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Simple Modelling of Undrained Shear Response of Granular Materials

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RESEARCH ARTICLE

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Abstract
A vast amount of past experimental investigations reported that the internal peak angle of sand was jointly governed by the density and effective stress level. Several relationships were proposed between these elements. The dependence of dilatancy characteristics on the internal state of a granular material was examined and revealed. A simple constitutive model framework was established on a basis of several well-proven and experienced relationships for granular materials to simulate their undrained shear behaviour. A basic hardening law connecting the varying tendency of the stress ratio with shear strain was employed. This model is capable of predicting the undrained monotonic stress-strain relationship of granular materials at different densities and various confining pressures. A series of parametric studies are conducted to investigate the susceptibility of the simulation results to the selected parameters. The simulation results also confirm the influential influences of dilatancy and deformability on the shear characteristics of granular materials at the critical state.

Keywords
undrained shear behaviour, granular material, stress-dilatancy, constitutive relation, critical state

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1 Introduction

The shear strength and deformation characteristics of granular soils subjected to shearing is of great interest in geomechanics and geoengineering practice [1]. The sands during shearing tend to contract at loose state and show dilative tendency at dense state. Thus, the density state is regarded as an influential factor governing the shear response of sands. The mean stress level is also an important parameter affecting the shear response of sand. The past research data revealed that a continue rise in the mean effective stress suppressed the dilation tendency and intensified the contraction behaviour [2–6].

In undrained conditions, flow failure of loose sand resulting from the steady state deformation following the unstable behaviour is liable to occur. The steady state [7] is a state of deformation of soil without effective stress increment or decrement and with migration of pore water. The steady state is regarded as anonymous to the critical state in many experimental investigations [8, 9]. The latter term is employed in the following section. The shear strength at the critical state was believed to be the undrained strength of granular soils with a flow failure. The flow deformation would continue infinitely once the outer force exceeds the undrained shear at the critical state [10]. It was realized that the determination of shear strength at the critical state was more important than the decision of the stress triggering liquefaction during associated evaluations.

In the past two decades, a great number of constitutive models were proposed to represent the shear strength and deformation properties of granular soils to consider the influences of density and confining pressure [11–15]. They made great contribution to the deep understanding of soil behaviour and the further development of theoretical research [16–19] Even so, the constitutive model of granular soils with simple calculation implementation is also desirable.

Some well-proven equations acquired from the drained triaxial tests are combined to formulate a simple constitutive model with the capacity of simulating the shear response of grain soils in this investigation. This study intends to provide a better understanding of the shear properties of granular materials with particular emphasis on the existence of critical state.

The establishment of this simple model is also expected to be employed to evaluate the liquefaction phenomena of sand in laboratorial test.

2 Framework of simple constitutive model

This simple model is constructed in terms of conventional triaxial compression space using stress and strain increment variables. The determination methods of the mean stress p, deviatoric stress q, volumetric strain increment $d\varepsilon_v$ and shear strain increment $d\varepsilon_s$ adopted in the simple constitutive model are expressed in Eqs. (1)–(4).

$$p = \frac{1}{3}(\sigma_a + 2\sigma_r) \tag{1}$$

$$q = \sigma_a - \sigma_r \tag{2}$$

$$d\varepsilon_v = d\varepsilon_a + 2d\varepsilon_r \tag{3}$$

$$d\varepsilon_s = \frac{2}{3}(d\varepsilon_a + d\varepsilon_r) \tag{4}$$

where σ_a is the axial stress, σ_r indicates the radial stress, $d\varepsilon_a$ and $d\varepsilon_r$ are the axial and radial strain increments, respectively. Stress ratio $\eta = q/p$ is obtained by the deviatoric stress q divided by the mean stress p. Actually, the framework of simple model is established to simulate the shear response of granular soils subjected to triaxial compression conditions. In the model, p and q correspond to the volumetric strain increment $d\varepsilon_v$ and shear strain increment $d\varepsilon_s$.

Some well-known equations associated with the strength and deformation characteristics of granular materials were summarized based on a large amount of experimental data in past investigations. The relevant relationships for this simple constitutive model are reviewed as below.

