Periodica Polytechnica
Civil Engineering

62(4), pp. 1001–1006, 2018
https://doi.org/10.3311/PPci.11257

RESEARCH ARTICLE

A Semi-Analytical Equation to Estimate Hydraulic Jump Length

Seyed Asadollah Mohseni Movahed[1*], Javad Mozaffari[1], Davood Davoodmaghami[1], Mahmood Akbari[2]

Received 13 July 2017; Revised 27 March 2018; Accepted 09 May 2018

Abstract

Empirical methods for determining the length of hydraulic jump, including USBR graphical method are not accurate. In addition to reduced errors, providing an analytical equation will result in the possibility of programming and development of hydraulic software application. In this research, based on the principles of dimensional analysis, an analytical equation was achieved for estimating the length of hydraulic jump of E and classical type. Dimensionless coefficient of this equation was determined as a function of slope and Froude number using Table Curve software. For this purpose, 387 data series (258 calibration data and 129 validation data) of experiments conducted in this research, USBR studies, results of a research in the laboratory of Ohio University and in an appropriate range of the flume width, discharge, Froude number, the velocity before jump as well as the slope were used. At calibration stage, a simple equation was provided that indicates the hydraulic jump length in values L_j/y_1 *depends on the slope of the flume and* Fr_2 *with determination coefficient of 94% and standard error less than 8%. Also at validation stage, the observed data and computed values was close to identity line with a determination coefficient of 0.97. Presenting the hydraulic jump length in values* L_j/y_1 *versus* Fr_1 *(which is very similar to USBR graphical method) was not satisfactory even with zoning* Fr_1*.*

Keywords

dimensional analysis, classical hydraulic jump, hydraulic jump of E type, hydraulic jump length

1 Department of Water Sciences & Engineering
Faculty of Agriculture & Natural Resources,
Arak University, Arak, 38156-8-8349, Iran
2 Ph.D. Graduated, Department of Water Engineering,
Isfahan University of Technology, Isfahan, Iran
* Corresponding author, email: movahed244@yahoo.com

1 Introduction

Hydraulic jump is one of the types of rapidly varied flows occurring when the flow changes from supercritical to subcritical state in a part of its path based on channel conditions. Hydraulic structures for energy dissipation and downstream stilling basins are considered to eliminate flow energy, to control hydraulic jump, and to create conditions for its occurrence in a specific location. For designing the above mention structures, it is needed to know the hydraulic jump length [1].

Air-water mixture on the water surface which transfers to downstream is considered as the feature of this phenomenon. Based on the quantity of upstream Froude number, the jump is classified into a variety of undulating, weak, oscillating, steady and strong jump [1]. In the sloping bed, hydraulic jump is classified to the types A, B, C, D, E and F. In hydraulic jump of E type the bed slope is constant but in other types is variable. In study of the hydraulic jump, conjugate depths, jump length and energy dissipation are more important. The most economical mode of stilling basins is created when downstream depth and jump length have the least and energy dissipation has the most value in jumps. Regarding the classical hydraulic jump length, significant studies was performed by the United States Department of Interior Bureau of Reclamation (USBR) [2], Hager & Bremen [3] and Hager [4]. The length and location of hydraulic jump was investigated by Chen-Fen Li in a sloping bed [5].

Furthermore, Pagliara et al.[6], Chanson [7], Alikhani et al. [8], Nasr Esfahani and Shafai Bajestan [9], Imran and Akib [10] and Riazi & Jafari [11] have conducted studies in recent years on the hydraulic jump using dimensional analysis. These studies were about effect of drop and barrier height, divergence angle of stilling basins, rough and corrugated bed on jump specifications and energy dissipated in this phenomenon.

Abdelhaleem et al. [12] and Ahmed et al. [13] investigated the various forms of corrugated beds to reduce downstream depth and jump length. In these researches, since the number of the desired parameters is high, a simple relationship has not acquired and sometimes, the results are provided graphically. In addition, these researches have been done in a horizontal bed and in a limited range of discharge and Froude number.

Kumar and Lodhi [14] investigated the effect of bed roughness heights on characteristics of a hydraulic jump in sloping channels. They found that the bed roughness height in sloping channels has no considerable effect on the hydraulic jump characteristics but, the channel slope significantly affected its characteristics.

