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RESEARCH ARTICLE

Analytical and Reliability Study of the Tunnel with Rockbolts in Rock Masses

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Abstract

Rockbolts are a critical reinforcement ways which widely used in tunnel engineering. In this paper, an analytical solution of circular tunnel with rockbolts was proposed based on homogenization method, and then the stability of a circular tunnel was investigated by considering the uncertainty based on the proposed analytical solution. Elastoplastic analytical solution for unsupported circular tunnel was presented briefly in hydrostatic stress field with a linear Mohr-Coulomb yield criterion and a non-associated flow rule. An analytical solution of circular tunnel with rockbolts was proposed through considering rock mass and rockbolts as a new homogeneous, isotropic, parameters strengthened equivalent composite material. A numerical example is used to verify the proposed analytical solution. The results show that the proposed solution can effectively characterize the mechanical behavior of rock mass and rockbolts in tunnel. Then, the proposed solution is adopted to calculate reliability index and failure probability of tunnel. The results show that the proposed method can also be effectively used to perform the stability and reliability analysis of tunnel and rockbolts have an important effect on plastic zone size and displacement of tunnel.

Keywords

tunnel, analytical solution, rockbolt, homogenization method, reliability analysis

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1 Introduction

Rockbolts have been widely used to reinforce the surrounding rock mass in tunnel engineering. A proper design of rockbolts, which depends on the full understanding of the mechanism and effectiveness of rockbolts, is very important to reinforcement, stabilization and safety construction of rock tunnel [1–3]. Meanwhile, there are lots of uncertainties in rock tunnel. Uncertainties are critical to design and safety construction due to the complexity of rock mass [4]. Reliability analysis is a widely developed method to determine the uncertainty in engineering system [5]. The stresses and displacements around tunnel is an important problem considering the uncertainty in tunnel engineering.

Various analytical, experimental, and numerical methods have been developed to analyze and understand the mechanism of rockbolts in tunnel based on different assumptions and conditions. Analytical models are able to define the stresses and the deformations and have been widely used due to the computational simplicity [6,7]. Hoek and Brown (1980) had presented analytical solutions of tunnel with rockbolts [8]. Brown et al (1983) proposed an analytical solution based on elastic-brittle-plastic material behavior and Hoek-Brown yield criterion [9]. Li and Stillborg (1999) developed three analytical models for rockbolts based on the mechanical coupling at the various interface of the rockbolts, the grout medium and the rock mass [2]. Cai et al. (2004) described the interaction mechanism between the rockbolts and rock mass and proposed an analytical model to analyze the supporting behavior of rockbolts in tunnel [10]. Guan et al. (2007) considered the interaction relationship between rockbolts and rock mass and proposed a framework to analyze the elastoplastic ground response of tunnel with rockbolts [11]. Oreste (2008) proposed a calculation procedure to determine the stress and strain state of rock mass in tunnel with rockbolts [12]. Indraratna and Kaiser (1990) extended the general solutions based on the Mohr–Coulomb criterion and developed an elastic-brittle plastic model [13]. Fahimifar and Soroush (2005) presented a new approach based on non-linear strength criterion for rock mass and the brittle and strain softening stress–strain behavior models [14]. Carranza-Torres (2009) analyzed the mechanical contribution of rockbolts reinforcement based on elastoplastic model and proved that rockbolts can also have a critical effect in controlling the extent of the plastic failure zone and the convergences of the tunnel [15]. Bobet and Einstein (2011) analyzed the reinforcement mechanism of different types of bolts and proposed an analytical solution for circular tunnel with rockbolts based on coupling analysis [16].

The above models have been used in design of rockbolts and stability analysis of tunnel. But these models didn't deal with the uncertainties in variables such as rock mass strength and the parameters of the rockbots. The reliability method has been developed for stability analysis of tunnels to consider the uncertainty [5, 17–21]. Hoek (1998) applied reliability method to stability analysis of circular tunnel through integrating analytical solutions and Monte Carlo simulation [22]. Li and Low (2010) analyzed the stability of circular tunnels subjected to hydrostatic stress by combining an analytical solution with first order reliability method (FORM) [18]. Zhang and Goh (2012) estimated the stability of underground rock cavern using reliability method [23].

In this paper, an elastoplastic analytical solution was proposed for circular tunnel without/with rockbolt using Mohr-Coulomb criterion and then the stability of circular tunnel was analyzed considering the uncertainty based on the analytical solution. An elastoplastic analytical solution for circular tunnel without rockbolt was presented in detail. Based on this analytical solution, the analytical solution of circular tunnel with rockbolt was proposed by considering rock mass and rockbolt as a new equivalent material with strengthened strength parameters such as equivalent Young's modulus, cohesion and friction angle. Then both the above solutions are applied to estimate stability of circular tunnel with permissible limit displacement of tunnel wall as instability criterion based on reliability method. The results show rockbolts have good effect on reducing surrounding rock mass displacement of tunnel and the proposed method can be effectively used to perform the stability and reliability analysis of tunnel.

