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An Efficient Entropy-based Method for Reliability Assessment by Combining Kriging Meta-models

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Abstract

Meta-models or surrogate models are convenient tools for reliability assessment of problems with time-consuming numerical models. Recently, an adaptive method called AK-MCS has been widely used for reliability analysis by combining Mont-Carlo simulation method and Kriging surrogate model. The AK-MCS method usually uses constant regression as a Kriging trend. However, other regression trends may have better performance for some problems. So, a method is proposed by combining multiple Kriging meta-models with various trends. The proposed method is based on the maximum entropy of predictions to select training samples. Using multiple Kriging models can reduce the sensitivity to the regression trend. So, the propped method can have better performance for different problems. The proposed method is applied to some examples to show its efficiency.

Keywords

reliability, meta-model, simulation, Kriging, adaptive method

1 Introduction

The basic aim of reliability analysis methods is to estimate the failure probability by taking into account different uncertainties in the properties of real engineering structures. The failure probability can be defined by the following integral [1]

$$P_f = \mathrm{P}(\mathrm{G}(\mathbf{X}) \le 0) = \int_{\mathrm{G}(\mathrm{X}) \le 0} \mathrm{f}_{\mathbf{X}}(\boldsymbol{x}) \mathrm{d}\boldsymbol{x} , \quad (1)$$

where $\mathbf{X} = [\mathrm{X}_1, \ldots, \mathbf{X}_n]$ denotes the vector of random variables, $\mathrm{G}(\mathbf{X})$ indicates the performance function, and $\mathrm{f}_{\mathbf{X}}$ is the joint probability density function. Moreover, $\mathrm{G}(\mathbf{X}) \le 0$ denotes the failure region, and $\mathrm{G}(\mathbf{X}) = 0$ is called the limit state surface (LSF).

Reliability methods can be classified into two groups [2]: moment methods and simulation approaches. FORM and SORM methods are popular moment methods, which use the linear and quadratic approximation of the LSF, respectively [3]. The main difficulty of moment methods is related to the calculation of the gradient for problems with non-differentiable or highly nonlinear LSFs. On the other hand, simulation methods are powerful tools to approximate the failure probability for each arbitrary LSF [4]. The high computational effort is the main drawback of the simulation methods, especially for implicit LSFs with computationally expensive models. For instances, reliability analysis of a nonlinear finite element model (FEM) by simulation method may require several days or months. To overcome this drawback, some variance reduction methods have been proposed such as importance sampling [5, 6] and subset simulation [7, 8]. In spite of these improvements, simulation methods may require high computation cost for expensive-to-evaluate problems.

Usually, meta-models or surrogate models are used instead of expensive-to-evaluate functions to reduce the computational cost in simulation methods [9–11]. Kriging is a popular surrogate model, which has been used widely for different applications [12]. The main feature of the Kriging model is to provide the variance of predictions. Therefore, samples with high variance can be selected sequentially as training samples. Defining training samples with sequential manner is called adaptive sampling or active learning methods. The AK-MCS is a recently developed method that combines **A**ctive learning method with the **K**riging model and **M**onte **C**arlo **S**imulation for reliability analysis [13]. In the AK-MCS method, the constant

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regression has been used as a trend of the Kriging model. However, other regression trends may be a good choice for some problems. So, Kriging models with various regression trends can be built. The proposed method in this paper is based on the maximum entropy of predictions with multiple Kriging models. Indeed, a new learning function is introduced to select training samples from the MC population using the maximum entropy of predictions.

This paper is organized as follows: Section 2 introduces the AK-MCS method. Section 3 presents the proposed method for combining multiple Kriging models. Three examples are presented in Section 4 to show the efficiency of the proposed entropy-based method. Section 5 is the conclusion.

2 Reliability analysis using AK-MCS model

2.1 Kriging model

Kriging meta-model approximates the function as a combination of regression terms and departures as [12]

$$\mathrm{G}(\boldsymbol{x}) = \boldsymbol{f}(\boldsymbol{x})^{\mathrm{T}}\boldsymbol{\beta} + \mathrm{Z}(\boldsymbol{x}) , \tag{2}$$

where $\boldsymbol{\beta} = [\beta_1, \ldots, \beta_p]^{\mathrm{T}}$ indicates the vector of regression coefficients, $\boldsymbol{f}(\boldsymbol{x}) = [\boldsymbol{f}_1(\boldsymbol{x}),\ldots, \boldsymbol{f}_p(\boldsymbol{x})]^{\mathrm{T}}$ represents the basic functions, and $\mathrm{Z}(\boldsymbol{x})$ denotes a Gaussian process [14, 15]. To construct a Kriging model, training samples or design of experiments (DoE) $[\boldsymbol{x}_1,\ldots, \boldsymbol{x}_k]$ with $\boldsymbol{x}_i = \in \mathbb{R}^n$, and response values $\boldsymbol{Y}$ with $\mathrm{Y}_i = \mathrm{G}(x_i) \in \mathbb{R}$ should be defined.