In the previous researches, the experimental results have been used to provide an empirical relationship and or a physical relationship which is complex. In those studies the hydraulic jump length is calculated in a specific range of Froude number, or by providing graphs, the researchers try to describe this parameter. However, a relatively simple and general formula has not been provided yet to calculate the hydraulic jump length in both sloped and horizontal bed resulting from a wide range of Froude number.

The main objective of the research is to provide a general and simpler analytical-empirical relationship that can be used to calculate the E-type and classical hydraulic jump length in a wide range of Froude number. Comparing the results with USBR graphical method showed that the proposed equation in this research is more accurate.

2 Materials and methods

Effective parameters on hydraulic jump length in dimensional analysis:

Prior to any dimensional analysis, the factors that affect the desired physical phenomenon should be identified. In this section, using momentum equation which is a physical relationship, the parameters affecting the hydraulic jump length will be determined by argument. For this purpose, in Figure 1, which shows the E-type hydraulic jump and is turned into the classical jump with zero angle θ, an equation can be written as follows:

$$F_1 + W\sin\theta - F_2 - F_D - F_f = \rho Q(v_2 - v_1) \qquad (1)$$

Where F_1 and F_2 as hydrostatic pressure force on the sides of control volume between sections 1 and 2, $W\sin\theta$ as weight component of control volume in flow direction, F_f as friction force and F_D as drag force due to the barriers in the flow path or deformation of channel section, are the effective forces.

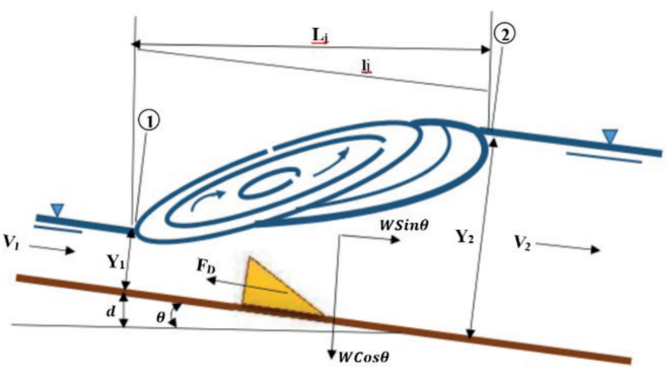

Fig. 1 Schematic figure of *E*-type hydraulic jump

Assuming a rectangular cross-section with a width B and without barrier in the flow path, equation 2 is written as follows:

$$\frac{\gamma y_1^2}{2}B + W\sin\theta - \frac{\gamma y_2^2}{2}B - \tau_0 l_j \bar{P} = \rho B y_1 v_1 (v_2 - v_1) \qquad (2)$$

Where y_1 and y_2 as the depths perpendicular to flow direction, $\bar{P}$ as average wetted perimeter perpendicular to flow direction, l_j as the jump length parallel to the channel bed and τ_0 as average shear stress between the two sections 1 and 2 are considered. If instead of γ, its equivalent (ρg) and instead of average shear stress, its equivalent ($a\rho v^2$) is placed, equation 3 will obtain as:

$$\begin{aligned} &\rho g \frac{y_1^2}{2}B + kk'\rho g l_j\left(\frac{y_1+y_2}{2}\right)B\sin\theta - \rho g\frac{y_2^2}{2}B \\ &-kk'a\rho\left(\frac{v_1+v_2}{2}\right)^2\left(2\frac{y_1+y_2}{2}+B\right)l_j = \rho B y_1 v_1 (v_2 - v_1) \end{aligned} \qquad (3)$$

In the above equation, in terms of the curvature of water surface between the two sections 1 and 2 and for calculation of the control volume at this distance, correction factor k is taken into account. The correction factor k' is also considered in order to take into account the effect of air mixing and the reduced water volume at this distance. On the other hand, with the constant discharge in the fixed slope, the flow depth y is a function of the flow width B. So, in the fourth term of relation 3, y_1 and y_2 can be replaced by MB and NB respectively.