2.1 Peak shear strength estimation

Past investigations revealed that the peak friction angle ϕ_p of granular materials was jointly governed by the levels of initial density and confining pressure [1, 11, 20, 21]. A rise in density and a reduction in confining pressure enhance the peak friction angle ϕ_p of granular soils. The peak friction angle ϕ_p is composed of the frictional angle at the critical state ϕ_{cr} and the portion due to dilation behaviour. The friction angle ϕ is correlated with the stress ratio η by the equation $\sin\phi = 3\eta/(6 + \eta)$ at any state during triaxial shearing. Analogous to this relationship, the peak stress ratio of granular materials η_p could be expressed using Eq. (5).

$$\eta_p = \eta_{cr} + CD_r \ln\left(\frac{p_{cr}}{p}\right) \tag{5}$$

where η_p is the peak stress ratio, η_{cr} is the stress ratio at the critical state, D_r is the relative density and C is a material parameter related to the dilatancy characteristics of soils. p_{cr} refers to the mean stress when the peak stress ratio reduces to the stress ratio at the critical state. The peak stress ratio η_p only emerges for the soil at the dense state [22–24]. η_p approaches to η_{cr} at the critical state as the mean stress p is increased to be identical to p_{cr}. It is indicated that the peak stress ratio η_p is independent on the density and confining pressures once the mean stress p exceeds p_{cr}. Besides, p_{cr} is also greatly affected by the level of density.

2.2 Stress-dilatancy

Stress-dilation relationship is also an important mechanical characteristic to describe the shear response of soils. This relationship captures the evolution of strain increment in accompany with the stress ratio of granular materials in drained shearing test. Well prediction of the stress-dilatancy characteristic plays a major role in the usefulness of a constitutive model. Some classical expressions were proposed to characterize the stress-dilation behaviour [14, 25–29]. The empirical equation in Cam-clay model [27] is written in Eq. (6).

$$D = M - \eta, \tag{6}$$

where $D = d\varepsilon_{vd}/d\varepsilon_s$ is the rate of dilation, $d\varepsilon_{vd}$ is the volumetric strain increment due to dilation and $d\varepsilon_s$ is the shear strain increment. M represents the stress ratio at the critical state and is regarded as synonymous to η_{cr} in Eq. (5). It is noted that the summary of rate of dilation D and stress ratio η is a constant in Eq. (6). For simplicity, the plastic strains are approximately regarded as the same to the corresponding total strains. For $\eta < M$, $d\varepsilon_{vd}$ is positive value and expresses contractive tendency. For $\eta > M$, $d\varepsilon_{vd}$ is negative value and indicates dilative tendency. However, the influence of the material internal state (density) is neglected in this unique equation between the stress ratio and dilatancy ratio for granular materials. Li and Dafalias [29] proposed a state-dependent stress-dilatancy form using the state parameter ψ as the state variable in Eq. (7).

$$D = d_o\left(e^{m\psi} - \frac{\eta}{M}\right) = \frac{d_o}{M}\left(Me^{m\psi} - \eta\right) \tag{7}$$

where m and d_o are two positive parameters. It is noticed that the Eq. (6) is the specific case of Eq. (7) when m is equal to zero and d_o is identical to M. At the critical state, the $\psi = 0$ and $\eta = M$ lead to a zero dilatancy. Thus, Eq. (7) is positively included in this simple model.

To adopt the stress-dilatancy relationship in Eq. (7), it is important to decide the location of the critical state line in the specific volume and mean stress space so as to calculate the state variable ψ. $\psi = e - e_c$ is defined as the difference between the current void ratio e and the critical void ratio e_c on the critical state line at current stress level p. Additionally, the critical state line is mathematically expressed using the form $e = e_\Gamma - \lambda(p/p_a)^\xi$ [30]. e_Γ, λ_{csl} and ξ are the material parameters determining the position of critical state line. p_a indicates the atmosphere pressure.

2.3 Distortion hardening law

The distortion relationship linking the development of stress ratio η with shear strain $d\varepsilon_s$ was proposed and well-proven in the hyperbolic stress-strain model [20, 31]. This simple hardening law assumes that granular soils are constantly approaching to the peak shear stress ratio η_p with increasing shear strain ε_s in Eq. (8).

$$\eta = \frac{\eta_p \varepsilon_s}{A + \varepsilon_s} \tag{8}$$

A is a parameter associated with the initial shear modulus of soils.

2.4 Model construction

Particular expressions reviewed above are combined and formulated to create a new simple constitutive model. The specific procedure to implement the calculation using the proposed simple model is explained in this section. The total volumetric strain increment $d\varepsilon_v$ is supposed to be composed of the portion due to consolidation $d\varepsilon_{vc}$ and the portion induced by dilation $d\varepsilon_{vd}$. The volumetric strain increment due to consolidation $d\varepsilon_{vc}$ in Eq. (9) could be calculated from the relationship between the mean stress p and the void ratio e during isotropic consolidation loading.

$$d\varepsilon_{vc} = \frac{\lambda}{1+e_o}\frac{dp}{p} \tag{9}$$

where λ is the slope of isotropic consolidation line, e_o represents the initial void ratio of sand.