In the Kriging model, the mean and variance of prediction by can be expressed as

$$\mu_{\hat{G}}(\boldsymbol{x}_0) = \mathbf{f}(\boldsymbol{x}_0)^{\mathrm{T}}\beta + \mathbf{r}_0^{\mathrm{T}}(\boldsymbol{x}_0)\mathbf{R}^{-1}(\mathbf{Y} - \mathbf{F}\beta), \tag{3}$$

$$\sigma_{\hat{G}}^2(\boldsymbol{x}_0) = \sigma^2\left[1 - \mathbf{r}_0^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{r}_0 + \mathbf{u}^{\mathrm{T}}\left(\mathbf{F}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{F}\right)^{-1}\mathbf{u}\right], \tag{4}$$

where $\mathbf{R}$ denotes the correlation between training samples, and $\mathrm{F}_{ij} = \mathrm{f}_j(\boldsymbol{x}_i)$, i = 1, …, k, j = 1, …, p shows the information matrix. Moreover, $\mathbf{u} = \mathbf{F}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{r}_0 - \mathbf{f}(\boldsymbol{x}_0)$, σ^2 is the variance of the process, and $\mathbf{r}_0$ denotes the correlation between the sample point $\boldsymbol{x}_0$ and training samples $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_k$.

The coefficients of regression and the variance can be computed as [16]

$$\boldsymbol{\beta} = (\mathbf{F}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{F})^{-1}\mathbf{F}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{Y} , \tag{5}$$

$$\sigma^2 = \frac{1}{\boldsymbol{k}}(\boldsymbol{Y} - \boldsymbol{F}\beta)^{\boldsymbol{T}}\boldsymbol{R}^{-1}(\boldsymbol{Y} - \boldsymbol{F}\beta) . \tag{6}$$

In the current study, the implementation of the Kriging model in Matlab toolbox DACE [15] is used to construct Kriging models.

2.2 AK-MCS method

The AK-MCS [13] combines Monte Carlo simulation and adaptive Kriging model for reliability as follows:

1. Generate Monte Carlo population according to joint probability density function.
2. Define an initial design of experiments (DoE), and evaluate these points on the original performance function (G).
3. Construct a Kriging model using the training samples and corresponding response values.
4. Predict the MC population (S) by the Kriging model and then estimate P_f as

$$\hat{P}_f = \frac{n_f}{N_{\mathrm{MC}}}, \tag{7}$$

where n_f is the number of samples with failed status ($\mathrm{G}(\mathbf{X}) \leq 0$), and N_{MC} is the size of the MC population.

5. Identify the best training sample by minimizing the learning function as

$$\mathrm{U}(\boldsymbol{x}) = \left|\mu_{\hat{G}}(\boldsymbol{x})\right| / \sigma_{\hat{G}}(\boldsymbol{x}). \tag{8}$$

This means that the sample with the minimum value of learning function is considered as the best training sample.

6. Check the stopping criterion by

$$\min\left[\mathrm{U}(\boldsymbol{x})\right] \geq 2 \quad \forall \boldsymbol{x} \epsilon \mathrm{S} . \tag{9}$$

7. If the stopping criterion is not satisfied, update the previous DoE with the best sample and call the original performance function. Then, repeat stages 3 to 7 until the stopping condition in Eq. (9) is satisfied.
8. Compute the coefficient of variation as

$$COV_{\hat{P}_f} = \sqrt{\frac{1 - \hat{P}_f}{N_{MC} . \hat{P}_f}}. \tag{10}$$

9. If $C.O.V_{Pf} > 0.05$, more samples should be added to MC population. Then go back to stage 4.
10. End AK-MCS.

According to Eqs. (3) and (4), the computation time for prediction of mean value and variance depends on the size of the MC population (N_{MC}), the dimension of the problem (n), the number of regression coefficients (p) and size of DoE (k). Moreover, according to steps 9 and 10 of the AK-MCS algorithm, the required size of MC population (N_{MC}) will be increased for problems with a very low probability of failure to satisfy the criterion of $C.O.V_{Pf} > 0.05$. Furthermore, the dimension of the problem (n) will be increased in high dimensional problems. So for problems with a very low probability of failure and high dimensions, the computation cost will be increased.