Finally, after sorting and simplifying the relation 3, l_j can be written as the following form:

$$l_j = \frac{2y_1v_1(v_2-v_1) - gy_1^2 + gy_2^2}{kk'g(y_1+y_2)\sin\theta - \frac{1}{2}kk'a(v_1+v_2)^2(M+N+1)} \qquad (4)$$

Although in the above equation the values of parameters a, k, k', M and N are unknown but the significance of this equation is that shows the factors affect the jump length. In equation 4, dimensionless parameter α mainly depends on bed roughness k_s. Both dimensionless parameters k and k' mainly depend on viscosity and water flow velocity or actually depend on Reynolds number. Dimensionless parameters M and N depend on the flow depth and velocity in the sections 1 and 2. As shown in Figure 1, $\sin\theta$ is equal to d divided by l_j while d is the height difference of canal bed in sections 1 and 2 as well as l_j is hydraulic jump length in direction of channel slope.

Regarding components of relation 4 and dependence of the parameters a, k, k', M, and N on the roughness, Reynolds number, depth and velocity, it can be written as follows:

$$f(v_1, y_1, v_2, y_2, d, g, l_j, k_s, R_e) = 0 \qquad (5)$$

If L_j is in the desired horizontal direction, accordingly L_j is equal to $l_j\cos\theta$, so L_j will be another function of the same physical parameters, so that it can be written as follows:

$$F(v_1, y_1, v_2, y_2, d, g, L_j, k_s, R_e) = 0 \quad (6)$$

In functions 5 and 6, assuming a smooth bed, the effect of roughness k_s can be ignored. In open channel flows, when Reynolds number is greater than 2000, the effect of it can be avoided [3]. In this research, the effect of Reynolds number is neglected because it was between 4938 and 575280 in all experiments; therefore, functions 5 and 6 have become simpler, and they can be written as follows:

$$f(v_1, y_1, v_2, y_2, d, g, l_j) = 0 \quad (7)$$

$$F(v_1, y_1, v_2, y_2, d, g, L_j) = 0 \quad (8)$$

Dimensional analysis through Buckingham theorem:

Relation 8 is considered for dimensional analysis. Using continuity equation, the relation also becomes simpler as follows:

$$v_1 y_1 = v_2 y_2 \Rightarrow v_1 = \frac{v_2 y_2}{y_1} \quad (9)$$

Therefore, by reducing the characteristic parameters, the function 8 becomes still simpler as follows:

$$c(y_1, v_2, y_2, d, g, L_j) = 0 \quad (10)$$

Without removing the variables, by dividing identical dimension variables, two dimensionless variables can be created easily:

$$d\left(\frac{L_j}{d}, \frac{L_j}{y_1}, v_2, y_2, g\right) = 0 \quad (11)$$

In the above function, only two main dimensions T and L are exist; therefore, the number of other dimensionless groups which should be formed are equal to the difference between the number of non-identical dimension variables (g, y_2, v_2) and the number of main dimensions. Hence, another dimensionless factor should be formed. Among the variables (g, y_2, v_2), variables v_2 and y_2 are selected as iterative variables and dimensionless factors are formed as follows:

$$\pi_1 = \frac{L_j}{d} \quad , \quad \pi_2 = \frac{L_j}{y_1}$$

$$\pi_3 = v_2{}^m . \ y_2{}^n . \ g = \left(LT^{-1}\right)^m (L)^n \left(LT^{-2}\right) = L^0 T^0$$

Where in the above relation, n and m are unknown powers of iterative variables which by solving the system of equations will obtained:

$$\left(L^{m+n+1}\right)\left(T^{-m-2}\right) = L^0 T^0 \Rightarrow \begin{Bmatrix} m+n+1=0 \\ -m-2=0 \end{Bmatrix} \Rightarrow \begin{Bmatrix} n=1 \\ m=-2 \end{Bmatrix}$$

Therefore, by determining unknown parameters, the third dimensionless factor will be equal to:

$$\pi_3 = v_2{}^{-2} . y_2 . g \Rightarrow \pi_3 = \frac{g y_2}{v_2{}^2}$$

According to Buckingham theorem [15]:

$$e(\pi_1, \pi_2, \pi_3) = 0 \Rightarrow e\left(\frac{L_j}{d}, \frac{L_j}{y_1}, \frac{g y_2}{v_2{}^2}\right) = 0$$
$$\Rightarrow \frac{L_j}{y_1} = f\left(\frac{L_j}{d}, \frac{g y_2}{v_2{}^2}\right) \quad (12)$$