The total volumetric strain increment $d\varepsilon_v$ could be acquired using Eqs. (7) and (9).

$$d\varepsilon_v = d\varepsilon_{vc} + d\varepsilon_{vd} = \frac{\lambda}{1+e_o}\frac{dp}{p} + \frac{d_o}{M}\left(Me^{m\psi} - \eta\right)d\varepsilon_s \tag{10}$$

For undrained condition, the total volumetric strain increment $d\varepsilon_v$ should be zero for a saturated sample when the pore fluid and soil particle compressibility are neglected. Thus, the expression in Eq. (10) is rearranged to display the mean stress increment dp,

$$dp = -\frac{p(1+e_0)}{\lambda} d_o \left(e^{m\psi} - \frac{\eta}{M}\right) d\varepsilon_s \tag{11}$$

Putting the Eqs. (8) and (5) into the Eq. (11), we further obtain the exact form of mean stress increment dp,

$$dp = -\frac{p(1+e_0)}{\lambda} d_o \left[e^{m\psi} - \frac{\varepsilon_s}{\varepsilon_s + A}\left(1 + \frac{CD_r}{M}\ln\frac{p_{cr}}{p}\right)\right] d\varepsilon_s \tag{12}$$

Eq. (12) descripts the variation in mean stress dp with the rise in the shear strain increment $d\varepsilon_s$ during undrained shearing in triaxial shearing test. The calculation implementation of dp is integrated numerically. Firstly, dp could be decided using Eq. (12) for a given increment $d\varepsilon_s$. Secondly, the mean stress ($p \leftarrow p + dp$) and shear strain ($\varepsilon_s \leftarrow \varepsilon_s + d\varepsilon_s$) could be updated using the superposition method. Then, the peak stress ratio η_p could be re-determined using the new value of p and Eq. (5). The stress ratio η could also be updated using Eq. (8) so as to decide the new deviatoric stress q. Subsequently, the iteration calculation would continue with the same given increment $d\varepsilon_s$. The whole calculation would be completed once the desire level of shear strain is attained.

3. Simulation results

The simulation capacity of this simple constitutive model is demonstrated in this section. Figs. 1–3 show the measured and predicted stress-strain response of Toyoura sand at loose, medium-dense and dense states at various confining pressures. In each figure, subtitle (a) describes the effective stress paths of sands and (b) shows the corresponding stress-strain curves of sands. The solid symbols are the measured results and the solid lines represent the predicted values. Verdugo and Ishihara [32] conducted an extensive set of monotonic drained and undrained triaxial tests on Toyoura sand at a wide range of stresses and densities to determine the ultimate condition of sand. The initial void ratios of Toyoura sand are prepared as 0.907 (loose), 0.833 (medium-dense), 0.735 (dense). A set of high quality data was reported. The maximum and minimum void ratios of the Toyoura sand are 0.977 and 0.597, respectively. Toyoura sand is a uniform fine sand composed of subrounded to subangular particles and contain 75% quartz, 22% feldspar and 3% magnetite [33].

