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# Discrete Size and Shape Optimization of Truss Structures Based on Job Search Inspired Strategy and Genetic Operations

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**Abstract**

A meta-heuristic algorithm for discrete size and shape optimization of trusses via a job search inspired strategy together with genetic operators of mutation, selection, and crossover is proposed. The alternation of movements with respect to objective function and load bearing capacity of constructive decisions is provided. Being introduced is an intermediate search goal connected in terms of posed limitations with heightened suitability levels of individuals meeting the current requirements for the initial objective function. As soon as these conditions allow achieving a structure type which meets task limitations, requirements for the function value are redefined. This technique does not demand penalty functions that provide strict control of limitations in any algorithm usage, greater stability of the results received, and finding better solutions. The efficiency of this approach in terms of solution accuracy is demonstrated through five benchmark design examples, in comparison with other methods of discrete truss structure optimization.

**Keywords**

trusses, size and shape optimization, meta-heuristic algorithms, job search inspired strategy

## 1 Introduction

Meta-heuristic algorithms are in widespread use for solving truss optimization problems. These computing methods are defined as derivative-free, robust, and efficient for global optimum searches. Such meta-heuristic approaches as Genetic Algorithms [1], Simulated Annealing [2], Particle Swarm Optimization [3], Harmony Search [4], and Ant Colony Optimization [5] have been successfully tested throughout truss structure optimization. The application of relatively new computing methods such as Big Bang-Big Crunch Algorithm [6], Imperialist Competitive Algorithm [7], Ray Optimization [8], Mine Blast Algorithm [9], Firefly Algorithm [10], Dolphin Echolocation [11], Teaching-Learning-Based Optimization [12], Chaotic Swarming of Particles [13], Bat-Inspired Algorithm [14], Colliding Bodies Optimization [15], Enhanced Colliding Bodies Algorithm [16], Search Group Algorithm [17], Water Evaporation Optimization [18], Vibrating Particles System Algorithm [19], and Cyclical Parthenogenesis Algorithm [20] should also be mentioned. Detailed information regarding the usage of meta-heuristic algorithms for these problems can be found in reviews [21, 22].

In meta-heuristic algorithms for optimal truss design, limitations are usually taken into account by penalty functions [22], which in many cases result in distortion of the problem statement and significant instability in the final results. It should be noted that one of the most suitable ways to solve the problem of reducing the negative effects of limitation control can be the complete cancellation of using penalty functions altogether. At the same time, a number of approaches considering the limitations of meta-heuristic algorithms without utilizing penalty functions by providing implementation of each limitation step-by-step [23], repairing infeasible individuals [24], searching the boundaries of a feasible region [25], homomorphous mapping [26], as well as solution classification on infeasible, semi-feasible, and feasible situations [27], are not versatile enough.

In this article, the problem of developing a meta-heuristic algorithm without penalty functions is solved for discrete minimization of truss weight according to a job search inspired (JSI) strategy recently proposed by the author [28] using genetic operators of mutation, selection,

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and crossover [29] for implementing its steps. The truss design is interpreted both as a vacancy of workplace and as an individual of the population. The process of minimization of truss system weight under the given limitations is consistent with the actions of a person searching for a job with the highest salary while meeting both his individual preferences and employer demands.

The general case provides variations in the cross-sectional areas of bars and the coordinates of individual nodes under limitations on bar stresses and stability, as well as on displacements of nodes. When forming the optimization procedure, we take into consideration that the computational costs for determining structure weight are negligible compared to the complex calculation of the stress-strain states of trusses. This approach involves a series of sequential searches of individuals satisfying the problem limitations on the basis of improving the degree of compliance with the limitations for design options. Each search takes into account strictly those individuals whose weight corresponds to the current minimal value of this quantity. As soon as it becomes possible to find an individual not in violation of any of the limitations, its weight is accepted as the minimum required for continuing with optimization. This individual is then also considered as a current optimization result.

The efficiency of the proposed algorithm is tested on the standard examples of discrete optimization according to the parameters of 10- and 200-bar plane trusses, 25-bar space truss and 354-bar braced dome and the optimization according to the parameters and shape of 18-bar truss. For all examples, the limitations on stresses have been taken into account. In addition, limitations on stiffness for 10-and 25-bar trusses, as well as limitations on rod stability for 18-bar truss are taken into consideration. The comparison of the obtained optimization results with the data from references has shown that the JSI strategy has a sufficiently high efficiency in terms of solution accuracy.

## 2 Statement of the problem

We consider the problem of minimizing the weight of plane and space trusses. In a general case, the search is carried out on discrete sets of cross-sectional areas of bars and the coordinates of nodes. Limitations on strength, stability of members, and node displacements can be taken into account. To calculate the stress-strain state of trusses, the finite element method is used according to a displacement approach. Each of the bars is represented by one finite element. Thus, the optimization problem may be posed as:

Minimize

$$W(\boldsymbol{A},\boldsymbol{T}) = \sum_{i=1}^{n} \gamma_i l_i A_i \tag{1}$$

subject in general case to stress, displacement a stability constraints. Here $W$ is the weight of all bars, $\boldsymbol{A} = \{A_1, ..., A_n\}^T$, $\boldsymbol{T} = \{T_1, ..., T_k\}^T$ are the numerical vectors of the cross-sectional areas and nodal coordinates, respectively, $n$ is the number of the bars, $k$ is the number of the coordinates, $\gamma_i$, $l_i$ are the material density and the length of the $i^{\text{th}}$ member.

The following constraint conditions are considered for non-specialized truss structures:

$$\Phi_{\sigma b} = \max_{\substack{i=1,...,n \\ j=1,...,J}} f_{ij} \le 1, \tag{2}$$

$$f_{ij} = \begin{cases} \dfrac{\sigma_{ij}}{\sigma_{ij}^t} & for\ \sigma_{ij} \le 0 \\ \dfrac{|\sigma_{ij}|}{\min\left(\sigma_{ij}^c, \sigma_i^b\right)} & for\ \sigma_{ij} < 0 \end{cases}, \tag{3}$$

$$\Phi_{\delta} = \max_{\substack{m=1,...,M \\ r=1,...,(2 or 3) \\ j=1,...,J}} \frac{|\delta_{mrj}|}{\delta_{mrj}^{\max}} \le 1, \tag{4}$$

where $\Phi_{\sigma b}$ is the value associated with meeting assumed limitations on stresses and stability for the truss, $f_{ij}$ is the value used to describe meeting limitations on stresses and stability for bar $i$ with loading $j$, $J$ is the number of loadings, $\sigma_{ij}$ is the stress of bar $i$ with loading $j$, $\sigma_{ij}^t$, $\sigma_{ij}^c$ are the allowable normal stresses if there is the strength of bar $i$ with loading $j$ in tension and compression, respectively, $\sigma_i^b = cA_iE/l_i^2$ is the buckling stress of the $i^{\text{th}}$ bar (Euler formula), $c$ is a shape constant, $E$ is the Young's modulus of the material, $\Phi_\delta$ is the value which describes meeting limitations on displacements of truss nodes, $|\delta_{mrj}|$, $\delta_{mrj}^{\max}$ are the displacement module of node $m$ with loading $j$ in the direction of the axis under number $r$ with consecutive numbering of $x$, $y$, and $z$ axes and the assumed value of this displacement, respectively, $M$ is the number of the nodes. During optimization only some of the truss parameters can vary. The bars can be combined into groups, in each of which a certain parameter receives the same value.