By reversing the variables in parentheses, Equation 12 can be written as follows:

$$\frac{L_j}{y_1} = g\left(\frac{d}{L_j}, \frac{v_2{}^2}{g y_2}\right) = g\left(tan\theta, Fr_2^2\right) = C_{1j} \quad (13)$$

$$L_j = C_{1j} . y_1 \quad (14)$$

As it is clear in the above relation, the hydraulic jump length is a function of the square of Froude number, bed slope and water depth at the upstream of the jump in a section perpendicular to flow direction. Relation 14 is the initial form to determine the hydraulic jump length as a function of water depth before the jump. The next step for completing the above relation is to calculate the coefficient C_{1j} achieved from the equality of relation 13:

$$C_{1j} = g\left(tan\theta, Fr_2^2\right) = \frac{L_j}{y_1} \quad (15)$$

$$L_j = C_{2j} . y_2 \quad (16)$$

$$C_{2j} = m\left(tan\theta, Fr_1^2\right) = \frac{L_j}{y_2} \quad (17)$$

Determining dimensionless C coefficient using experimental data:

First, C_{1j} or C_{2j} values are obtained at the calibrated stage using experimental data and relations 15 and 17, by dividing the jump length into depth. Then, these values are modeled with the corresponding values of Froude number and channel bottom slope using multiple non-linear regression in three-dimensional version of Table Curve software so that the best fitted equation is obtained. Finally, by placing C values in the above equations, final equations are obtained to calculate the jump length. Then, the status of observed and calculated values of these coefficients will be taken into account on the identity line at the validation stage. For hydraulic jump in horizontal bed, the slope can be considered equal to zero in the final equation.

Fig. 2 Laboratory flumes of Arak University

Laboratory data used:

Some part of the data used in this research has obtained from an investigation carried out in the hydraulic laboratory of Ohio University [5], some part from the USBR investigations [2] and other part from the experiments in two hydraulic laboratory flumes in departments of water engineering and civil engineering of Arak University have been obtained. These two laboratory flumes with 20 and 50 centimeters wide as well as adjustable slop are shown in Figure 2. These widths (20&50 Cm.) not used between the data of USBR and Ohio laboratory. Increasing range of the data used can raise the validity of calibration and validation stages.

In the experiments conducted in this study using the two above-mentioned flumes, $tan(\theta)$ was measured between zero to 0.1509, discharge was measured between 7 to 63.36 cubic meters per hour and Fr_1 was calculated between 1.231 to 9.19. Finally, from the data of the United States Department of Interior Bureau of Reclamation [2], the results of Chen Fen research [5] in the sloped flume of the hydraulic laboratory of Ohio University and flumes of the hydraulic laboratories of Arak university in Iran, a total of 387 data sets were extracted. The data are sorted as follows: the horizontal flume data of USBR (120 data), the sloped flume data of USBR (94 data), the results of Chen Fen research in a sloped flume (88 data), and the hydraulic laboratory flumes of Arak University (85 data). In all of the data used, dip to 0.263 and Fr_1 to 19.61 have been increased. It should be noted that if the dimensions of a model is less than an extent, the effect of surface tension force will be effective. Research has shown that if the flow width in the model is more than 10 cm, and if the velocity on the water surface to be more than 230 mm/Sec and if surface wavelengths are more than 17 mm, the effect of surface tension and capillary force can be avoided [2]. In this research, all the above appropriate conditions have been established.

First series of experiments:

In this case, in every step with constant slope, the hydraulic jump conditions were taken into consideration in different discharges. The experiments were conducted in eight different slopes and in each slope, with a maximum of five discharges. In total, given that in some cases, the hydraulic jump was not created, about 33 modes were tested with regard to multiplication principle pattern. In all test period, the gate opening at the upstream and downstream of the hydraulic jump remained constant equal to 15 and 46 mm respectively.

Second series of experiments:

In this series of experiments, in every step with constant upstream gate opening, the jump conditions were investigated in different discharges in a determined slope; then, with the same opening, the slope was changed and repeated again in different discharges of the test. After all the slopes were tested, the gate opening was changed and the above process was repeated. In total, 60 cases were tested and the jump was created in 52 cases.