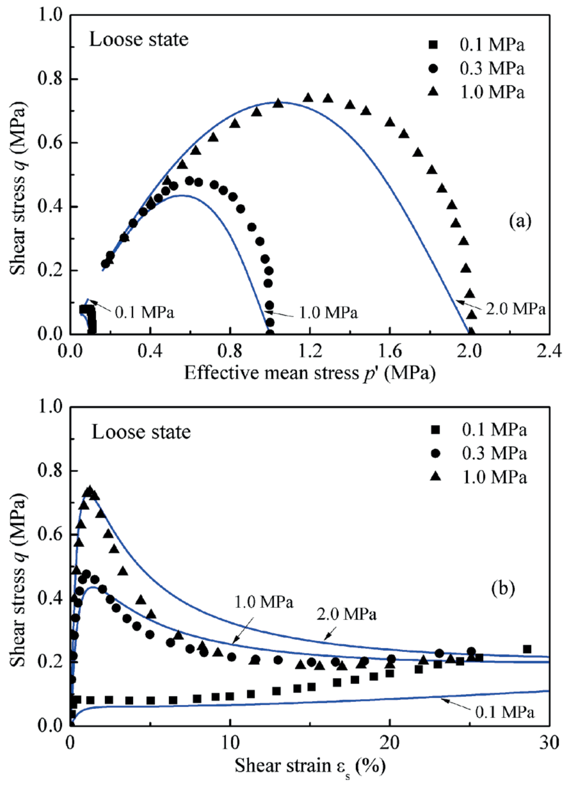

Fig. 1 Predicted versus measured stress-strain response of loose sand

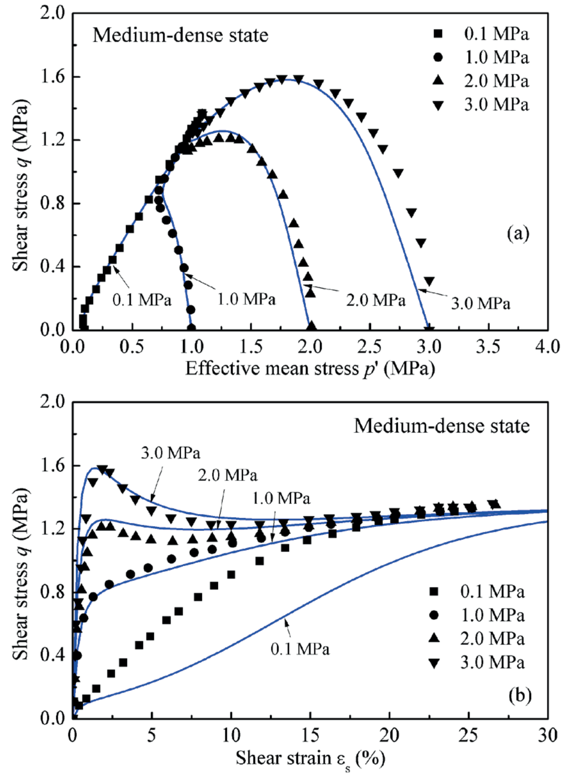

Fig. 2 Predicted versus measured stress-strain response of medium-dense sand

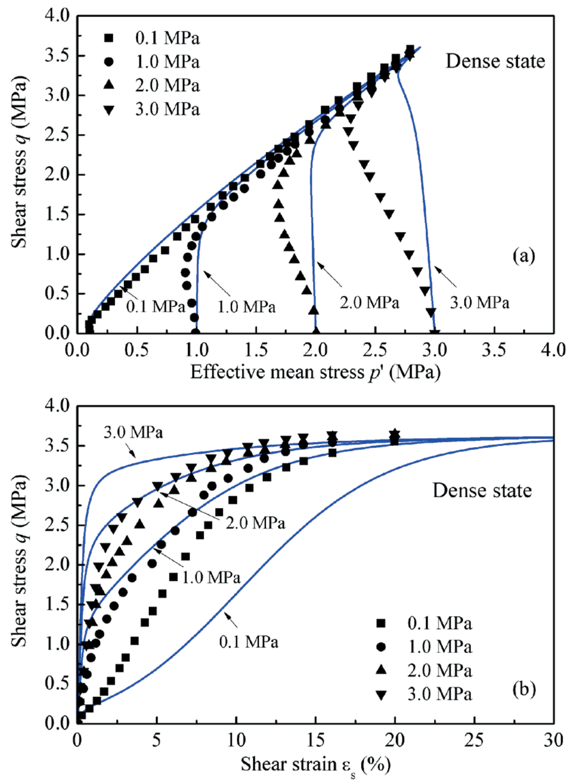

Fig. 3 Predicted versus measured stress-strain response of dense sand

The prediction values have a good agreement with the measured results of Toyoura sand subjected to undrained shearing in Figs. 1–3. This constitutive model has capacity of describing the pure contractive behaviour of loose samples at confining pressures of 1.0 and 2.0 MPa. This simple model is capable of predicting dilative behaviour of medium-dense sand at confining pressures of 0.1 MPa and 1.0 MPa and contractive behaviour of medium-dense sand at confining pressures of 2.0 MPa and 3.0 MPa in Fig. 2. Both of predicted and measured results show that the deviatoric stresses of medium-dense sand at confining pressures of 0.1 MPa and 1.0 MPa monotonically increase with the shear strain progresses. However, medium-dense sand rapidly grains the peak shear strength and gradually loses the deviatoric stress with the rise in shear strain. For dense sand, simulated values are generally consistent with the observation results in Fig.3. The slight deviation of predicted result from the measured value at a confining pressure of 0.1 MPa in Fig.3 is probably resulted from the strong dilation estimation at low confining pressures by the selected stress-dilatancy relation in Eq. (7).

It is noted that the simulation results of sand specimens at different confining pressures and initial densities approach to the constant deviatoric stress q which is the hint of the critical state as the axial strain progresses. It is indicated that the critical state could be attained at larger shear strains around 20% through the simulation using this simple constitutive model.