2 Analytical solution of circular tunnel

2.1 Analytical solution of circular tunnel without rockbolts

A circular tunnel in hydrostatic pressure field was analyzed, horizontal in-situ stress coefficient is 1, a is radius of circular tunnel, p_0 is hydrostatic presure, R is radius of plastic zone, as shown in Fig.1. Rock mass is assumed to be continuous, homogeneous and isotropic, tunnel face effect is neglected, thus the problem described above is a planar axisymmetric problem.

An elastic-brittle-plastic material model for rock mass was adopted in this study (See Fig. 2). Fig.2 shows the post yield strength softening behavior which strength suddenly drops and keeps an invariant level after reaching peak strength. The strength of after-peak is called residual strength which is very different from perfectly plastic material.

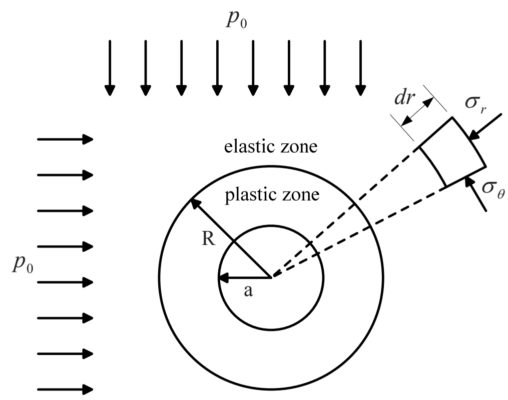

Fig. 1 Definition of the circular tunnel model

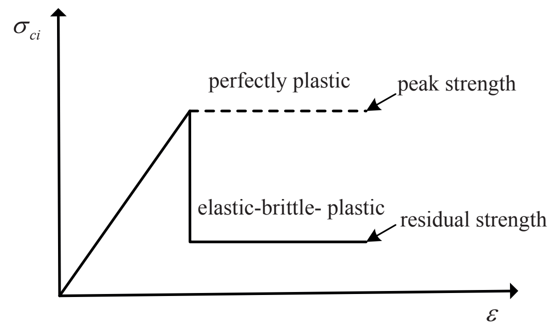

Fig. 2 Material model of rock mass

Polar coordinate was adopted and the center of tunnel was taken as origin point. The tangential stresses are zero for planar axisymmetric problem, thus the differential equation of equilibrium can be presented the following expression in polar coordinates.

$$\frac{\sigma_\theta - \sigma_r}{r} - \frac{d\sigma_r}{dr} = 0 \tag{1}$$

Linear Mohr-Coulomb criterion is given by the following equation:

$$\sigma_\theta - \sigma_r = \frac{2\sin\phi}{1-\sin\phi}\sigma_r + \frac{2c\cos\phi}{1-\sin\phi} \tag{2}$$

where c and ϕ are peak cohesion and peak internal friction angle of rocks respectively. Residual cohesion c_r and residual internal friction angle ϕ_r were used to present the mechanical behavior of rock mass after yielding. So, the linear Mohr-Coulomb criterion in terms of residual parameters can be expressed the following form.

$$\sigma_\theta^p - \sigma_r^p = \frac{2\sin\phi_r}{1-\sin\phi_r}\sigma_r^p + \frac{2c_r\cos\phi_r}{1-\sin\phi_r} \tag{3}$$

where σ_r^p and σ_ϕ^p are radial stress and tangential stress in plastic zone respectively. Equation (3) was substituted into equation (1) to get the following equation.

$$\frac{d\sigma_r^p}{c_r\cot\phi_r + \sigma_r^p} = \frac{2\sin\phi_r}{1-\sin\phi_r}\cdot\frac{dr}{r} \tag{4}$$

Equation (4) is a first order differential equation and can be solved by taking account of boundary condition.

$$r = a, \sigma_r^p = 0 \tag{5}$$

σ_r^p and σ_ϕ^p can be obtained.

$$\begin{cases} \sigma_r^p = c_r \cot\phi_r \left(\dfrac{r}{a}\right)^{\frac{2\sin\phi_r}{1-\sin\phi_r}} - c_r \cot\phi_r \\ \sigma_\theta^p = \dfrac{1+\sin\phi_r}{1-\sin\phi_r} c_r \cot\phi_r \left(\dfrac{r}{a}\right)^{\frac{2\sin\phi_r}{1-\sin\phi_r}} - c_r \cot\phi_r \end{cases} \tag{6}$$

Elastic stresses distributing problem can be considered as that of a tunnel with radius R and uniform internal supporting stress σ_r^R in the same hydrostatic pressure field. Elastic radial stress σ_r^e and elastic tangential stress σ_θ^p can be obtained by classical Lame's solution as the following.

$$\begin{cases} \sigma_r^e = p_0 - (p_0 - \sigma_r^R)\left(\dfrac{R}{r}\right)^2 \\ \sigma_\theta^e = p_0 + (p_0 - \sigma_r^R)\left(\dfrac{R}{r}\right)^2 \end{cases} \tag{7}$$

The following equation was obtained by substituting $r = R$ into equation (7).