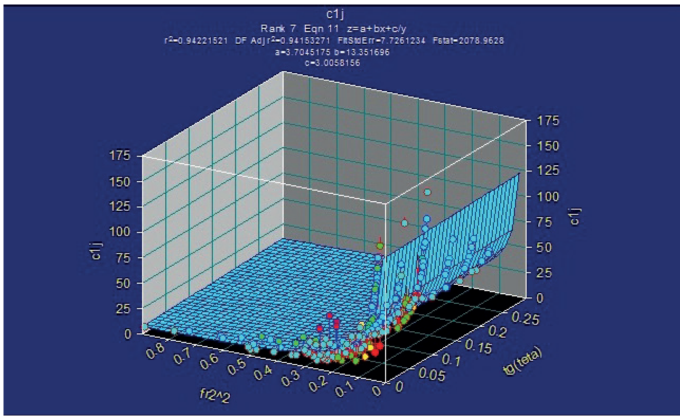

Fig. 3 C_{1j} coefficient versus Fr_2^2 and $\tan(\theta)$

Data allocation for calibration and validation stages:

Among the 387 data series collected, data allocation was made to the calibration and validation stages with the ratio of two to one. Thus, the two first data was allocated to calibration, the third data to validation, the fourth and fifth data to calibration, the sixth data to validation and so forth. Its advantage is to use all the data series in both stages, and consequently, lack of bias in equations; therefore, 258 data were used for calibration and 129 data for validation.

The selected data were processed by Table Curve software for the calibration stage. At the calibration stage, the three-dimensional version of software was used to determine surface regression equations; therefore, considering the channel bed slope as the x-axis values, square of Froude number as the y-axis and the measured values of C as the z-axis values, the best fitted curve equation (with regard to its statistical parameters) was determined.

3 Results and discussion

Determining dimensionless coefficient of equations:

In Figure 3 the best surface fitted on $C1_j$ coefficient versus Fr_2^2 and $tan(\theta)$ is shown.

The selective equation for fitted surface is as follows:

$$C_{1j} = 3.7 + 13.35\tan(\theta) + \frac{3}{Fr_2^2} \tag{18}$$

As it can be seen, the fitted surface is very accurate and the determination coefficient which provided by software is more than 94% and the standard error is less than 8%. Due to high correlation of the equation, the fitted surface is conform to data points as well. By placing the equation 18 instead of C1j value in the equation 14, the hydraulic jump length can be achieved. So, the final equation of the hydraulic jump length is as follows:

$$L_j = \left(3.7 + 13.35\tan(\theta) + \frac{3}{Fr_2^2}\right)y_1 \tag{19}$$

Equation 19 shows the hydraulic jump length is a function of the slope and hydraulic conditions before and after the jump. At the validation stage, using 129 chosen data, the observed