Table 1 shows the set of simple model parameters for Toyoura sand at three densities. The determination method of the associated parameters is simply explained. The difference in the stress ratios at the peak state and the critical state are plotted against the effective mean stress to determine p_{cr} using Eq. (5) in Fig. 4. It is seen that the rise in density increases the material parameter C and level of p_{cr}. With regard to the parameters related to the stress-dilatancy and the location of critical state line, they are independent on the density of sand. Fig. 5 presents the stress-dilatancy relationship of granular soils using Eq. (7) at different confining pressures and densities. The parameters m, d_o, M adopt the values shown in Table 1. The simulated curves express that a rise in density and reduction in confining pressure intensify the dilation tendency. Fig. 6 shows the plots of Toyoura sand determined at the critical state on the void ratio and effective mean stress space and its approximate simulation expression. λ can obtained from the isotropic consolidation curve and A can be acquired from the measured stress-strain curve at a small strain region.

Table 1 Constitutive model parameters for sand with different densities

	C	D_r (%)	p_{cr} (MPa)	m	d_o	M	e_Γ	λ_{csl}	ξ	λ	A
Loose (e_o = 0.907)	0.13	18.5	0.28	3.5	0.88	1.25	0.934	0.019	0.7	0.028	0.007
Medium-dense (e_o = 0.833)	0.37	37.9	1.2	3.5	0.88	1.25	0.934	0.019	0.7	0.035	0.005
Dense (e_o = 0.735)	0.47	63.7	3.0	3.5	0.88	1.25	0.934	0.019	0.7	0.050	0.002

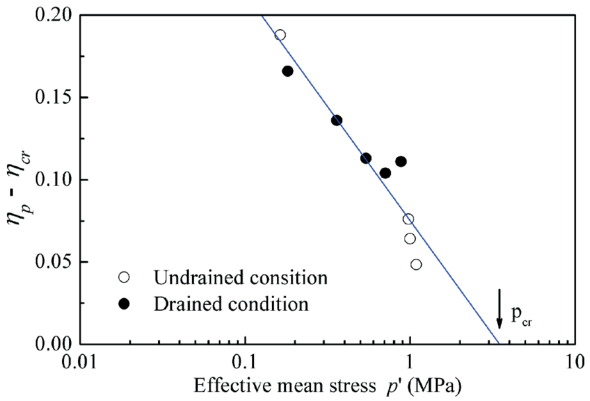

Fig. 4 Determination of the mean stress at the critical state p_{cr}

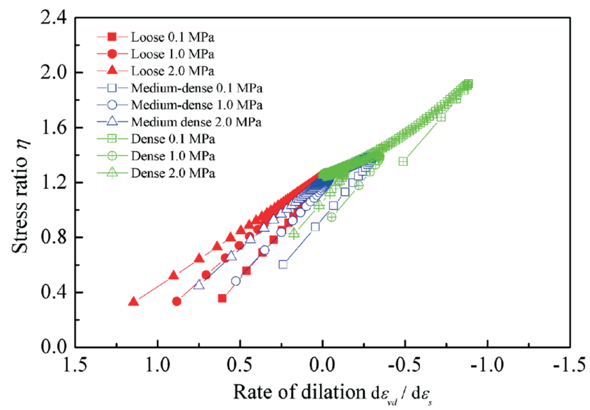

Fig. 5 Stress-dilatancy relationship of sand at different confining pressures and densities

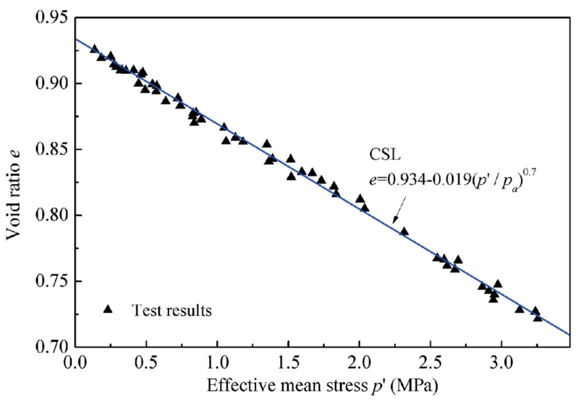

Fig. 6 Determination of the critical state line of sand

4. Parametric study

A series of parametric studies on the proposed constitutive model are conducted for a better understanding of the undrained shear characteristics of granular soils. The influences of the dilatancy characteristic parameters m and d_o in Eq. (7) and C in Eq. (5), shear deformability property A in Eq. (8) and compressibility characteristic λ in Eq. (9) on the simulated undrained shear behaviour are comprehensively investigated. It is known that parameters M, $e_{\Gamma l}$, λ_{csl}, ξ are the inherent parameters of granular soils. Accordingly, the discussion on the influences of these inherent parameters are excluded in this numerical investigation. Figs. 7–11 show the different simulation results of granular soils at loose state with relative density $D_r = 20\%$ using various parameters of the simple model presented in previous section. The selected parameters adopt various values to examine the susceptibility of the predicted undrained shear strength and behaviour of granular soils. The investigated parameters vary and are entitled with four different values for each simulation case. The remain parameters of the simple constitutive model are also shown in each figure and kept constant during each parametric study.