$$\sigma_\theta^R - \sigma_r^R = 2\left(p_0 - \sigma_r^R\right) \tag{8}$$

Mohr-Coulomb criterion at elastic-plastic interface can be

$$\sigma_\theta^R - \sigma_r^R = \frac{2\sin\phi}{1-\sin\phi}\sigma_r^R + \frac{2c\cos\phi}{1-\sin\phi} \tag{9}$$

expressed as follow.

$$\sigma_r^R = p_0\left(1-\sin\phi\right) - c\cos\phi \tag{10}$$

σ_r^R can be obtained from equation (8) and (9),

σ_r^R can be also obtained by substituting $r = R$ into equation (6),

$$\sigma_r^R = c_r \cot\phi_r \left(\frac{R}{a}\right)^{\frac{2\sin\phi_r}{1-\sin\phi_r}} - c_r \cot\phi_r \tag{11}$$

So, plastic zone radius R can be obtained by solving the equation which right components of equation (10) and equation (11) is equal,

$$R = a\left[\frac{p_0\left(1-\sin\phi\right) - c\cos\phi}{c_r \cot\phi_r} + 1\right]^{\frac{1-\sin\phi_r}{2\sin\phi_r}} \tag{12}$$

Strains in elastic zone can be determined by generalized Hook's law

$$\begin{cases} \varepsilon_r^e = \dfrac{1-v^2}{E}\left[\sigma_r^e - \left(\dfrac{v}{1-v}\right)\sigma_\theta^e\right] \\ \varepsilon_\theta^e = \dfrac{1-v^2}{E}\left[\sigma_\theta^e - \left(\dfrac{v}{1-v}\right)\sigma_r^e\right] \end{cases} \tag{13}$$

where E and v are Young's modulus and Poisson's ratio respectively, ε_r^e is elastic radial strain, ε_θ^e is elastic tangential strain. Elastic strains can be obtained by substituting elastic stresses equation (7) into equation (13).

$$\begin{cases} \varepsilon_r^e = \dfrac{\left(1-2v\right)}{2G} p_0 + \dfrac{\left(\sigma_r^R - p_0\right)}{2G}\left(\dfrac{R}{r}\right)^2 \\ \varepsilon_\theta^e = \dfrac{\left(1-2v\right)}{2G} p_0 - \dfrac{\left(\sigma_r^R - p_0\right)}{2G}\left(\dfrac{R}{r}\right)^2 \end{cases} \tag{14}$$

where G is shear modulus of rocks. The relationship between E, v and G is given by $G = \dfrac{E}{2\left(1+v\right)}$.

Total radial strain ε_r^{pt} and total tangential strain ε_θ^{pt} in plastic zone are made up with two components respectively i.e. elastic strain ε_r^{pe}, ε_θ^{pe} plastic strain ε_r^{pp}, ε_θ^{pp}

$$\begin{cases} \varepsilon_r^{pt} = \varepsilon_r^{pe} + \varepsilon_r^{pp} \\ \varepsilon_\theta^{pt} = \varepsilon_\theta^{pe} + \varepsilon_\theta^{pp} \end{cases} \tag{15}$$

Elastic strains components in plastic zone still can be obtained by generalized Hook law.

$$\begin{cases} \varepsilon_r^{pe} = \dfrac{1-v^2}{E}\left[\sigma_r^p - \left(\dfrac{v}{1-v}\right)\sigma_\theta^p\right] \\ \varepsilon_\theta^{pe} = \dfrac{1-v^2}{E}\left[\sigma_\theta^p - \left(\dfrac{v}{1-v}\right)\sigma_r^p\right] \end{cases} \tag{16}$$

Plastic constitutive relation is necessary to solve the plastic strains components in plastic zone. In this study, incremental theory was adopted. The flow rule is given by

$$d\varepsilon_{ij}^{pp} = \lambda \frac{\partial Q}{\partial \sigma_{ij}} \tag{17}$$

where $d\varepsilon_{ij}^{pp}$ is plastic strain increment, Q is plastic potential, is non-negative constant. The flow rule is called associated flow rule if yield function (Mohr-Coulomb yield function in this study) is equal to plastic potential, otherwise the flow rule is called non-associated flow rule. The ratio of dilation for yielding rocks will usually be overestimated if associated flow rule is used. So, non-associated flow rule is adopted by making yield function unequal to plastic potential. Generally plastic potential is assumed to have the same form with yield function. But internal friction angle ϕ was replaced by dilation angle ψ, thus the plastic potential can be expressed in the following form.

$$Q = \sigma_\theta - \frac{1+\sin\psi}{1-\sin\psi}\sigma_r - \frac{2c\cos\psi}{1-\sin\psi} \tag{18}$$

Plastic strain increment $d\varepsilon_r^p$, $d\varepsilon_\theta^p$ can be obtained by combining equation (17) with equation (18).

$$\begin{cases} d\varepsilon_r^{pp} = \lambda \dfrac{\partial Q}{\partial \sigma_r} = -\lambda \dfrac{1+\sin\psi}{1-\sin\psi} \\ d\varepsilon_\vartheta^{pp} = \lambda \dfrac{\partial Q}{\partial \sigma_\theta} = \lambda \end{cases} \tag{19}$$