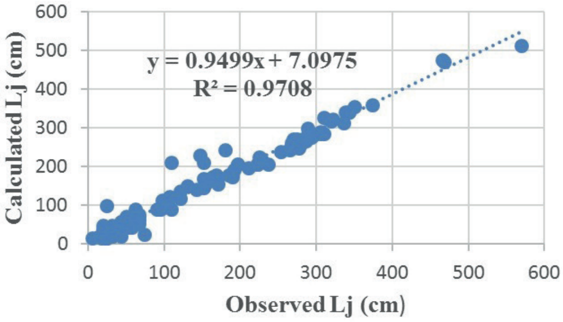

Fig. 4 Comparison of computed and observed values of hydraulic jump length

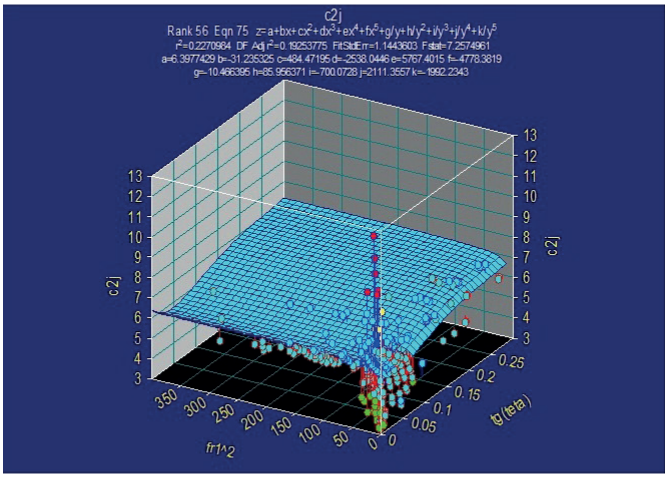

Fig. 5 C_{2j} coefficient versus Fr_1^2 and $\tan(\theta)$

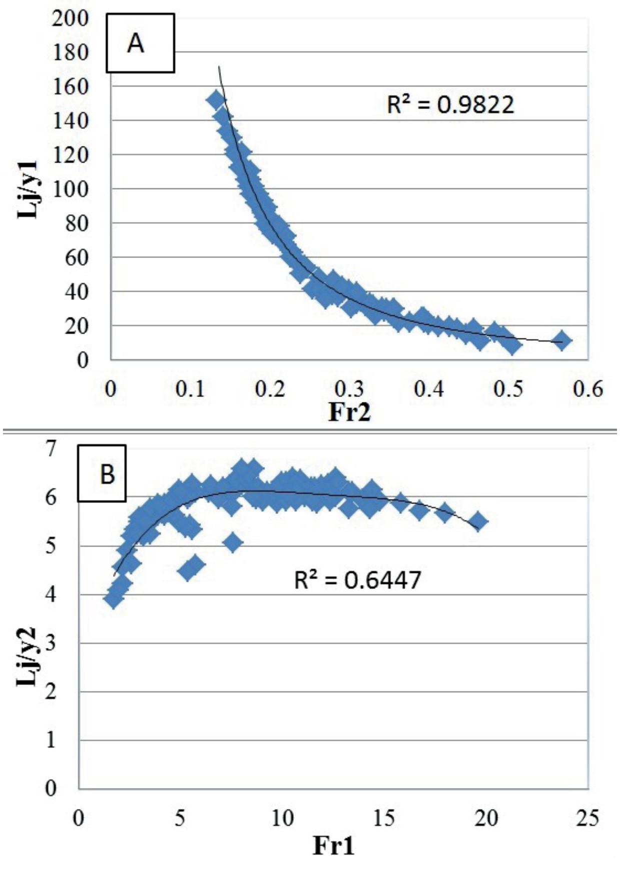

Fig. 6 Hydraulic jump length based on A) proposed method and B) USBR graph

hydraulic jump length was put in the horizontal axis, and the jump length computed from equation 19 was transferred to the vertical axis. Figure (4) shows the results.

From Figure 4, it is quite clear the results with determination coefficient of 97% is very close to the identity line ($y = x$). In Figure 5 the best surface fitted on C_{2j} coefficient versus Fr_1^2 and $tan(\theta)$ is shown.

According to figure 5 the determination coefficient is about 22% and standard error is about 1%. Because of achieved complex equation and poor correlation coefficient, the relevant procedure is not so accurate. Due to inefficiency of the calibration stage equation, validation is not required. To improving complexity of equation and poor correlation, zoning the upstream Froude number was examined. So, the Froude number range was divided into five zones and an equation was obtained for each zone. The determination coefficient of equations was between 19 and 70 percent, and equations were very complicated. So, even with zoning of Froude number, no good results achieved.

It should be noted the USBR graph is $\frac{L_j}{y_2}$ versus Fr_1, but our equation indicates $\frac{L_j}{y_1}$ versus Fr_2. The figure 6-A is based on the proposed method in this research and the figure 6-B is like USBR graph. According to figure 6 using 120 data series, the hydraulic jump length in the form of $\frac{L_j}{y_1}$ versus Fr_2 will have better result than $\frac{L_j}{y_2}$ versus Fr_1 (USBR graph [2]).

4 Conclusions

In this paper, estimating the hydraulic jump length of E and classical type was investigated. The effective variables on hydraulic jump length are derived from momentum equation. Using the outcomes of USBR studies, the empirical results in the laboratory flume of Ohio University and experimental results in this study, based on dimensional analyses, a semi-analytical equation to calculate the hydraulic jump length, with very good accuracy was obtained. In the calibration step, the coefficient of determination was 94% and the standard error was less than 8%. In the validation step, the computational and observational data points were close to identity line with a determination coefficient of 97%. These results indicate the accuracy and reliability of the proposed equation. The equation can be used in modeling software for design of hydraulic structures, prediction of hydraulic jump length in stilling basins after gates as well as in downstream diversion dams and spillways. Also, the results show expressing the hydraulic jump length in values $\frac{L_j}{y_2}$ versus Fr_1 (which is very similar to USBR graph) was not satisfactory even with zoning Fr_1.