LRFD-AISC requirements [30] are taken into consideration for steel trusses. At the same time, the Eq. (2) is also used for stress constraints. However, the function $f_{ij}$ is represented in the form

$$f_{ij} = \max\left(\frac{\lambda_i}{\lambda_{\max}}, \frac{P_{uij}}{\phi P_{ni}}\right), \tag{5}$$

where $\lambda_i = k_i l_i / r_i$ is the slenderness ratio of $i^{th}$ member, $k_i$, $r_i$ are its effective length factor and radius of gyration, respectively ($k_i = 1$ for all truss members), $\lambda_{max}$ is the maximum allowed value of $\lambda_i$ (for tension $\lambda_{max} = 300$, for compression $\lambda_{max} = 200$), is the axial strength of bar $i$ with loading $j$, $\phi$ is the resistance factor (for tension $\phi = \phi_t = 0.9$, for compression $\phi = \phi_c = 0.85$), $P_{ni}$ is the nominal strength of member $i$.

The nominal strength for tensile $P_{ni} = F_y A_{gi}$, where $F_y$ is the specified yield stress and $A_{gi}$ is the gross area of member $i$. For compression $P_{ni} = F_{cr} A_{gi}$, where $F_{cr}$ is the critical stress computed depending on the value

$$\lambda_c = \frac{k_i l_i}{r_i \pi} \sqrt{\frac{F_y}{E}}. \tag{6}$$

For $\lambda_c \leq 1.5$

$$F_{cr} = 0.658^{\lambda_c^2} F_y, \tag{7}$$

for $\lambda_c > 1.5$

$$F_{cr} = \frac{0.877}{\lambda_c^2} F_y. \tag{8}$$

Displacement condition is represented as

$$\Phi_\delta = \max_{\substack{m=1,\dots,M \\ j=1,\dots,J}} \frac{\left|\delta_{mj}\right|}{\delta_{mj}^{\max}} \leq 1, \tag{9}$$

where $\left|\delta_{mj}\right|$, $\delta_{mj}^{\max}$ are the displacement module of node $m$ with loading $j$ in any direction and the assumed value of this displacement, respectively.

## 3 Interpreting job search as meta-heuristic procedure

Let an applicant set the task to find a job with the highest salary $F$ based on his preferences and abilities to meet the requirements specified for applicants for this vacancy. The set of vacancies $V$ represents a discrete set for the search. Let us determine the relatively rapid phases of the search $S_a$ (study of advertisements, CV distribution, phone calls, etc.) and those phases of the interview (possibly including an exam) $S_b$. We assume that within phases $S_a$, the applicant receives information on vacancies for value $F$, as well as on the implementation of his own conditions (or a percentage of them) and the requirements of employers (limitations of optimization $T_1$). Testing other limitations $T_2$ is carried out during the interview.

We define the set of vacancies $V_1 \subset V$, which satisfies limitations $T_1$. Let the applicant during the initial stage choose by way $n$ vacancies $v_i$ from the set $V_1 \subset V$ that meet the salary condition $F > F_A$, where $F_A$ is a set value, which in the future may change. Then for each cycle (iteration) of the JSI strategy, the following sequence of steps is provided:

Step 1: Random variations is performed to replace a part of vacancies $v_i$ with new vacancies matching requirement $v_i \in V_{1A}$.

Step 2: This group of vacancies is tested for satisfaction of condition $T_2$. If any vacancy $v_i$ meets these conditions, we assume $F_A = F_i$, where $F_i$ is the value of the salary for this vacancy.

Step 3: According to the results of interviews, the occupational fitness of an applicant relative to vacancies $v_i$ is estimated. On the basis of these results, the group of vacancies $v_i^c$ is chosen from set $V_{1A}$. This group should be close to vacancies $v_i$ for which occupational fitness will be the greatest.

Step 4: For vacancies $v_i^c$ step 2 is implemented.

The result of this search for the current iteration is an individual corresponding to the last found value $F_A$ during step 2.

## 4 Truss optimization algorithm

According to the task, to optimize trusses we assume that $F = 1/W$, a vacancy is a set of values of an individual parameter, limitations $T_1$ provide defining discrete sets of values of cross-sectional areas and node coordinates where the search is implemented, $T_2$ is a test to satisfy inequalities Eqs. (2), (4), and (9). The JSI strategy assumes usage of different approaches to carry out its steps. Let us form the algorithm based on this strategy and the technique of genetic algorithms [29].

We assume that a set of admissible values for each varied parameter is arranged in the order of their increasing. We will operate using the main population Π with the size $N_\Pi$ and auxiliary elite population Π, the size of which depends on the result of the iteration process but does not exceed value $N_\Psi$ (see [31]). Primarily we define $F_A = 0$, we form population Π from maximum values relative to all varied parameters, and we leave population Ψ empty at this stage. The steps of the JSI strategy are implemented during each iteration $s \geq 1$ in the following way:

Step 1: Mutation of individuals in population Π is implemented. If the iteration number exceeds some number $s_1$, then this procedure is performed for a randomly selected $n_1 = \max\left(1, \lfloor \lambda n_o \rfloor\right)$ parameter for each individual of the population, where $\lambda$ is the specified value on the segment ($0 \leq \lambda < 1$), $n_o$ is the total number of parameters. If $s \leq s_1$ then $n_1$ can be defined by a great number of multiplications $\lambda$ by the value $d\left(1 < d \le \lfloor n_o/n_1 \rfloor\right)$. For each parameter subject to changing, we choose value $p_a$ with the help of a random number generator on the segment (0, 1) with a uniform law of distribution, and then it is compared with the mutation control number $m_a (0 < m_a < 1)$. If $p_a > m_a$, any of the admissible parameter values is chosen randomly with equal probability, otherwise the number of the current parameter position in the set of its acceptable ones randomly changes into 1–2 units. A mutation operation can be performed for an individual many times, until condition $F_i \ge F_A$ is not satisfied.

Step 2:

2.a: Implementation of limitations $T_2$ for individuals of population Π is tested. For this purpose, we determine the value of the occupational fitness coefficient for each individual $i$ in terms of the JSI strategy:

$$k_p = \frac{1}{\max\left(\Phi_{\sigma b}, \Phi_\delta\right)}. \tag{10}$$

After achieving the condition $k_p \ge 1$ and $F_i > F_A$, the new value $F_A = F_i$ is set. It shall be noted that such an approach provides strict adherence to the limits of the problem.

2.b: To population Ψ we gradually add each individual $i$ of population Π, which has greater value $k_p$ than the worst individual in population Ψ, and this population lacks the gene pattern of $i$ individual. If the size of population Ψ equals $N_\Psi + 1$, then an individual with the minimum value of $k_p$ will be excluded from it.

2.c: The individuals of population Ψ are checked on implementing condition $F_i \ge F_A$. If this condition is not implemented, then the individual is excluded from the population. If value $F_A$ changed during stage 2a, then at this stage population Ψ can include only the individuals which satisfy condition $F_i = F_A$.

Step 3: The operation of selection and single-point crossover is performed. Those individuals having coefficient $k_p$ of greater value are considered more adapted. To choose individual pairs, we use the roulette wheel method with defining segment length for individual $r$ on a unitary numerical interval in the following way:

$$\Delta_r = t_r / \sum_{n=1}^{N_\Pi} t_n, \tag{11}$$

where

$$t_n = \alpha k_{pn}^{\beta}. \tag{12}$$

Here $k_{pn}$ is the value of $k_p$ for individual $n$, $\alpha$, $\beta$ are prescribed constants.