The relation between plastic strains ε_r^{pp} and ε_θ^{pp} can be obtained by transforming equation (19) into one formula and integrating.

$$\varepsilon_r^{pp} + k\varepsilon_\theta^{pp} = 0 \tag{20}$$

where $k = \dfrac{1+\sin\psi}{1-\sin\psi}$. Compatibility conditions for planar axisymmetric problem will be as follows.

$$\begin{cases} \dfrac{du_r}{dr} = \varepsilon_r^{pt} \\ \varepsilon_\theta^{pt} = \dfrac{u_r}{r} \end{cases} \tag{21}$$

Radial displacement u_r can be obtained from equation (15), (20) and (21).

$$\frac{du_r}{dr} + k\frac{u_r}{r} = \varepsilon_r^{pe} + k\varepsilon_\theta^{pe} \tag{22}$$

By equation (6), (16) and (22), u_r can be also obtained.

$$\frac{du_r}{dr} + k\frac{u_r}{r} = Lr^B + M \tag{23}$$

$$\begin{cases} L = \dfrac{C}{E}\left(\dfrac{1}{a}\right)^B \left[(-k-1)(1+A)v^2 - (k+A)v + kA + 1\right] \\ M = \dfrac{C}{E}\left[2(k+1)v^2 + (k+1)v - k - 1\right] \\ A = \dfrac{1+\sin\phi_r}{1-\sin\phi_r} \\ B = \dfrac{2\sin\phi_r}{1-\sin\phi_r} \\ C = c_r \cot\phi_r \end{cases} \tag{24}$$

Equation (23) is a first order nonhomogeneous differential equation. Boundary condition at the elastic-plastic interface is given by

$$r = R, u_r = \frac{R\left(p_0 - \sigma_r^R\right)}{2G} \tag{25}$$

In order to simplify computation, defining $T = \dfrac{R\left(p_0 - \sigma_r^R\right)}{2G}$ and solving equation (23), radial displacement u_r can be obtained.

$$u_r = \frac{L}{B+k+1}r^{B+1} + \frac{M}{k+1}r + \left(-\frac{L}{B+k+1}R^{B+k+1} - \frac{M}{k+1}R^{k+1} + TR^k\right)r^{-k} \tag{26}$$

Equation (26) is different from existed solutions. The detailed example and explanation will be given in section 3.

2.2 Analytical solution of tunnel with rockbolts

Rockbolts are widely used in rock tunnel because of their excellent performance in reducing surrounding rock mass deformation and plastic zone radius. For the tunnel with

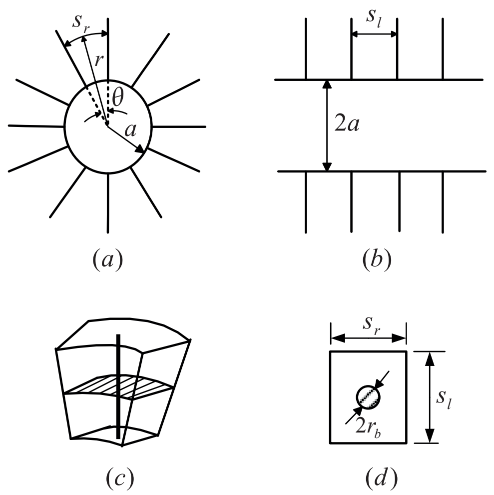

Fig. 3 The tunnel with pattern rock

rockbolts, a coupling support system will be formed by combing rockbolts with surrounding rock. In this study, homogenization method was applied to simplify the problem. Fig.3 shows a tunnel with pattern rockbolts. If rockbolt spacing is small enough (namely the rockbolt density is large), the coupling support system was considered as a new homogeneous, isotropic composite material whose parameters are strengthened to equivalent material on macroscale.

The strength parameters of equivalent material are influenced by both rock mass parameters and rockbolts parameters. Fig. 3 shows bolts parameters i.e. tangential angle θ, tangential space s_r, longitudinal space s_l, radius of bolt r_b. Rockbolt density parameter α is defined as follows.

$$\alpha = \frac{2\pi r_b\eta}{s_l s_r} = \frac{2\pi r_b\eta}{s_l r\theta} \Rightarrow \frac{2\pi r_b\eta}{s_l a\theta} \tag{27}$$

where η is a friction coefficient between rockbolts and rock mass. It is relate to the roughness of bolts. In order to keep the equivalent material as properties of continuous, homogeneous and isotropic, $r = a$ was applied in equation (27).

By considering rock mass and rockbolts proportion of cross area (Fig. 3(d)), equivalent Young's modulus of equivalent material can be obtained.

$$E^* = \frac{E_b\pi r_b^2 + E\left(s_l s_r - \pi r_b^2\right)}{s_l s_r} \Rightarrow \frac{E_b\pi r_b^2 + E\left(s_l a\theta - \pi r_b^2\right)}{s_l a\theta} \tag{28}$$

where E_b is Young's modulus of rockbolt, likewise $r = a$ was applied in equation (28).

