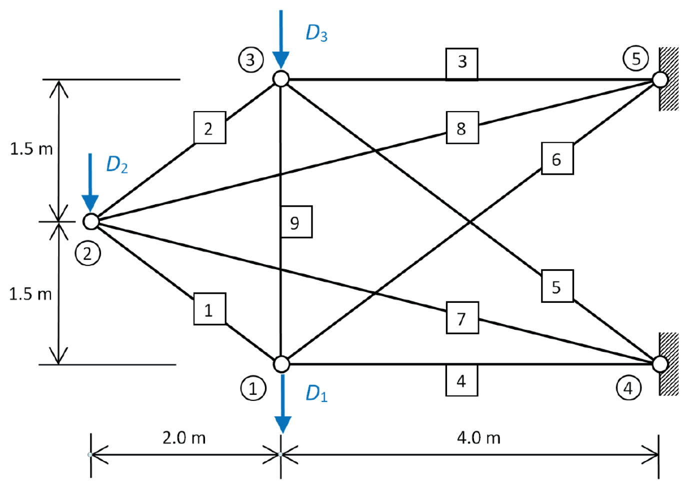

Fig. 6 Planar 9-bar truss structure

Table 1 Optimal structural mass and cross section areas for different allowable displacements

Allowable displacement u_A[m]	Optimal cross section areas A_e [cm^2] A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	Optimal structural mass M_s[kg]
0.1	2.233	2.233	3.332	3.332	2.233	2.233	3.665	3.665	0.033	82.9
0.2	1.132	1.132	1.689	1.689	1.132	1.132	1.858	1.858	0.017	42.0
0.3	0.757	0.757	1.130	1.130	0.757	0.757	1.244	1.244	0.011	28.1
0.4	0.571	0.571	0.852	0.852	0.571	.0571	0.937	0.937	0.009	21.2
0.5	0.457	0.457	0.681	0.681	0.457	0.457	0.750	0.750	0.007	17.0

Table 2 Stress intensities for optimal mode of plastic deformation

Bar no.	1	2	3	4	5	6	7	8	9
Stress intensity for optimal mode $Q_e/A_e\sigma_0$	93.5	-93.5	-100.0	100.0	95.5	-95.5	100.0	-100.0	99.9

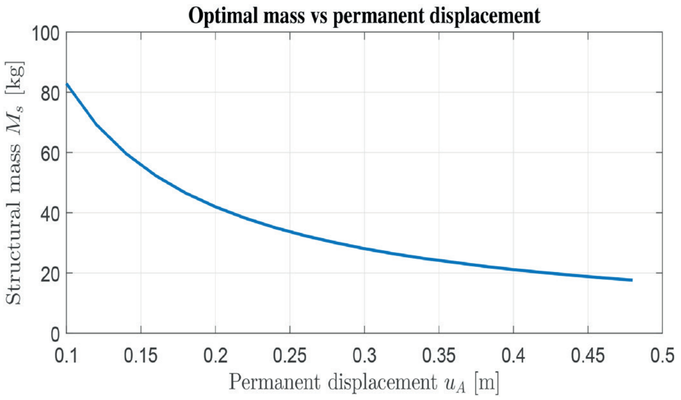

Fig. 7 Optimal solution as a function of allowable permanent plastic displacement.

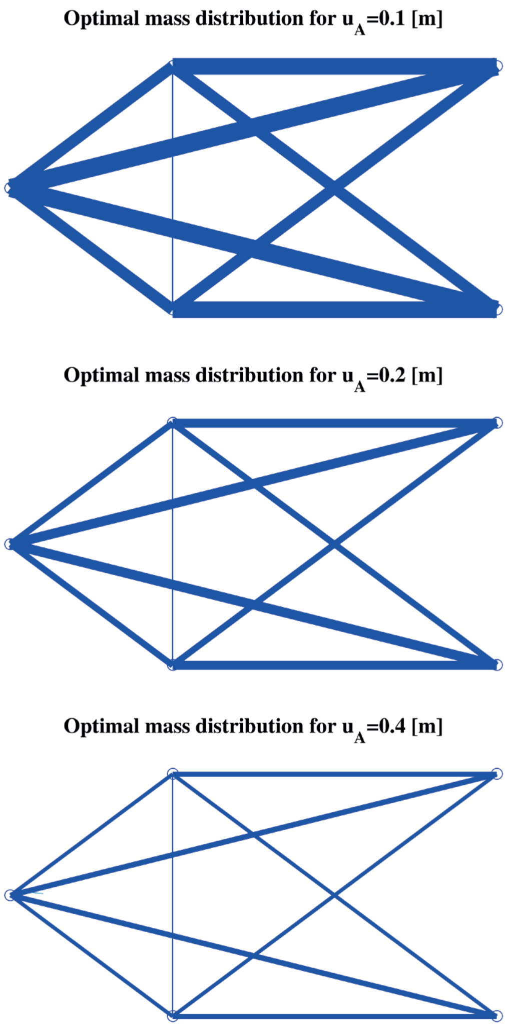

Fig. 8 Optimal mass distribution for different values of permanent plastic displacements.

5 Conclusions

The present study proposes an effective method for finding an optimal solution of a dynamically loaded plastic structures. The optimized structure can be subjected to sudden impact caused by a mass drop. Initial velocity of the impacting mass causes plastic deformation of the structure and the goal of optimization is to find the lightest structure satisfying requirements related to allowable stresses and permanent plastic displacements. Inertia forces caused by constant acceleration allow to avoid numerical time integration of the equations of motion and the whole problem can be treated as a quasi-static. Additionally, based on the presented numerical example one can conclude that after determining optimal mass distribution is needed only once for a single allowable permanent displacement. The remaining optimal solutions for other values of allowable displacements are scaled in such a way that ratios between individual cross section areas remain constant, while the overall structural mass is inversely proportional to applied permanent displacement.

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