Step 4:

4.a: Step 2 is implemented on the basis of population Π received as the result of the crossover. In this case, if population Ψ is replenished from population Π, then there is an additional check for satisfying condition $F_i \ge F_A$ by the individual as the crossover can result in its violation.

4.b: Implementation of condition $F_i \ge F_A$ is tested for all individuals of population Π. If this requirement is violated for the individual under consideration, then it is replaced by the best individual placed into population Ψ if there is no such individual in population Π. If there are no individuals in population Ψ for which this condition holds true, then a new individual is generated via random choice of design variable values.

A flowchart of the proposed algorithm is presented in Fig. 1, where $s_0$ is the specified total number of iterations, $k_{p\min}$ is the minimum value of $k_p$ for individuals currently in the population Ψ. The algorithm does not require the use of complex procedures for the parameter tuning. In general, values $N_\Pi = N_\Psi = 20$, $\alpha = 0.1$, $\beta = 120$, $\lambda = 0.1$, $d = 5$, $s_1 = 0.3N_\Pi n_o$ are practical.

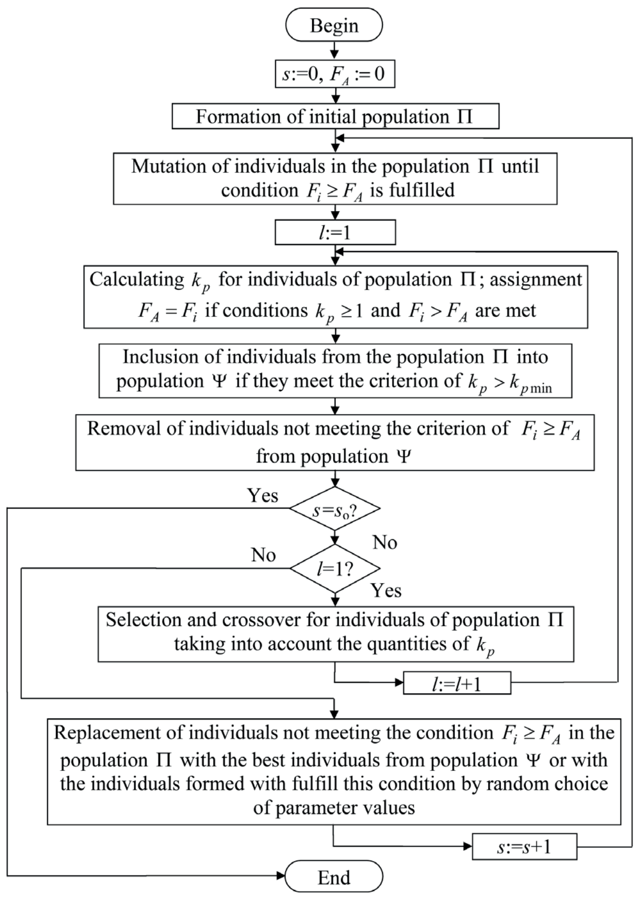


**Fig. 1** Flowchart of the JSI strategy

## 5 Numerical examples

For efficiency analysis of the provided strategy, some standard examples of size and size/shape optimization were considered. Dimensions such as inches, kips, ksi and lbs were used for convenience when comparing the received data with the results given in literature sources.

### 5.1 A 10-bar truss

The 10 plane truss shown in Fig. 2 has been optimized using discrete algorithms in [1, 32–40, etc]. Let us consider size optimization. We specify the following task conditions: material density $\rho = 0.1$ lb/in$^3$, $E = 10{,}000$ ksi, force $P = 100$ kips, and distance $L = 360$in. The stress limitations of the members are ±25 ksi, the displacement limitations of the nodes are ±2.0 in. in both $x$ and $y$ directions. A cross-sectional area of every member was varied independently. Two optimization cases shown in Table 1 are considered. In both cases, we performed 100 independent runs of the algorithm. In the first case, at each run we received the same vector $\{33.5, 1.62, 22.9, 14.2, 1.62, 1.62, 7.97, 22.9, 22, 1.62\}^T$ (in.$^2$) of designed variables corresponding with the weight 5490.7 lb. The same result for the best individual was achieved in [34, 36, 37, 39, etc.]. Fig. 3 shows the fastest and slowest convergence obtained here with the proposed algorithm. Table 2 represents a comparison between the statistical results achieved by different researchers for this case. This table shows that in terms of stability of the obtained result with the minimum weight, the given algorithm excels here the other compared methods.

In the second case, at each run we obtained the same weight value of 5067.33, however, this weight was obtained for 21 various task solutions. In Table 3, solution 1 obtained in our experiments is compared with the results

**Table 1** Permitted cross-sectional areas of members for the size optimization of the 10-bar truss

| Case | Areas (in.$^2$) |
|---|---|
| 1: Nanakorn and Meesomklin [33] | 1.62, 1.8, 1.99, 2.13, 2.38, 2.62, 2.63, 2.88, 2.93, 3.09, 3.13, 3.38, 3.47, 3.55, 3.63, 3.84, 3.87, 3.88, 4.18, 4.22, 4.49, 4.59, 4.8, 4.97, 5.12, 5.74, 7.22, 7.97, 11.5, 13.5, 13.9, 14.2, 15.5, 16, 16.9, 18.8, 19.9, 22, 22.9, 26.5, 30, 33.5 |
| 2: Li et al. [35] | 0.1, 0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 5.5, 6, 6.5, 7, 7.5, 8, 8.5, 9, 9.5, 10, 10.5, 11, 11.5, 12, 12.5, 13, 13.5, 14, 14.5, 15, 15.5, 16, 16.5, 17, 17.5, 18, 18.5, 19, 19.5, 20, 20.5, 21, 21.5, 22, 22.5, 23, 23.5, 24, 24.5, 25, 25.5, 26, 26.5, 27, 27.5, 28, 28.5, 29, 29.5, 30, 30.5, 31, 31.5 |