Mohr-Coulomb criterion is assumed to be value of the equivalent material. Equation (2) can be given by

$$\sigma_\theta = \frac{1+\sin\phi}{1-\sin\phi}\sigma_r + \frac{2c\cos\phi}{1-\sin\phi} \tag{29}$$

Fig.4 shows the yield locus of equation (29) in principal stress space. Mohr circle is shown in Fig. 5.

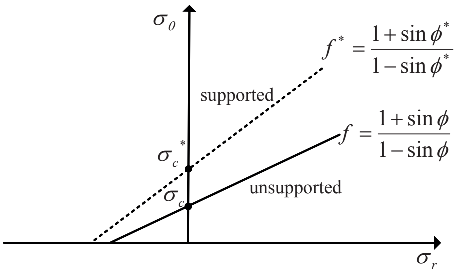

Fig. 4 Yield locus in principal stress space

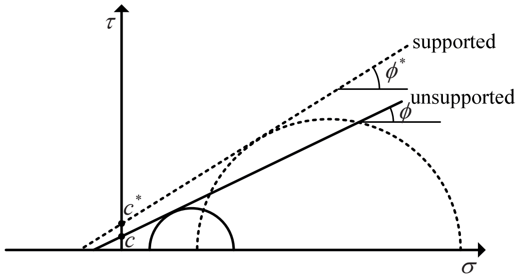

Fig. 5 Mohr circle

f and f^* are gradients of yield locus of rock mass in tunnel without or with rockbolts (Fig. 4).

$$f = \frac{1+\sin\phi}{1-\sin\phi}, \tag{30}$$

$$f^* = \frac{1+\sin\phi^*}{1-\sin\phi^*}. \tag{31}$$

where c^* and ϕ^* are cohesion and internal friction angle of equivalent material respectively (Fig. 5). The intercepts of yield locus (Fig. 4) are uniaxial compressive strengths for rock mass and equivalent material.

$$\sigma_c = \frac{2c\cos\phi}{1-\sin\phi}, \tag{32}$$

$$\sigma_c^* = \frac{2c^*\cos\phi^*}{1-\sin\phi^*}. \tag{33}$$

For tunnel reinforced by rockbolts, the gradient and intercept will increase, and cohesion and internal friction angle increased actually. The increase of gradient and intercept is related with the rockbolts density parameter α. The relations are obtained by the following equations.

$$\begin{cases} f^* = (1+\alpha)f \Rightarrow \dfrac{1+\sin\phi^*}{1-\sin\phi^*} = (1+\alpha)\dfrac{1+\sin\phi}{1-\sin\phi} \\ \sigma_c^* = (1+\alpha)\sigma_c \Rightarrow \dfrac{2c^*\cos\phi^*}{1-\sin\phi^*} = (1+\alpha)\dfrac{2c\cos\phi}{1-\sin\phi} \end{cases} \tag{34}$$

Equivalent cohesion and equivalent internal friction angle can be obtained by solving equation (34).

$$\begin{cases} \phi^* = \sin^{-1}\left[\dfrac{(1+\sin\phi)\alpha + 2\sin\phi}{(1+\sin\phi)\alpha + 2}\right] \\ c^* = \dfrac{c(1+\alpha)(1-\sin\phi^*)\cos\phi}{(1-\sin\phi)\cos\phi^*} \end{cases} \tag{35}$$

In this study, equivalent cohesion is given instead of equivalent uniaxial compressive strength. So more convenient calculations can be applied based on solution for tunnel without rockbolts.

3 Examples and comparison of solutions

An example is presented based on the above analytical solutions of tunnel. Some parameters are given by Table 1.

Table 1 Parameters of the example model

p_0(MPa)	a(m)	E(MPa)	υ	c(MPa)	c_r(MPa)	φ(o)
2	2	5000	0.2	0.276	0.055	35
φ_r(o)	ψ(o)	r_b(m)	s_l(m)	θ(rad)	E_b(MPa)	η^c
30	20	0.025	0.6	$\frac{\pi}{18}$	100000	$\tan\frac{\phi}{2}$

For tunnel without rockbolts, results were calculated based on the proposed analytical solution. Radial displacement of tunnel wall is 0.017238 m, radius of plastic zone is 5.5616 m. Fig. 6 shows the comparisons of solutions which proposed by Park and Kim [24], Ogawa and Lo[25] and Reed[26]. Radius of plastic zone in this study is the same as Park, Kim and Ogawa, Lo, which approach to 0.0550 m. But the value given by Reed is slightly small, which is 0.046974 m. All results show that the radial displacement reduces with the increase of distance form tunnel axis, and it will slow while reaching to elastic zone. The largest displacement was given by Park and Kim. The results of