4.1 Analysis of the influences of parameter d_o and m

Parameters d_o and m are two important parameters affecting the evolution of strain increment ratio with a rise in the stress ratio q/p. Fig. 7 shows the stress-strain relationship and associated stress path of granular soils when the parameter d_o is equal to 0.48, 0.88, 1.28 and 1.68. It is clearly seen that all simulation results display contractive behaviour followed by dilation behaviour after the soils passing the phase transformation point. The deviatoric stress q monotonically increases with increasing shear strain when d_o is 0.48. The deviatoric stresses q decided at d_o =1.28 and d_o =1.68 initially increase and subsequently decrease before attaining the phase transformation point, finally reverse to increase again. After the phase transformation point, the simulation results exhibit stronger dilation tendency with a larger d_o. Parameter d_o alters the slope of the stress-dilatancy relationship described in Eq. (7). Thus, a rise in d_o leads to a remarkable dilation behaviour of granular soils after attaining the phase transformation point.

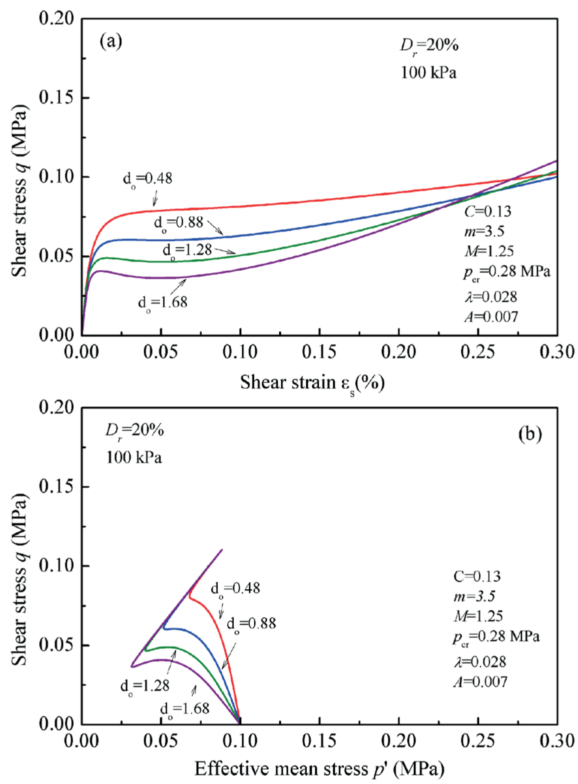

Fig. 7 Influence of parameter d_o on the computed undrained shear strength and behaviour of granular soils

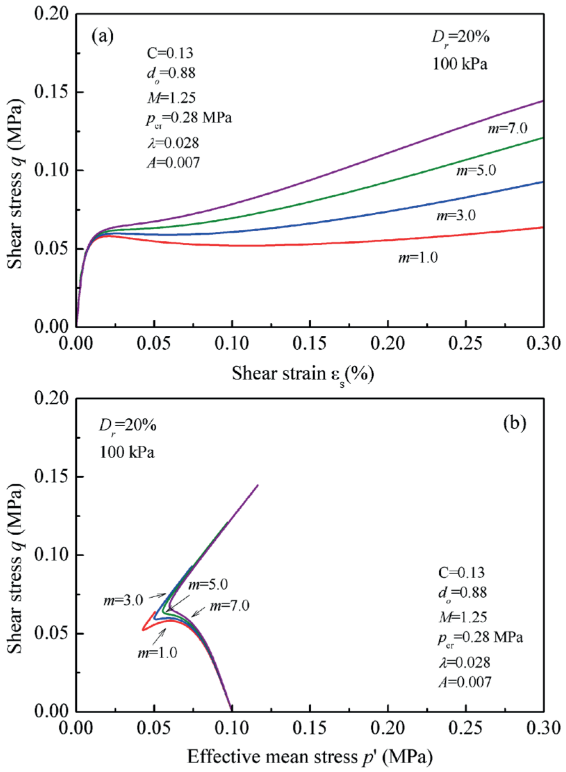

Fig. 8 Influence of parameter m on the computed undrained shear strength and behaviour of granular soils