The stopping criterion according to Eq. (9) is conservative [17]. So, in the present study, another stopping criterion is used instead of Eq. (9) as [17]:

$$\frac{\hat{P}_f^+ - \hat{P}_f^-}{\hat{P}_f^0} \leq \epsilon_{\hat{P}_f}, \tag{11}$$

where $\hat{P}_f^+$ and $\hat{P}_f^-$ indicates the upper and lower bounds of failure probabilities, respectively. Moreover, $\hat{P}_f^0$ is failure probability using the mean values of predictions, and $\epsilon_{\hat{P}_f} = 5\ \%$ indicates the acceptable threshold error. The mean failure probability and the bounds of failure probabilities can be defined as

$$\hat{P}_f^0 = \mathrm{P}\left[\mu_{\hat{G}}(\boldsymbol{x}) \leq 0\right], \tag{12}$$

$$\hat{P}_f^{\pm} = \mathrm{P}\left[\mu_{\hat{G}}(\boldsymbol{x}) \mp \mathrm{k}\sigma_{\hat{G}}(\boldsymbol{x}) \leq 0\right]. \tag{13}$$

The initial DoE has been defined by the random selection from the MC population in the AK-MCS method [13]. In the current study, the initial training samples are created by Latin Hypercube Sampling method in the region between the upper and lower bounds of variables [18].

3 Entropy-based proposed method

In this section, a new method for reliability analysis of structures is presented based on the combination of multiple Kriging models. As mentioned in section 1, the constant regression trend has been used in the AK-MCS method. Using other regression tends can reduce the number of evaluations on the original performance function. The basic aim of the current study is to combine multiple Kriging models, because using multiple Kriging models can reduce the sensitivity to the regression trend. In the proposed entropy-based method, the disagreement between multiple surrogate models is used to define training samples.

The prediction of the Kriging model follows a normal distribution as $\hat{G}(\boldsymbol{x}) \sim \mathrm{N}\left(\mu_{\hat{G}(\boldsymbol{x})}, \sigma^2_{\hat{G}(\boldsymbol{x})}\right)$. So, the probability of prediction for negative or positive values can be expressed as

$$\mathrm{P}\left(\hat{G}(\boldsymbol{x}) \leq 0\right) = \Phi\left(\frac{0-\mu_{\hat{G}(\boldsymbol{x})}}{\sigma_{\hat{G}(\boldsymbol{x})}}\right), \tag{14}$$

$$\mathrm{P}\left(\hat{G}(\boldsymbol{x}) > 0\right) = 1-\Phi\left(\frac{0-\mu_{\hat{G}(\boldsymbol{x})}}{\sigma_{\hat{G}(\boldsymbol{x})}}\right), \tag{15}$$

where Φ(.) indicates the cumulative density function for the normal distribution.

Reliability analysis can be considered as a classification problem, because the limit state surface separates the MC population into two groups of failed or safe. So, the label of samples (y_i) can be assumed equal to 1 or 0 for samples located in the failure or safety region, respectively. The probability of classification using Kriging model θ can be rewritten for each sample $\boldsymbol{x}$ according to Eqs. (14) and (15) as

$$P_\theta\left(y_i = 1 \mid \boldsymbol{x}\right) = \Phi\left(-\frac{\mu_{\hat{G}(\boldsymbol{x})}}{\sigma_{\hat{G}(\boldsymbol{x})}}\right), \tag{16}$$

$$P_\theta\left(y_i = 0 \mid \boldsymbol{x}\right) = 1-\Phi\left(-\frac{\mu_{\hat{G}(\boldsymbol{x})}}{\sigma_{\hat{G}(\boldsymbol{x})}}\right), \tag{17}$$

where the value of y_i varies between all possible labeling, and $P_\theta(y \mid \boldsymbol{x})$ is the probability of labeling y for a sample of $\boldsymbol{x}$ by the Kriging model θ.