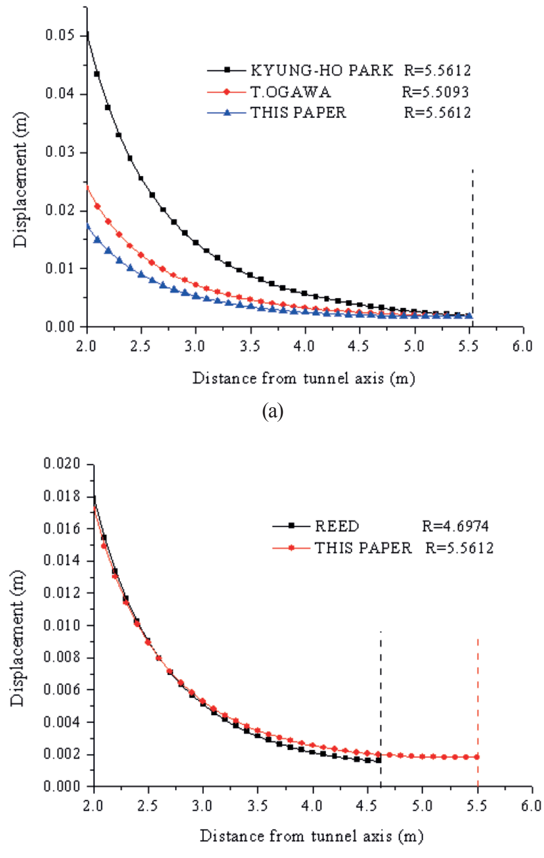

Fig. 6 Displacement comparison of tunnel

this study and Reed are the minimum. The differences of Park and Kim, Ogawa and Lo and this study are caused by different calculation of tangential elastic strain component of plastic zone. Initial hydrostatic stress was concluded in Park and Kim.

For tunnel with rockbolt, strength parameters of equivalent material can be obtained by substituting the parameters in Table 1 into equation (28) and equation (31), which were shown in Table 2.

Table 2 Strength parameters of equivalent material

Parameter	Value	Parameter	Value
E(MPa)	5000	E^*(MPa)	5893.4
c(MPa)	0.276	c^*(MPa)	0.307
c_r(MPa)	0.055	c_r^*(MPa)	0.061
Φ(o)	35	Φ^*(o)	39.8
Φ_r(o)	30	Φ_r^*(o)	34.9

Radius of plastic zone and displacement for tunnel without and with rockbolts were obtained. The comparisons are shown in Fig. 7. For tunnel with rockbolts, radius of plastic zone is 4.0283 m and displacement of tunnel wall is 0.005696 m. Dramatic decline will occur after the installation of rockbolts. It is an effective way to reinforcement of tunnels.

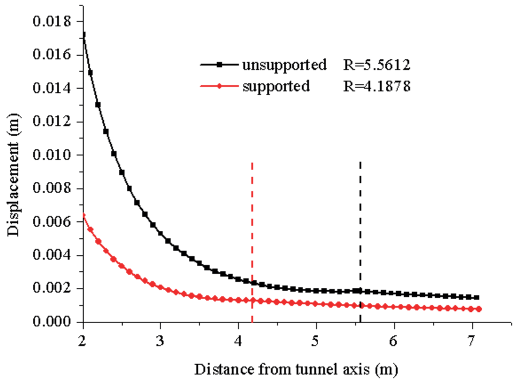

Fig. 7 Displacement comparison of tunnel with or without rockbolts

In order to investigate the influence of different parameters on the radius of plastic zone and displacement of tunnel, the comparisons were made in different r and η. When the radius of tunnel is 2 m, other parameters are listed in Table 1. The friction coefficient between bolts and rock mass η is $\tan\frac{\phi}{4}$, $\tan\frac{\phi}{2}$, $\tan\frac{2\phi}{3}$ and $\tan\phi$ respectively. The results were shown in Fig. 8 (a). The restriction effect by rockbolts will increase with the increasing of friction coefficient between bolts and rock mass. The displacements and radius of plastic zone will decrease. Fig. 8(b) shows the displacements and radius of plastic zone when radius of tunnel is 1 m, 2 m, 3 m and 4 m, respectively. Both will increase with the increasing of radius of tunnel.

A numerical solution was presented to verify the proposed solutions. The numerical model was built by finite FLAC (See

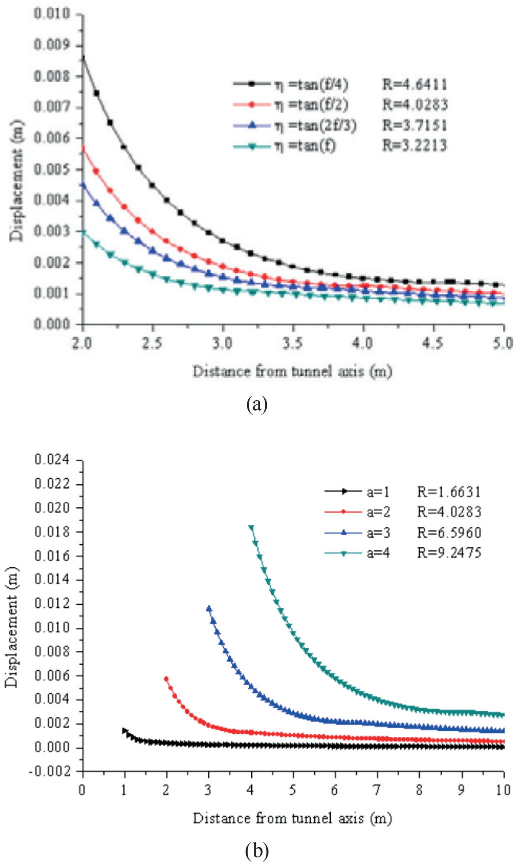

Fig. 8 Reinforcement effect of tunnel in different parameters

Fig. 9). For it's a planar axisymmetric problem, quarter model was adopted. The model size is 25 m × 25 m, the length of bolts is 3 m, ultimate tension of rockbolts is 1.0 × 10^6 N, the strength parameters of grouts were the same as the rock in this study. Other parameters were listed in Table 1.