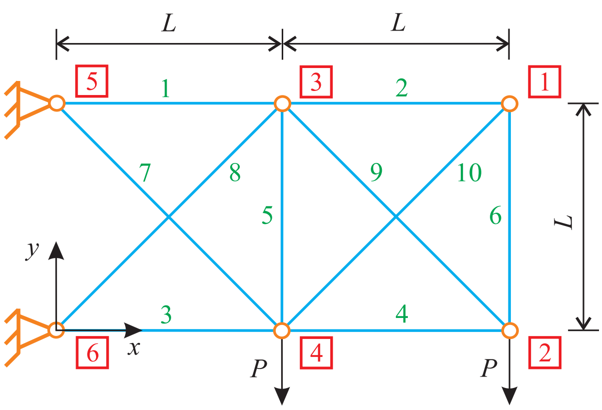


**Fig. 2** 10-bar truss

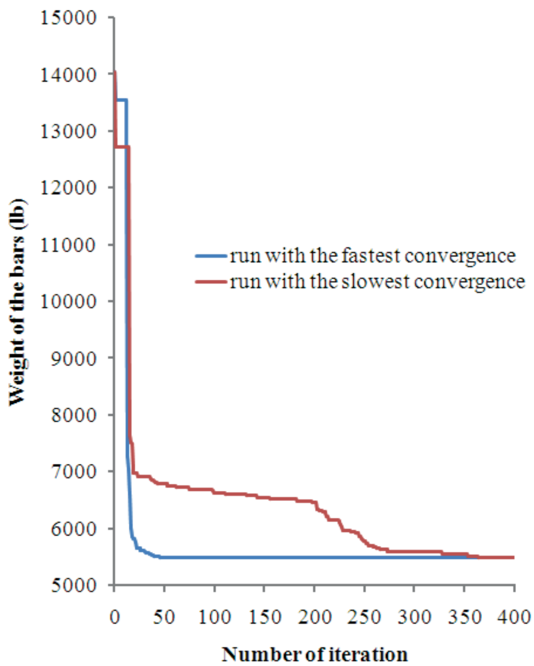


**Fig. 3** Convergence curves for the 10-bar truss problem in Case 1

**Table 2** Statistical performance for the 10-bar truss structure in Case 1

| Reference | Camp and Bichon [34] | Toğan [36] | Li and Ma [37] | | Ho-Huu et al. [39] | This work |
|---|---|---|---|---|---|---|
| | | | Setups 1–3, and 5 | Setup 4 | | |
| Best weight (lb) | 5,490.74 | 5,490.74 | 5,490.74 | 5,491.72 | 5,490.74 | 5,490.74 |
| Average weight (lb) | 5,491.24 | 5,510.54 | 5,497.25-5,653.23 | 5,634.06 | 5502.62 | 5,490.74 |
| Worst weight (lb) | – | – | 5,585.73-6,549.42 | 5,842.67 | 5549.20 | 5,490.74 |
| Standard deviation (lb) | 1.69 | 22.2 | 17.38-108.68 | 85.08 | 20.78 | 0 |
| Structural analyses | 10,000 | 8,040 | 5,050-50,000 | 8,020 | 2,380 | 2,280-15,960 |

**Table 3** Comparison for the size optimization of the 10-bar truss in Case 2

| Variables | Optimal cross-sectional areas (in.$^2$) | | | | | |
|---|---|---|---|---|---|---|
| | Ringertz [32] | Li et al. [35] | Kaveh and Zolghadr [40] | Li & Ma [37] Setups 1, 2, 4, and 5 | Li & Ma [37] Setup 3 | This work Solution 1 |
| A1 | 30.5 | 31.5 | 31.5 | 30 | 30.5 | 31 |
| A2 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 |
| A3 | 23 | 24.5 | 20.5 | 23.5 | 23 | 22 |
| A4 | 15.5 | 15.5 | 20.5 | 15 | 14.5 | 15.5 |
| A5 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 |
| A6 | 0.5 | 0.5 | 0.1 | 0.5 | 0.5 | 0.5 |
| A7 | 7.5 | 7.5 | 9 | 7.5 | 8 | 7.5 |
| A8 | 21 | 20.5 | 20.5 | 21.5 | 22 | 20.5 |
| A9 | 21.5 | 20.5 | 20.5 | 21.5 | 21 | 22.5 |
| A10 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 |
| Best weight (lb) | 5,059.9 | 5,073.51 | 5,171.5 | 5,067.33 | 5,074.79 | 5,067.33 |
| Structural analyses | – | – | – | 8,020-50,000 | 5,050 | 8,400 |
| Constraint violation | 0.044% | None | None | None | None | None |
| Average weight (lb) | – | – | – | 5,086.61-5,196.17 | 5,270.92 | 5,067.33 |
| Worst weight (lb) | – | – | – | 5,248.10-5,811.02 | 5,840.33 | 5,067.33 |
| Standard deviation (lb) | – | – | – | 27.82-157.37 | 183.45 | 0 |

from literature sources. Table 4 provides data on all 21 solutions. Tables 3 and 4 show that for case 2, in all experiments which meet the limitations the minimum weight obtained was as in [37] and by means of the JSI strategy. In this case, the iteration procedure worked out in this article excels [37] in terms of both the stability of the result to achieve the minimum weight as well as the number of solutions for this weight.

### 5.2 A 25-bar space truss

The optimization problem for the 25-bar transmission tower, shown in Fig. 4, was previously studied in [1, 34, 36, 37, 41, 42, etc.]. Let us specify $\rho$ = 0.1 lb/in.$^3$ and $E$ = 10,000 ksi. We also assume the following distances: $L_1$ = 75 in., $L_2$ = 100 in., $L_3$ = 200 in.. The stress and displacement limitations are ±40 ksi for each member, and ±0.35 in. for each node in the $x$, $y$, and $z$ directions, respectively. The design variables are selected from 34 discrete values, which are uniformly distributed over a numerical interval [0.1–3.4] (in.$^2$) with a step of 0.1 in.$^2$. The members are divided into following 8 groups: (1): A1, (2): A2-A5, (3): A6-A9, (4): A10-A11, (5): A12-A13, (6): A14-A17, (7): A18-A21, (8): A22-A25. We accepted the loading, shown in Table 5. We performed 100 independent runs. Table 6 compares the results of the JSI strategy and other methods. It can be seen that in [1, 41] the results obtained were of the minimum weights 486 lb and 493 lb, respectively. In all other works, they managed to achieve the same best result with the weight 484.85 lb. At the same time, in our algorithm only this solution was obtained in all runs performed.

### 5.3 200-bar plane truss

The structure of this well-known benchmarking problem [39, 43–45, etc.] is shown in Fig. 5. We used the task conditions in accordance with [43]. We assumed: $L_1$ = 240 in., $L_2$ = 144 in., $L_3$ = 360 in., $\rho$ = 0.283 lb/in.$^3$, $E$ = 30,000 ksi and specified stress limitations in members with limits of ±10 ksi. The truss is subjected to three independent loading conditions (Table 7). The members were linked into 29 groups. The available set of 30 discrete cross-sectional areas values $R$ = {0.1, 0.347, 0.44, 0.539, 0.954, 1.081, 1.174, 1.333, 1.488, 1.764, 2.142, 2.697, 2.8, 3.131, 3.565, 3.813, 4.805, 5.952, 6.572, 7.192, 8.525, 9.3, 10.85, 13.33, 14.29, 17.17, 19.18, 23.68, 28.08, 33.7} (in.$^2$).

We performed 30 independent runs of the algorithm. The minimum $W_{min}$, maximum $W_{max}$, and average $W_{avg}$ values and standard deviation of the weight obtained during the performance of 2,000, 4,000, and 8,000 iterations are presented in Table 8 . Comparison of the best individuals with a minimum weight obtained in some researches is shown in Table 9. The results obtained by the JSI strategy have a better value in terms of weight than those determined in other compared works.