If the number of Kriging models is equal to C, the average probability of classification using multiple Kriging models can be defined as

$$P_C\left(y_i \mid \boldsymbol{x}\right) = \frac{1}{C}\sum_{\theta \in C} P_\theta\left(y_i \mid \boldsymbol{x}\right), \tag{18}$$

where $P_C(y_i \mid \boldsymbol{x})$ indicates the average probability that y_i is the exact label for sample $\boldsymbol{x}$. The entropy of probability distribution of P_C can be defined as [19]

$$H(X) = -\sum_y P_C(y_i \mid \boldsymbol{x}) \log P_C(y_i \mid \boldsymbol{x}) . \tag{19}$$

According to the maximum entropy principle, a sample with the largest value of entropy is the most informative sample [20]

$$\boldsymbol{x}_{\mathrm{H}}^* = \operatorname{argmax} - \sum_y P_C(y_i \mid \boldsymbol{x}) \log P_C(y_i \mid \boldsymbol{x}) . \tag{20}$$

So the above equation can be considered as the learning function for adaptive multiple Kriging methods.

In the current study, Kriging models are defined using different regression trends. The regression term of Kriging model can be defined in the form of $\boldsymbol{f}(\boldsymbol{x})^{\mathrm{T}}\boldsymbol{\beta}$. Hence, four regression trends can be expressed as

Constant Regression ($p = 1$):

$$\boldsymbol{f}(\boldsymbol{x})^{\mathrm{T}}\boldsymbol{\beta} = \beta_0 \tag{21}$$

Linear Regression ($p = n + 1$):

$$\boldsymbol{f}(\boldsymbol{x})^{\mathrm{T}}\boldsymbol{\beta} = \beta_0 + \sum_{i=1}^{n} \beta_i x_i \tag{22}$$

Quadratic Regression ($p = 2n + 1$):

$$\boldsymbol{f}(\boldsymbol{x})^{\mathrm{T}}\beta = \beta_0 + \sum_{i=1}^{n}\beta_i x_i + \sum_{j=1}^{n}\beta_j x_j^2 \tag{23}$$

Full Quadratic Regression ($p = \frac{(n+1)(n+2)}{2}$):

$$\boldsymbol{f}(\boldsymbol{x})^{\mathrm{T}}\beta = \beta_0 + \sum_{i=1}^{n}\beta_i x_i + \sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij} x_i x_j \tag{24}$$

where p is the number of terms in the regression trend, and n indicates the number of random variables (i.e., dimension of the problem). The coefficients β_i can be computed using Eq. (5). The number of regression coefficients (β_i) is equal to p. So, to compute $\boldsymbol{\beta}$ using Eq. (5), the size of DoE shall not be less than p.

Based on the proposed learning function and above multiple Kriging models, a method is presented for structural reliability analysis. Fig. 1 illustrates the flowchart of the proposed method. Differences of the proposed entropy-based method with the original AK-MCS method are shown with different colors. These differences can be summarized as follows:

1) The number of Kriging models is defined according to the size of DoE and the number of random variables, whereas the number of Kriging models is equal to one in the AK-MCS method.

2) Kriging models are constructed with different regression trends.