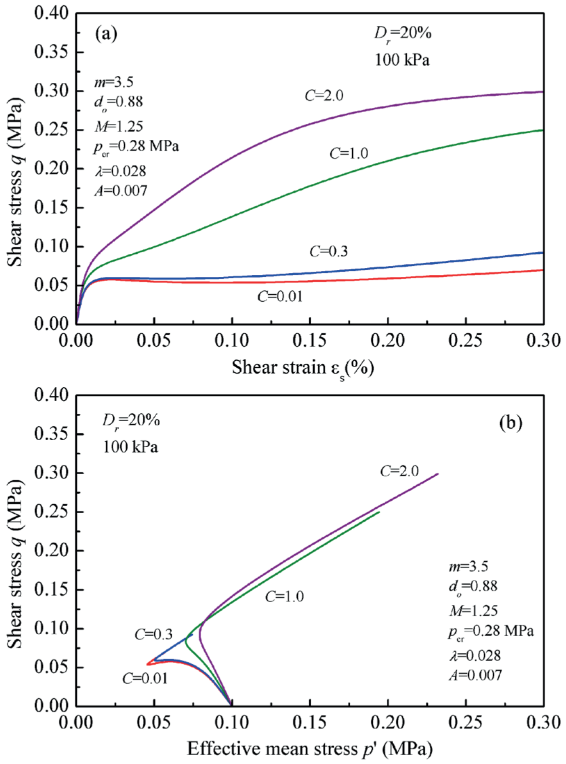

Fig. 9 Influence of parameter C on the computed undrained shear strength and behaviour of granular soils

The simulation results representing the stress path are convergent to a single line regardless of the magnitude of d_o. Fig. 8 shows the undrained shear response of the granular soils corresponding to the values of m as 1.0, 3.0, 5.0 and 7.0. The magnitude of m almost has very slight influence on the initial stiffness and the undrained shear strength at initial shearing stage. The simulation results demonstrate that the influence of m is limited at a small level of shear strain less than 0.025. It is distinctively seen that a rise in m markedly prompts the dilation behaviour of simulation results. It is noted that m is an exponential parameter expressed in Eq. (7). Accordingly, the influence of parameter m is minimal at small shear strains and becomes greater as the shear strain progresses. Besides, it is highlighted here that the evaluation of the influences of the parameters m and d_o should be jointly examined. These two parameters are simultaneously decided from the laboratory test data.

4.2 Analysis of the influence of parameter *C*

Fig. 9 shows the predicted undrained shear response of granular soils for various magnitudes of C identical to 0.01, 0.3, 1.0 and 2.0. It was mentioned in Eq. (7) that C was also a parameter related with the dilatancy characteristics of soils. The simulation results clearly express that granular soils exhibit initial contraction behaviour followed by strong dilation behaviour for C = 1.0 and 2.0 and display initial contraction behaviour followed by weak dilation behaviour for C = 0.01 and 0.3. The deviatoric stress q continually gains undrained shear strength with increasing shear strain for simulation results obtained using C = 1.0 and 2.0. It is demonstrated that the increase in parameter C promotes the dilation behaviour of granular soils for mean stress less than p_{cr}. Besides, the convergence of different simulation results for granular soils toward the steady state line is also seen in Fig. 9(b).

4.3 Analysis of the influence of shear ability parameter *A*

The parametric study on the shear deformability parameter A for granular soils is conducted to further clarify its physical meaning. The undrained shear stress-strain curve and associated stress path using A = 0.001, 0.01, 0.1 and 0.2 are also depicted in Fig. 10. It is distinctively seen that the increasing parameter A enhances both the initial stiffness and the undrained shear strength in Fig. 10(a). The simulation results exhibit very slight contractive but strong dilation behaviour for A = 0.001 due to its strong initial stiffness. Only the pure contraction behaviour of granular soils is seen when A takes the values of 0.1 and 0.2. The deviatoric stress increment is much smaller than the mean stress increment. The simulation response cannot mobilize an increase in deviatoric stress q due to the low initial stiffness, leading to a pure contractive behaviour. A larger parameter A gives a rise to a stronger dilation behaviour of granular soils under increasing shear strain.

4.4 Analysis of the influence of compressibility parameter *λ*

The dependence of simulation undrained shear response on the magnitude of the slope of the isotropic consolidation line λ is shown in Fig. 11. The slope of isotropic consolidation line λ