**Table 4** Optimal cross-sectional areas (in.$^2$) obtained for the 10-bar truss in Case 2

| Variables | Solutions | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| A1 | 31 | 29.5 | 30.5 | 29.5 | 29.5 | 30.5 | 29.5 | 31 | 31 | 29.5 | 30 |
| A2 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 |
| A3 | 22 | 23 | 23.5 | 24 | 23.5 | 23 | 24 | 23.5 | 23 | 23.5 | 24 |
| A4 | 15.5 | 16 | 14.5 | 15 | 15.5 | 15 | 15 | 14 | 14.5 | 15.5 | 14.5 |
| A5 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 |
| A6 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |
| A7 | 7.5 | 7.5 | 7.5 | 7.5 | 7.5 | 7.5 | 7.5 | 7.5 | 7.5 | 7.5 | 7.5 |
| A8 | 20.5 | 21 | 21.5 | 22 | 21 | 20.5 | 21 | 21 | 21 | 21.5 | 21.5 |
| A9 | 22.5 | 22 | 21.5 | 21 | 22 | 22.5 | 22 | 22 | 22 | 21.5 | 21.5 |
| A10 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 |
| Weight (lb) | 5,067.33 | | | | | | | | | | |
| Structural analyses | 8,400 | 4,440-37,520 | 4,480-13,560 | 6,080-12,560 | 4,280-9,000 | 7,440-11,360 | 3,160-13,960 | 11,120 | 4,520-11,760 | 6,800-60,720 | 5,520-19,000 |
| Constraint violation | None | | | | | | | | | | |
| Variables | Solutions | | | | | | | | | | |
| | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | |
| A1 | 29.5 | 30.5 | 30 | 30.5 | 30 | 30.5 | 29.5 | 30 | 30 | 31 | |
| A2 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | |
| A3 | 23 | 23.5 | 23.5 | 23 | 24 | 24 | 24 | 23 | 23.5 | 22.5 | |
| A4 | 16 | 14.5 | 15 | 15 | 14.5 | 14 | 15 | 15.5 | 15 | 15 | |
| A5 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | |
| A6 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | |
| A7 | 7.5 | 7.5 | 7.5 | 7.5 | 7.5 | 7.5 | 7.5 | 7.5 | 7.5 | 7.5 | |
| A8 | 21.5 | 21 | 21.5 | 21 | 21 | 21.5 | 21.5 | 21 | 21 | 20.5 | |
| A9 | 21.5 | 22 | 21.5 | 22 | 22 | 21.5 | 21.5 | 22 | 22 | 22.5 | |
| A10 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | |
| Weight (lb) | 5,067.33 | | | | | | | | | | |
| Structural analyses | 2,960-10,000 | 5,520-7,600 | 2,640-9,000 | 11,240-23,840 | 19,520 | 7,160-16,600 | 7,800-33,760 | 7,440-10,200 | 4,680 | 3,120-9,800 | |
| Constraint violation | None | | | | | | | | | | |

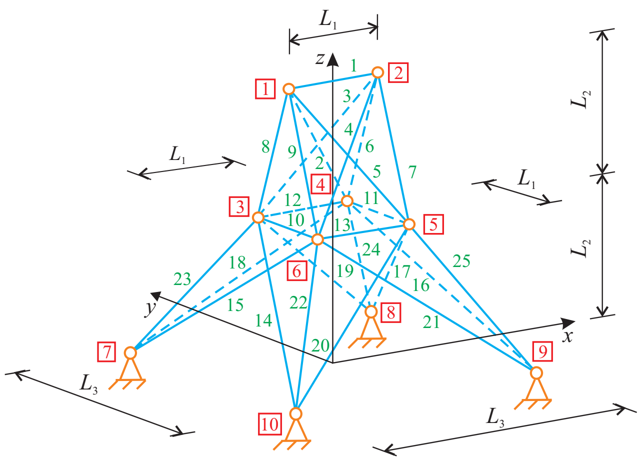


**Fig. 4** 25-bar space truss

**Table 5** Loading for the 25-bar space truss

| Node | Axial force (kips) | | |
|---|---|---|---|
| | *x* | *y* | *z* |
| 1 | 1.0 | -10.0 | -10.0 |
| 2 | 0 | -10.0 | -10.0 |
| 3 | 0.5 | 0.0 | 0.0 |
| 6 | 0.6 | 0.0 | 0.0 |

## 5.4 An 18-bar planar truss

The initial geometry of the 18-bar cantilever truss is shown in Fig. 6. This standard example [46] is frequently used to test the efficiency of new algorithms related to size and shape optimization. We consider discrete optimization with limitations of stress and stability

**Table 6** Comparison for the 25-bar space truss

| Variables | Optimal cross-sectional areas (in.$^2$) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Wu and Chow [41] | Erbatur et al. [1] | Camp and Bichon [34] | Kaveh et al. [42] | Toğan [36] | Sonmez [38] | Li and Ma [37] (Setup 5) | Ho-Huu et al. [39] | This work |
| A1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 |
| A2 | 0.5 | 1.2 | 0.3 | 0.3 | 0.3 | 0.3 | 0.3 | 0.3 | 0.3 |
| A3 | 3.4 | 3.2 | 3.4 | 3.4 | 3.4 | 3.4 | 3.4 | 3.4 | 3.4 |
| A4 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 |
| A5 | 1.5 | 1.1 | 2.1 | 2.1 | 2.1 | 2.1 | 2.1 | 2.1 | 2.1 |
| A6 | 0.9 | 0.9 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| A7 | 0.6 | 0.4 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |
| A8 | 3.4 | 3.4 | 3.4 | 3.4 | 3.4 | 3.4 | 3.4 | 3.4 | 3.4 |
| Best weight (lb) | 486.29 | 493.8 | 484.85 | 484.85 | 484.85 | 484.85 | 484.85 | 484.85 | 484.85 |
| Average weight (lb) | – | – | 486.46 | 484.90 | 486.54 | 484.94 | 484.98 | 485.01 | 484.85 |
| Worst weight (lb) | – | – | – | – | – | 485.05 | 485.91 | 486.10 | 484.85 |
| Standard deviation (lb) | – | – | 4.71 | – | 2.74 | – | 0.180 | 0.273 | 0 |
| Structural analyses | 40,000 | – | Min. – 5,200, avg. – 7,700 | 925 | 2420 | 24,250 | 50,000 | Min. – 1,440, avg. – 1,678 | Min. – 2,800, avg. – 8,838 |

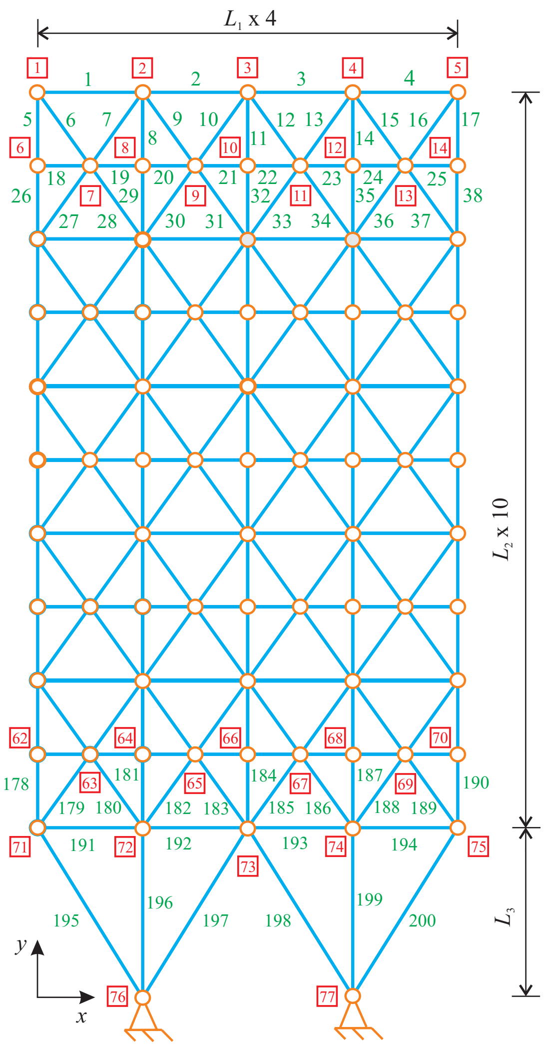


**Fig. 5** 200-bar truss

using the following data: $\sigma_{ij}^t = \sigma_{ij}^c = 20$ kps, $c$ = 4, force $P$ = 20 kips, $\rho$ = 0.1 lb/in.$^3$, $E$ = 10,000 ksi, and size $L$ = 250 in. The member cross-sections are placed into four groups as follow: (1) A1 = A4 = A8 = A12 = A16, (2) A2 = A6 = A10 = A14 = A18, (3) A3 = A7 = A11 = A15, (4) A5 = A9 = A13 = A17. For the cross-sections, 81 discrete values are used for every group. The values are uniformly distributed as presented in Table 10. The coordinates x and y corresponding to nodes 3, 5, 7, and 9 are taken as geometric variables. For these variables, the discrete values were also uniformly distributed on the specified ranges (see Table 10).