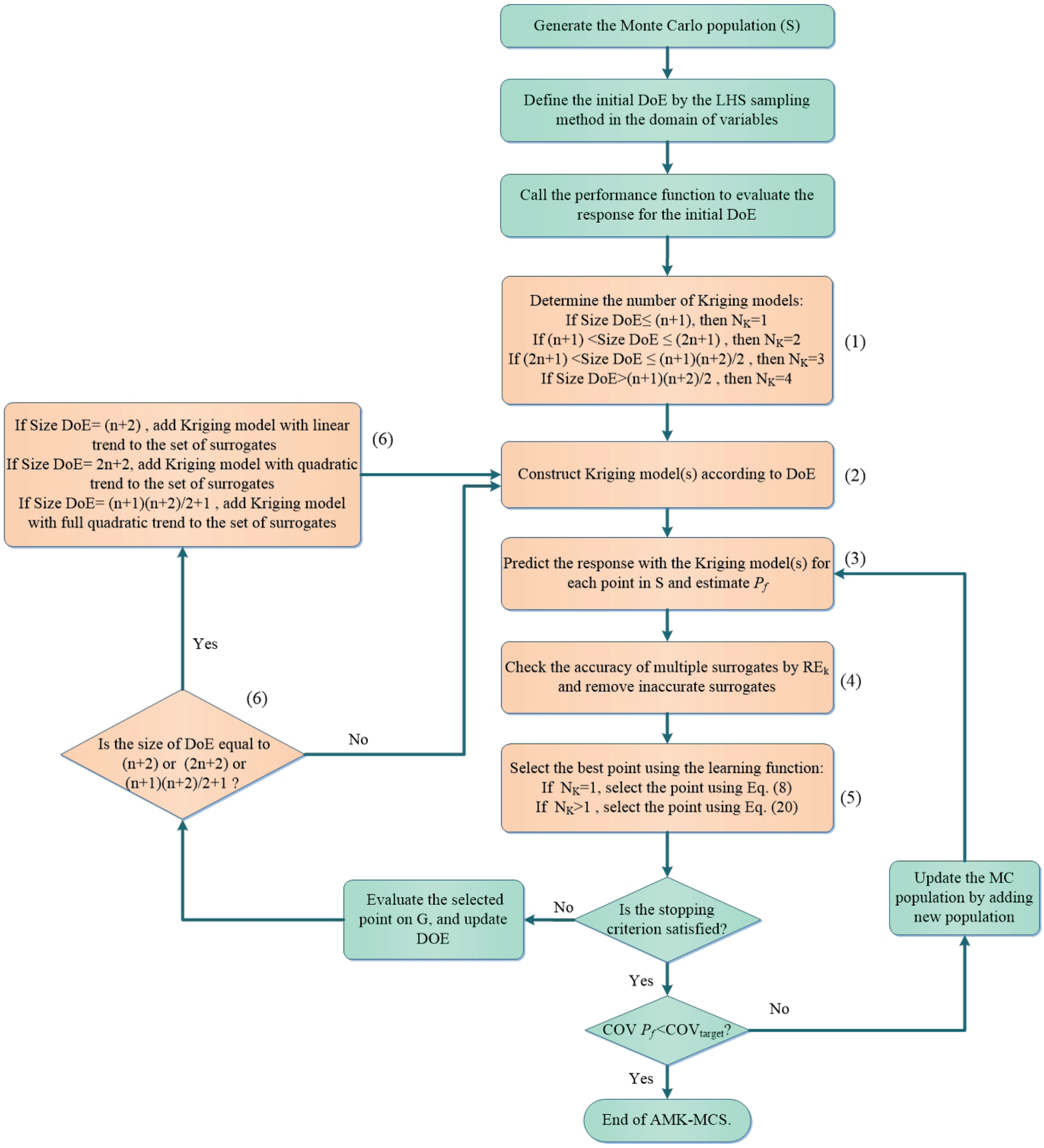


Fig. 1 Flowchart of the proposed method

3) Multiple Kriging models provide multiple predictions for each sample, but only one prediction is required. So, the presented method in Ref. [21] is employed in the current study to use the results of multiple Kriging models. U($\boldsymbol{x}$) in Eq. (8) shows the reliability index of misclassification [13]. So, for each sample, a Kriging model with the maximum value of learning function can be considered as the most reliable local model. After the prediction of the response of all samples, the probability of failure can be computed using Eq. (7). If the number of Kriging models is equal to one, the prediction will be the same as the AK-MCS method with the corresponding regression trend.

4) It is important to remove inaccurate Kriging model from the set of Kriging models. So, the presented method in Ref. [21] is employed to filter out inaccurate Kriging models. According to the stopping criterion in Eq. (9), samples with U($\boldsymbol{x}$) < 2 doesn't provide exact predictions. Thus, an error measurement criterion for comparing the accuracy of Kriging models can be expressed as

$$E_i = \frac{N_{U(\boldsymbol{x})<2}}{N_{MC}}, \tag{25}$$

where E_i indicates the prediction error of surrogate i, and $N_{U(\boldsymbol{x})<2}$ indicates the number of samples with U($\boldsymbol{x}$) < 2. So, the average error of multiple surrogates ($\bar{E}_{ms}$) can be defined as

$$\bar{E}_{ms} = \frac{\sum_{i=1}^{N_k} E_i}{N_k}, \tag{26}$$

where N_k denotes the number of Kriging meta-models. The relative error of surrogate i to the average error can be defined as

$$RE_i = \frac{E_i}{\bar{E}_{ms}}. \tag{27}$$

If RE_i becomes less than the predefined value of 1.2, the surrogate i maybe not exact enough. So, this surrogate model can be removed.

5) If the number of Kriging models is greater than one, Eq. (20) is used as the learning function to select the best point. In the case of using one Kriging model, the best point can be selected by Eq. (8).