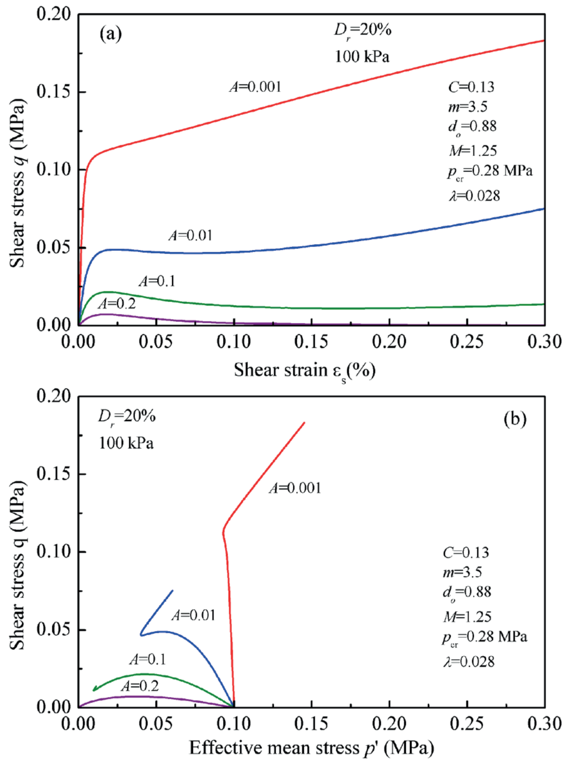

Fig. 10 Influence of shear deformability parameter A on the computed undrained shear strength and behaviour of granular soils

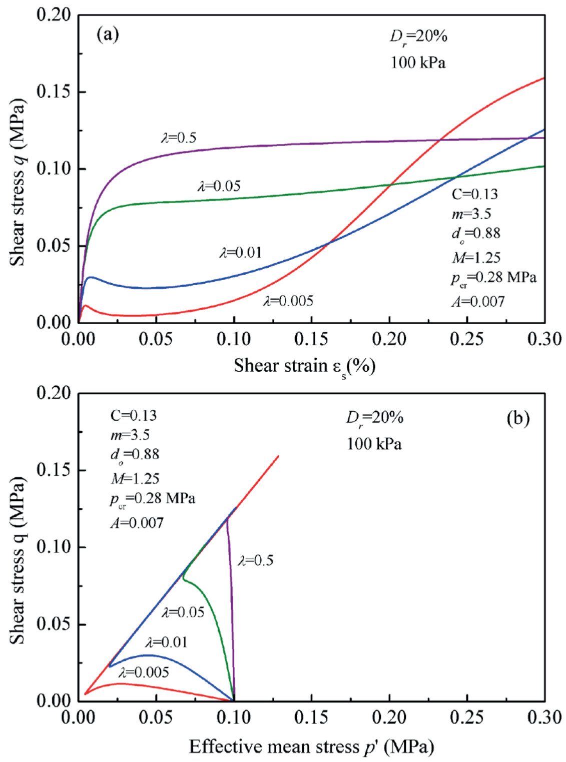

Fig. 11 Influence of compressibility parameter λ on the computed undrained shear strength and behaviour of granular soils

corresponds to the compressibility property and its reference value is selected as 0.005, 0.01, 0.05 and 0.5. Simulation results for λ of 0.005 display dramatic contraction behaviour at initial shearing strain region. A small compression index λ = 0.005 also reduces the undrained shear strength in advance of the soils attaining the phase transformation point. However, the granular soils immediately dilate with the stress path moving toward the right side. It is seen that the variation in the effective mean stress of simulation results for λ = 0.5 is very little during the entire shearing process. Thus, it illustrates that the pore pressure change is hard to mobilize for granular soils with a large λ.

5 Conclusions

A simple constitutive model was proposed based on some well-established relationships for sand obtained from drained shearing test to predict the undrain shear response and the static liquefaction of granular materials. The combined effects of density and mean stress level on the peak shear strength was considered and reflected in the relevant expressions. The influence of density on stress-dilatancy of granular material was captured in the framework of constitutive model. A rise in density and decline in the confining pressure lead to a marked dilation behaviour of granular material. These features are reflected by the stress-dilatancy relationship considering the state of granular materials. The parameters of constitutive model can be determined form the conventional triaxial shear test. The proposed simple constitutive model has the capacity of predicting density-dependent and stress-dependent undrained shear properties of granular material using a few parameters. The predicted results show good agreement with the measured values for sand at various densities and confining pressures.

The parametric studies demonstrate that a unique steady state line could be attained at large strains under different simulation conditions. From parametric studies, the deviatoric stress q with d_o = 0.48 monotonically increases with shear strain. A rise in d_o greatly enhances the dilation tendency of granular soils after passing the phase transformation point. The effect of parameter m is minimal at small shear strains and becomes greater as the shear strength progresses. A rise in dilatancy parameter C remarkedly prompts the dilation behaviour.

The remarked dependence of the simulation results on the shear deformability parameter A and compressibility parameter λ are also confirmed from parametric studies. A larger shear deformability A leads to a pure contraction behaviour. The larger the shear deformability A is, the stronger the dilation behavior exhibits. The simulation results for a small compressibility λ exhibit dramatic contraction behaviour followed by remarked dilation behaviour. On the other hand, the pore pressure experiences slight change during the entering shearing when a large λ is adopted. Thus, the parametric studies indicate that the liquefaction should be treated both as a deformation and strength problem.

Acknowledgement

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