**Table 7** Loading for the 200-bar truss

| Condition | Nodes | Axial force (kips) | |
|---|---|---|---|
| | | $x$ | $y$ |
| 1 | 1, 6, 15, 20, 29, 34, 43, 48, 57, 62, 71 | 1 | 0 |
| 2 | 1, 2, 3, 4, 5, 6, 8, 10, 12, 14, 15, 16, 17, 18, 19, 20, 22, 24, 26, 28, 29, 30, 31, 32, 33, 34, 36, 38, 40, 42, 43, 44, 45, 46, 47, 48, 50, 52, 54, 56, 57, 58, 59, 60, 61, 62, 64, 66, 68, 70, 71, 72, 73, 74, 75 | 0 | -10 |
| 3 | Combination of conditions 1 and 2 | | |

**Table 8** Weight results for the 200-bar truss

| Iteration | $W_{min}$ (lb) | $W_{max}$ (lb) | $W_{avg}$ (lb) | $S$ (lb) |
|---|---|---|---|---|
| 2000 | 27,131.8 | 30,045.2 | 27,712.5 | 550.9 |
| 4000 | 26,996.4 | 29,996.8 | 27,452.4 | 535.6 |
| 8000 | 26,996.4 | 28,198.8 | 27,343.7 | 288.9 |

**Table 9** Comparison for the 200-bar truss

| Group | Members | Optimal cross-sectional areas (in.$^2$) | | | | | |
|---|---|---|---|---|---|---|---|
| | | Toğan and Daloğu [43] | Talebpour et al. [44] | Flager et al. [45] | Ho-Huu et al. [39] | Serpik et al. [31] | This work |
| 1 | 1, 2, 3, 4 | 0.347 | 0.1 | 0.1 | 0.1 | 0.1 | 0.347 |
| 2 | 5, 8, 11, 14, 17 | 1.081 | 1.081 | 0.954 | 0.954 | 0.954 | 0.954 |
| 3 | 19, 20, 21, 22, 23, 24 | 0.1 | 0.347 | 0.1 | 0.347 | 0.1 | 0.1 |
| 4 | 18, 25, 56, 63, 94, 101, 132, 139, 170, 177 | 0.1 | 0.1 | 0.1 | 0.1 | 0.347 | 0.1 |
| 5 | 26, 29, 32, 35, 38 | 2.142 | 2.142 | 2.142 | 2.142 | 2.142 | 2.142 |
| 6 | 6, 7, 9, 10, 12, 13, 15, 16, 27, 28, 30, 31, 33, 34, 36, 37 | 0.347 | 0.347 | 0.347 | 0.347 | 0.347 | 0.347 |
| 7 | 39, 40, 41, 42 | 0.1 | 0.1 | 0.1 | 0.1 | 0.539 | 0.1 |
| 8 | 43, 46, 49, 52, 55 | 3.565 | 3.131 | 3.131 | 3.131 | 2.8 | 3.565 |
| 9 | 57, 58, 59, 60, 61, 62 | 0.347 | 0.1 | 0.1 | 0.347 | 0.539 | 0.1 |
| 10 | 64, 67, 70, 73, 76 | 4.805 | 4.805 | 4.805 | 4.805 | 3.813 | 4.805 |
| 11 | 44, 45, 47, 48, 50, 51, 53, 54, 65, 66, 68, 69, 71, 72, 74, 75 | 0.44 | 0.44 | 0.44 | 0.539 | 0.954 | 0.44 |
| 12 | 77, 78, 79, 80 | 0.44 | 0.1 | 0.347 | 0.347 | 0.1 | 0.1 |
| 13 | 81, 84, 87, 90, 93 | 5.952 | 5.952 | 5.952 | 5.952 | 5.952 | 5.952 |
| 14 | 95, 96, 97, 98, 99, 100 | 0.347 | 0.1 | 0.347 | 0.1 | 0.1 | 0.1 |
| 15 | 102, 105, 108, 111, 114 | 6.572 | 6.572 | 6.572 | 6.572 | 6.572 | 6.572 |
| 16 | 82, 83, 85, 86, 88, 89, 91, 92, 103, 104, 106, 107, 109, 110, 112, 113 | 0.954 | 0.539 | 0.954 | 0.954 | 0.539 | 0.539 |
| 17 | 115, 116, 117, 118 | 0.347 | 1.174 | 0.347 | 0.44 | 0.954 | 0.347 |
| 18 | 119, 122, 125, 128, 131 | 8.525 | 8.525 | 8.525 | 8.525 | 8.525 | 8.525 |
| 19 | 133, 134, 135, 136, 137, 138 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.347 |
| 20 | 140, 143, 146, 149, 152 | 9.3 | 9.3 | 9.3 | 9.3 | 9.3 | 9.3 |
| 21 | 120, 121, 123, 124, 126, 127, 129, 130, 141, 142, 144, 145, 147, 148, 150, 151 | 0.954 | 1.333 | 1.081 | 0.954 | 1.174 | 0.954 |
| 22 | 153, 154, 155, 156 | 1.764 | 0.539 | 0.347 | 1.081 | 0.44 | 0.1 |
| 23 | 157, 160, 163, 166, 169 | 13.33 | 13.33 | 13.33 | 13.33 | 13.33 | 13.33 |
| 24 | 171, 172, 173, 174, 175, 176 | 0.347 | 1.174 | 0.954 | 0.539 | 1.081 | 0.1 |
| 25 | 178, 181, 184, 187, 190 | 13.33 | 13.33 | 13.33 | 14.29 | 13.33 | 13.33 |
| 26 | 158, 159, 161, 162, 164, 165, 167, 168, 179, 180, 182, 183, 185, 186, 188, 189 | 2.142 | 2.697 | 1.764 | 2.142 | 2.142 | 0.954 |
| 27 | 191, 192, 193, 194 | 4.805 | 3.565 | 3.813 | 3.813 | 3.565 | 5.952 |
| 28 | 195, 197, 198, 200 | 9.3 | 8.525 | 8.525 | 8.525 | 8.525 | 10.85 |
| 29 | 196, 199 | 17.17 | 17.17 | 17.17 | 17.17 | 17.17 | 14.29 |
| Weight (lb) | | 28,544.0 | 28,030.2 | 27,151 | 27,858.5 | 27,701.7 | 26,996.4 |