Acknowledgement

The project presented in this article is supported by Vice President for Research and Technology of Arak University.

References

[1] Jain, S. C. "*Open-channel flow*". John Wiley & Sons. 2001.

[2] USBR. "*Hydraulic Laboratory Techniques: A Water Resources Technical Publication*". United States Department of the Interior Bureau of Reclamation. Denever, Colorado, Reprinted. 1986. https://www.usbr.gov/tsc/techreferences/mands/mands-pdfs/HydraulicLabTech.pdf

[3] Hager W. H., Bremen R. "Classical hydraulic jump; sequent depths". *Journal of Hydraulic Research*, 27(5), pp. 565–585.1989. https://doi.org/10.1080/00221688909499111

[4] Hager, W. H. "*Energy Dissipaters and the Hydraulic Jump*". Kluwer Academic Publishers, Dordrecht. 1992.

[5] Li, C-F. "Determining the location of hydraulic jump by model test and Hec-2 flow routing". MSc, Electronic Thesis or Dissertation, College of Engineering and Technology, Ohio University. p. 131.1995.

[6] Pagliara, S., Lotti I., Palermo, M. "Hydraulic jump on rough bed of stream rehabilitation structures". *Journal of Hydro-environment Research*, 2(1), pp. 29–38. 2008. https://doi.org/10.1016/j.jher.2008.06.001

[7] Chanson, H. "Current knowledge in hydraulic jumps and related phenomena. A survey of experimental results". *European Journal of Mechanics – B/Fluids,* 28(2), pp. 191–210. 2009. https://doi.org/10.1016/j.euromechflu.2008.06.004

[8] Alikhani, A., Behroozi-Rad, R., Fathi-Moghadam M. "Hydraulic jump in stilling basin with vertical end sill". *International Journal of Physical Sciences,* 5(1), pp. 25–29, 2010. http://www.academicjournals.org/journal/IJPS/article-abstract/67F603122160

[9] Esfahani, M. J. N., Bajestan, M. S. "Effect of Roughness Height on the Length of B-jump at an Abrupt Drop". *International Research Journal of Applied and Basic Sciences*, 3(S), pp. 2757–2762. 2012. http://www.irjabs.com/files_site/paperlist/r_584_121228132632.pdf

[10] Imran, H. M., Akib, S. "A Review of Hydraulic Jump Properties in Different Channel Bed Conditions". *Life Science Journal*, 10(2), pp.126–130. 2013. www.researchgate.net/publication/256475352_A_Review_of_Hydraulic_Jump_Properties_in_Different_Channel_Bed_Conditions

[11] Riazi, R., Jafari, S. "The characteristics of submerged hydraulic jump in sloped stilling basins with rough bed". *Bulletin of Environment, Pharmacology and Life Sciences*, 3(6), pp: 238–243. 2014. http://www.bepls.com/may_2014/38f.pdf

[12] Abdelhaleem, F. S., Amin, A. M., Esam, H. Y. "Effect of Corrugated Bed Shapes on Hydraulic Jump and Downstream Local Scour". *Journal of American Science*, 8(5), pp. 1–10. 2012. http://free-journal.umm.ac.id/journal-of-25-the-journal-of-american-science-year-2012.html

[13] Ahmed, H. M. A., Gendy, M. E., Mirdan, A. M. H., Ali, A. A. M., Haleem F. S. H. A. "Effect of corrugated beds on characteristics of submerged hydraulic jump". *Ain Shams Engineering Journal*, 5(4), 1033–1042. 2014. https://doi.org/10.1016/j.asej.2014.06.006

[14] Kumar, M., Lodhi, A. S. "Hydraulic jump over sloping rough floors". *ISH Journal of Hydraulic Engineering*, 22(2), pp. 127–134. 2016. https://doi.org/10.1080/09715010.2015.1088409

[15] Fox, M. "Fluid Mechanics". Wiley, London. pp. 296–305. 2014.