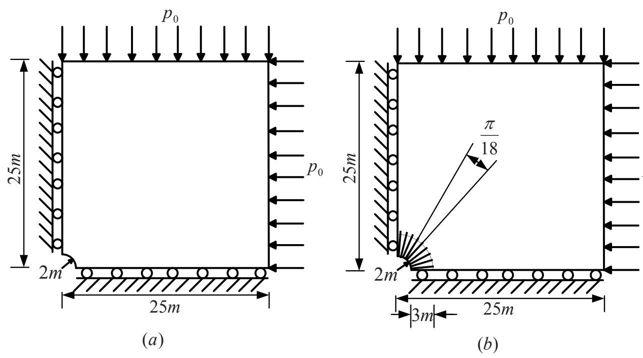

Fig. 9 Numerical model of tunnel

The comparison for analytical solutions and numerical solution were shown in Fig. 10. For the tunnel without rockbolts, the radius of plastic zone is about 5.5 m in numerical analysis. It is almost the agreement with Park and Kim, Ogagwa and Lo and this study. But for tunnel with rockbolts, the radius of plastic zone is about 3 m. It obviously differs from all the results. Displacement of numerical analysis is larger than all the analytical solutions. The results of numerical model will affected by lots of factors, but all the solutions are in the same order. It can provide a reference for practical projects on some degree.

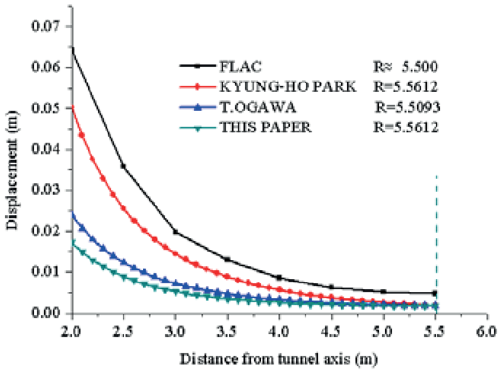

Fig. 10 Displacement comparison of tunnel with rockbolts in numerical analysis

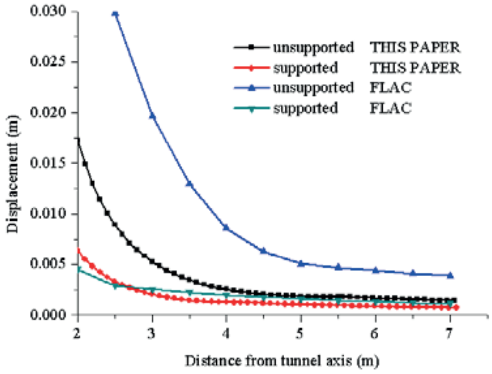

Fig. 11 Displacement comparison of tunnel with or without rockbolts in numerical analysis

Table 3 Computing results comparison for different methods

	KYUNG-HO PARK	T.OGAWA	REED	FLAC	THIS PAPER
Unsupport-displacement (m)	0.050233	0.023902	0.017885	0.064090	0.017238
Support-displacement (m)	-	-	-	0.004545	0.005696
Unsupport-radius of plastic zone (m)	5.5612	5.5093	4.6974	5.5000	5.5612
Support-radius of plastic zone (m)	-	-	-	3.0000	4.0283
Restriction percentage	-	-	-	↑92.91%	↑66.96%

Displacement comparison was shown in Fig. 11. As we have seen in the above section. The displacement of numerical mode for tunnel without rockbolts is larger than solution in this study. Both displacements will decrease after the installation of rockbolts. The values are approximately in the same level. To some extent, the proposed analytical solutions can indicate situations for tunnels before and after excavating.

Rockbolt is an effective reinforcement method on reducing radius of plastic zone and displacement for tunnel. The different results were listed in Table 3. The results show that restriction for displacement of tunnel increased 66.96% by rockbolts, while this value approached 92.91% while using numerical analysis.

4 Reliability analysis of tunnel

Due to the complexity of rock mass, reliability analysis is necessary and important to estimate the tunnel stability. Failure probability and reliability index are the measurements index for the stability of tunnels. The following is the performance function of the reliability analysis of tunnel.

$$z = g\left(x_1, x_2, \cdots, x_n\right) \qquad (36)$$

where $x_1, x_2, \ldots, x_n$ are basic random variables which influence the stability of tunnel. The tunnel is instability while $Z < 0$, it is stability while $Z > 0$, and it is in limit state while $Z = 0$.