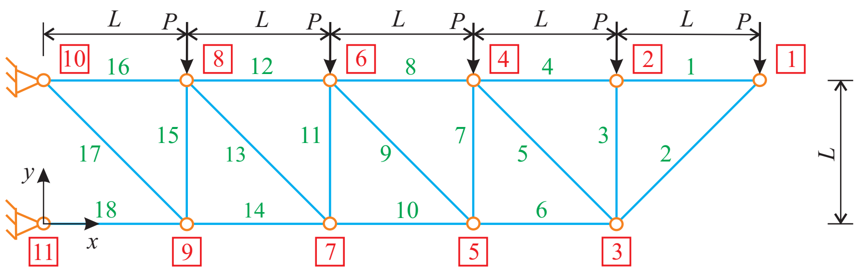


**Fig. 6** 18-bar truss

**Table 10** Allowable parameter values for the 18-bar truss

| Variables | Minimum | Maximum | Increment |
|---|---|---|---|
| A1 – A18 (in.$^2$) | 2 | 22 | 0,25 |
| $x_3$ (in.) | 775 | 1250 | 1 |
| $x_5$ (in.) | 525 | 1000 | 1 |
| $x_7$ (in.) | 275 | 750 | 1 |
| $x_9$ (in.) | 25 | 500 | 1 |
| $y_3, y_5, y_7, y_9$ (in.) | -225 | 250 | 1 |

In this task, the numbers of the members in the set of allowed values of cross-sectional areas were written in natural two-digit numbers in the number system with base 9, and for each set of allowed values of node coordinates, we used two-digit numbers with base 22. As well, each of the digit positions was varied independently. If the number of members for any of the varied coordinates exceeded the number of allowed values, then this individual was excluded. The total number of the varied parameters actually was 24. There were 30 independent runs performed. The weights obtained during the performance of 10,000, 30,000, and 60,000 iterations are represented in Table 11. The best result of the optimization using the proposed procedure is compared with those previously reported in literature sources for the discrete case in Table 12. This table shows that we obtained a smaller value of the objective function than in [46, 47]. The optimum geometry of the structure is shown in Fig. 7.

**Table 11** Weight results for the 18-bar truss

| Iteration | $W_{min}$ (lb) | $W_{max}$ (lb) | $W_{avg}$ (lb) | S (lb) |
|---|---|---|---|---|
| 10,000 | 4554.14 | 4909.13 | 4651.34 | 81.52 |
| 30,000 | 4536.83 | 4691.09 | 4593.95 | 46.92 |
| 60,000 | 4520.33 | 4673.65 | 4574.44 | 39.58 |

**Table 12** Comparison for the size and shape optimization of the 18-bar truss

| Design variables | Hasançebi and Erbatur [46] | Kaveh and Kalatjari [47] | This work |
|---|---|---|---|
| Cross-sectional areas design variables (in.$^2$) | | | |
| A1, A4, A8, A12, A16 | 12.25 | 13 | 12.5 |
| A2, A6, A10, A14, A18 | 17.5 | 18.25 | 17.75 |
| A3, A7, A11, A15 | 5.75 | 5.5 | 5.5 |
| A5, A9, A13, A17 | 4.25 | 3 | 3.75 |
| Geometric design variables (in.) | | | |
| $x_3$ | 910 | 913 | 911 |
| $y_3$ | 179 | 182 | 184 |
| $x_5$ | 638 | 648 | 642 |
| $y_5$ | 141 | 152 | 145 |
| $x_7$ | 408 | 417 | 412 |
| $y_7$ | 91 | 103 | 97 |
| $x_9$ | 198 | 204 | 201 |
| $y_9$ | 24 | 39 | 30 |
| Weight (lb) | 4,533.24 | 4,566.21 | 4,520.33 |

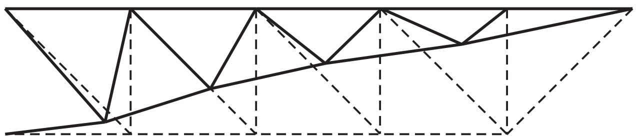

**Fig. 7** The optimum geometry of 18-bar truss

### 5.5 A 354-bar braced dome

A steel-braced dome with 8.28 m (27.165 ft) height and a diameter of 40 m (131.23 ft) [48–52] is considered in accordance with [51] as pin-jointed frame. The 3-D views, plan and elevation of the dome are shown in Fig. 8. It consists of 127 joints and 354 members. The members are grouped into 22 independent design variables as shown in Fig. 8(b) [51], which are selected from a set of 37 circular hollow sections in LRFD-AISC [30] steel profile list. The illustrations of the considered load cases are presented in Fig. 9 [51, 52]. For design purpose, the braced dome is subjected to following three various combinations of dead (*D*), snow (*S*) and wind (*W*) loads calculated according to the provisions of ASCE 7-98 [53]: (1) D + S, (2) D + S + W (with negative internal pressure), and (3) D + S + W (with positive internal pressure). While taking into account external wind pressure, the object is divided into three regions: a windward quarter, a center half, and a leeward quarter. The equivalent loads of these cases acting on nodes are given in Table 13 [52], were $P_x$, $P_z$ are the axial forces. The stress and stability constraints for members are specified according to LRFD-AISC [30], and the displacements are limited to 11.1 cm (4.37 in.) for all nodes in any direction. It was assumed: $E$ = 208 GPa (30,167.84 ksi) and $F_y$ = 250 MPa (36.26 ksi).

We performed 10 independent runs of the proposed algorithm. During execution of 50000 iterations different solutions with the weight ranging from 134.3 to 137.3 kN were obtained. The best individual is presented in Table 14 in comparison to results achieved in [51] with using the following meta-heuristic algorithms: Cuckoo Search Algorithm (CSA), Firefly Algorithm (FFA), Ant Colony Optimization (ACO), Particle Swarm Optimizer (PSO). and Artificial Bee Colony Algorithm (ABC). It is seen from the table that the weight of our design is 6.0–10.9 % lighter than the weights obtained with other algorithms.

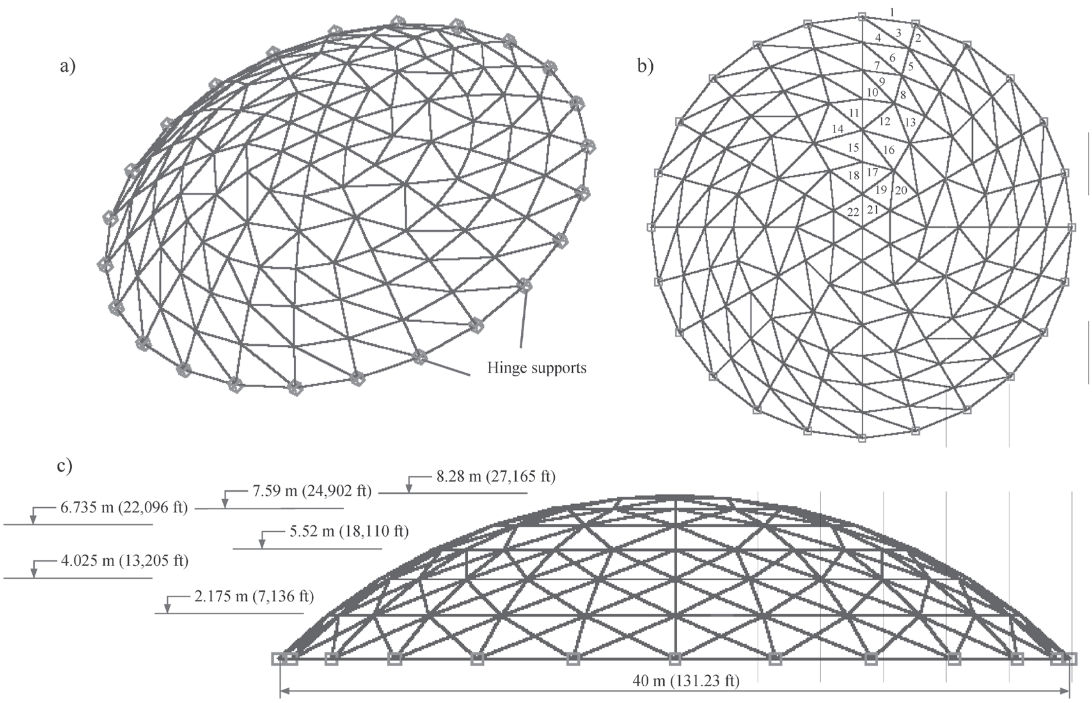


**Fig. 8** 354-bar steel-braced dome: (a) 3D view, (b) top view, (c) side view

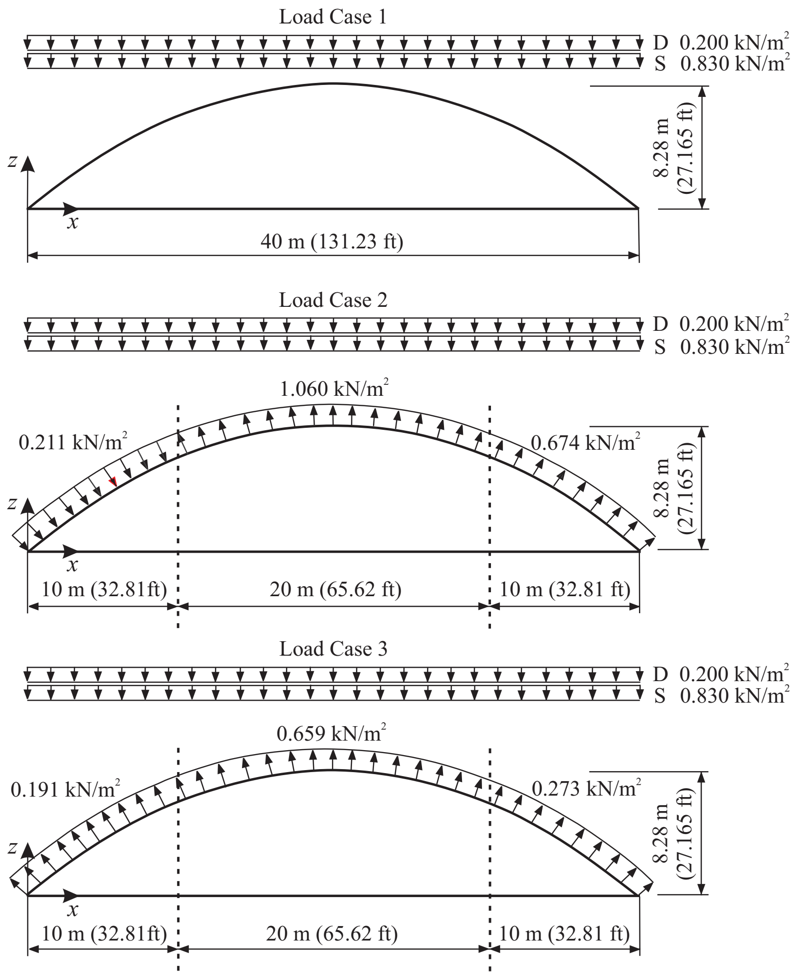


**Fig. 9** Loading on the 354-bar steel-braced dome

**Table 13** Loading conditions for the 354-bar braced dome truss

| Load case | Location of the nodes | $P_x$ (kN) | $P_z$ (kN) |
|---|---|---|---|
| 1 | All nodes | 0 | -10.195 |
| | Windward quarter | 1.257 | -11.388 |
| 2 | Center half | 0 | 1.708 |
| | Leeward quarter | 4.006 | -2.960 |
| | Windward quarter | -1.133 | -7.561 |
| 3 | Center half | 0 | -3.023 |
| | Leeward quarter | 1.627 | -6.786 |

## 6 Conclusions

The article presents a meta-heuristic algorithm for discrete size and shape optimization of plane and space trusses which combines the JSI strategy together with traditional genetic operations. The main distinction of the proposed methodology is to eliminate using penalty functions for limitation allowance. In fact, the sequence of optimization stages is implemented. Each stage provides an evolutionary search of the structure variant which meets the task limitations along with the interim requirement for weight value. This search provides using an auxiliary objective function which defines the degree of performing limiting conditions for components of a stress-strain state. After determining such an individual, the next stage is carried out with new requirements for the upper weight, on the basis of the obtained result. This algorithm makes possible strict meeting the proposed limitations. The performed numerical experiments for benchmark examples showed that the proposed computational procedure gives the opportunity for receiving new effective projects or better known solutions.

## Acknowledgement

This work was supported by the Russian Fundamental Research Fund (project No. 18-08-00567).

**Table 14** Optimum designs for the 354-bar steel-braced dome

| Group number | Optimal pipe sections | | | | | |
|---|---|---|---|---|---|---|
| | CSA | FFA | ACO | PSO | ABC | This work |
| 1 | P2 | P2 | P2 | P2 | P2 | P2 |
| 2 | P4 | P4 | P3 | P3 | P3 | PXX2 |
| 3 | P3.5 | P3.5 | P4 | P3.5 | P4 | P3 |
| 4 | P3.5 | P3.5 | P3.5 | P3.5 | P3 | P3 |
| 5 | P3.5 | P3.5 | P3 | P3 | P3 | PX2.5 |
| 6 | P3 | P3 | P3 | P3 | P3 | P3 |
| 7 | P3 | P3 | P3 | P3 | P3 | P3 |
| 8 | P3 | P2.5 | P2.5 | P3 | P2.5 | P2.5 |
| 9 | P3 | P3 | P3 | P3 | P3 | P2.5 |
| 10 | P3 | P3 | P3 | P3 | P3 | P2.5 |
| 11 | P2.5 | P2.5 | P2.5 | P2.5 | P2.5 | PX2 |
| 12 | P2.5 | P2.5 | P2.5 | P2.5 | P2.5 | P2.5 |
| 13 | P2.5 | P2.5 | P2.5 | P2.5 | P2.5 | P2.5 |
| 14 | P2.5 | P2.5 | P2.5 | P2.5 | P2.5 | P2.5 |
| 15 | P2.5 | P2.5 | PX2.5 | P2.5 | P2.5 | P2.5 |
| 16 | P2.5 | P2.5 | P2.5 | P2.5 | P2.5 | P3 |
| 17 | PX2 | PX2 | PX2 | PX2 | PX2 | P2 |
| 18 | P2.5 | PX2 | PX2 | P2 | P2 | PX2 |
| 19 | PX2 | PX2 | P2 | PX2 | PX2 | P2 |
| 20 | P2.5 | PX2 | PX2 | P2.5 | P2 | P2 |
| 21 | P2 | PX2 | P2 | P2 | P2 | P2 |
| 22 | P2 | P2 | P2 | P2 | P2 | P2 |
| Maximum no. of iterations | 50,000 | | | | | |
| Minimum weight (kN) | 150.78 | 148.67 | 146.65 | 144.53 | 142.87 | 134.3 |

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