The displacement is the most measurable information in practical projects. The magnitude of displacement concerns the safety, adaptability and durability of tunnel. In this study, permissible limit displacement of tunnel wall was adopted as instability criterion of tunnel. According to Chinese industry standard TB1003-2005 Code for Design on Tunnel of Railway [27], ultimate relative displacement of tunnel is listed in Table 4.

Table 4 Ultimate relative displacement of tunnel (%)

surrounding rock classification	tunnel depth h (m)		
	h ≤ 50	50 < h ≤ 300	300 < h ≤ 500
	relative settlement of vault		
II	-	0.01 ~ 0.05	0.04 ~ 0.08
III	0.01 ~ 0.04	0.03 ~ 0.11	0.10 ~ 0.25
IV	0.03 ~ 0.07	0.06 ~ 0.15	0.10 ~ 0.60
V	0.06 ~ 0.12	0.10 ~ 0.60	0.50 ~ 1.20

Permissible limit displacement of the proposed tunnel is given to be 0.03 m by combining with value from Table 1 and Table 4. So the performance function is obtained in the following.

$$\begin{cases} Z = 0.03 - u_r\left(c,\phi,c_r,\phi_r,E,\nu\right) & \textit{tunnel without rockbolts} \\ Z = 0.03 - u_r\left(c^*,\phi^*,c_r^*,\phi_r^*,E^*,\nu\right) & \textit{tunnel with rockbolts} \end{cases} \qquad (37)$$

Where $u_r(c, \phi, c_r, \phi_r, E, \nu)$ and $u_r(c^*, \phi^*, c_r^*, \phi_r^*, E^*, \nu)$ are the displacement of tunnel (equation (24) and (26)). The random variables and its statistics are listed in Table 5.

Table 5 Random variables and its statistics parameters

tunnel without rockbolts			tunnel with rockbolts		
random variable	mean	standard deviation	random variable	mean	standard deviation
c (Pa)	276000	41400	c^* (Pa)	307000	51340
Φ (rad)	0.61	0.0183	Φ^* (rad)	0.69	0.0122
c_r (Pa)	55000	6050	c_r^* (Pa)	61000	7620
Φ_r (rad)	0.52	0.0140	Φ_r^* (rad)	0.61	0.0463
E (Pa)	5 × 109	0.28 × 109	E^*(Pa)	5.9 × 109	0.43 × 109
υ	0.2	0.0024	υ	0.2	0.0024

Note: All random variables are normal distribution.

Monte Carlo Simulation (MCS) is adopted to calculate the reliability of tunnel in this study. Firstly, n random numbers were generated based on the distribution properties of random variable. Then, these random numbers were substituted into performance function and the displacements were obtained. Finally, the failure probability of tunnel P_f can be expressed as

$$P_f = \frac{n_0}{n} \tag{38}$$

Reliability index β can be obtained based on failure probability.

$$\beta = -\Phi^{-1}\left(P_f\right) \tag{39}$$

In which Φ^{-1} is inverse function of standard normal distribution function. Table 6 listed the failure probability and the reliability index of tunnel. When coefficient of variation for the random variable is the same, failure probability of tunnel without rockbolts was larger than tunnel with rockbolts. The reliability index was smaller. Its show the displacement of tunnel wall was reduced by rockbolts, tunnel was much reliable than tunnel without rockbolts.

Table 6 Failure probability and reliability index of tunnel

sample times	tunnel without rockbolts		tunnel with rockbolt	
	P_f	β	P_f	β
10^4	2.78%	1.9142	0.0600%	3.2389
10^5	2.53%	1.9542	0.0360%	3.3818
10^6	2.59%	1.9447	0.0361%	3.3811

5 Conclusions

In this paper, an elastoplastic analytical solution of circular tunnel with rockbolt was proposed based on homogenization method. Rock mass and rockbolt are considered to be a new homogeneous, isotropic, parameters strengthened equivalent composite material. A numerical example is used to verify the proposed analytical solution. The reliability method was adopted to estimate the stability of the circular tunnel. The reliability index and failure probability were calculated based on the proposed analytical solutions. The results show rockbolts have good effect on reducing surrounding rock mass displacement of tunnel. The proposed method can also be effectively used to perform the stability and reliability analysis of tunnel.

(1) Homogenization method was applied to tunnel with rockbolts. The composite material of rock mass and rockbolts was considered as a new homogeneous, isotropic, parameters strengthened equivalent material on macroscale. A proposed method was proposed to obtain the equivalent mechanical parameters such as Young's modulus, cohesion and internal friction angle.

(2) The displacements and radius of plastic zone for tunnel without rockbolts was calculated using a classic circular tunnel. The results show that the proposed analytical solutions dramatically reflect displacement of tunnel and reinforcement effect by rockbolts. It is important to understand and analyze the mechanical mechanism of rockbolts in tunnel.

(3) Failure probability and reliability index were calculated based on MCS. The results show that tunnel became much reliable and safety with rockbolts than without rockbolts.

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