6) If the size of DoE is equal to the minimum required to construct the Kriging model by linear, quadratic or full quadratic regression trends, the corresponding Kriging model will be added to the set of surrogate models.

Table 1 Example 1- Random variables of the cantilever tube

Variable	Distribution	Parameter 1*	Parameter 2*
t (mm)	Normal	5	0.1
d (mm)	Normal	42	0.5
L_1 (mm)	Uniform	119.75	120.25
L_2 (mm)	Uniform	59.75	60.25
F_1 (N)	Normal	3000	300
F_2 (N)	Normal	3000	300
P (N)	Gumbel	12,000	1200
T (N.mm)	Normal	90,000	9000
S_y (MPa)	Normal	220	22

* Parameters 1 and 2 are the lower and upper bounds for uniform distribution, and the mean and standard deviation for normal and Gumbel distributions.

4 Application examples

In this section, three analytical examples are presented to compare the proposed entropy-based method with the AK-MCS method. In the studied examples, Eq. (11) is considered as the stopping criterion, and the size of initial DoE is assumed to be equal to 12. The initial training samples are defined by LHS method.

In the examples, the relative error ($\in_{Pf}$) is computed as follows

$$\in_{P_f}(\%) = \frac{\left|\hat{P}_f - P_{f,MC}\right| * 100}{P_{f,MC}}, \tag{28}$$

where $\hat{P}_f$ is the predicted failure probability, and $P_{f,MC}$ is calculated by the original performance function for the same MC population.

The reduction of computational cost in problems with expensive-to-evaluate functions is compared using the number of calls to the performance function (N_{call}). Therefore, the reduction in N_{call} will decrease the time of calling the performance function. The studied examples have explicit performance function for comparison with benchmark results, so the time of calling the performance function is negligible. In real engineering problems with implicit performance function, the time of calling the performance function is considerable, so the reduction in the number of calls to the performance function is so important.

4.1 Example 1: Cantilever tube problem

This example includes a cantilever tube [22, 23], as shown in Fig. 2. External forces F_1, F_2, and P, and torsion T are applied to the tube. The performance function is defined based on the yield strength S_y and the maximum stress $\sigma_{\max}$ as follows

$$G = S_y - \sigma_{\max}, \tag{29}$$

where $\sigma_{\max}$ is the maximum stress of the tube given by

$$\sigma_{max} = \sqrt{\sigma_x^2 + 3\tau_{zx}^2}, \tag{30}$$

where σ_x and τ_{zx} are the normal stress and torsional stress, respectively which can be calculated by

$$\sigma_x = \frac{P + F_1 \sin\theta_1 + F_2 \sin\theta_2}{A} + \frac{Md}{2I}, \quad \tau_{zx} = \frac{Td}{2J}, \tag{31}$$

where A is the cross-sectional area, I is the moment of inertia, and M indicates the bending moment which can be respectively expressed by

$$\begin{aligned} &A = \left(\pi/4\right)\left[d^2 - (d-2t)^2\right], \\ &I = \left(\pi/64\right)\left[d^4 - (d-2t)^4\right], \quad J = 2I, \\ &M = F_1 L_1 \cos\theta_1 + F_2 L_2 \cos\theta_2, \end{aligned} \tag{32}$$

where $\theta_1 = 5^\circ$ and $\theta_2 = 10^\circ$ are constants. The nine independent random variables of the problem are presented in Table 1.

The obtained results including the number of evaluations on the original performance function (N_{call}), the failure probability (P_f), and the relative errors of failure probability ($\in_{P_f}$) are provided in Table 2. The results show the significant deprecating of N_{call} by the proposed method. The error of prediction with the proposed method is negligible, and it is less than the acceptable threshold of $\in_{\hat{P}_f} = 5$ % in Eq. (11).

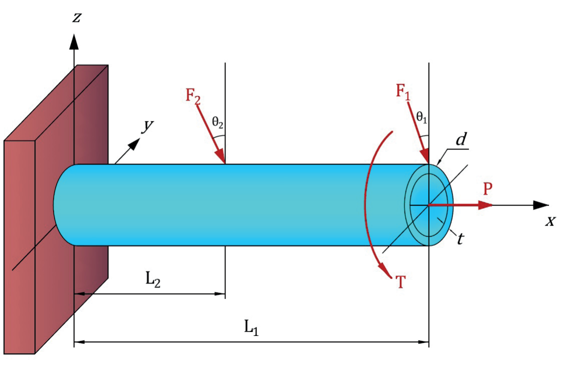
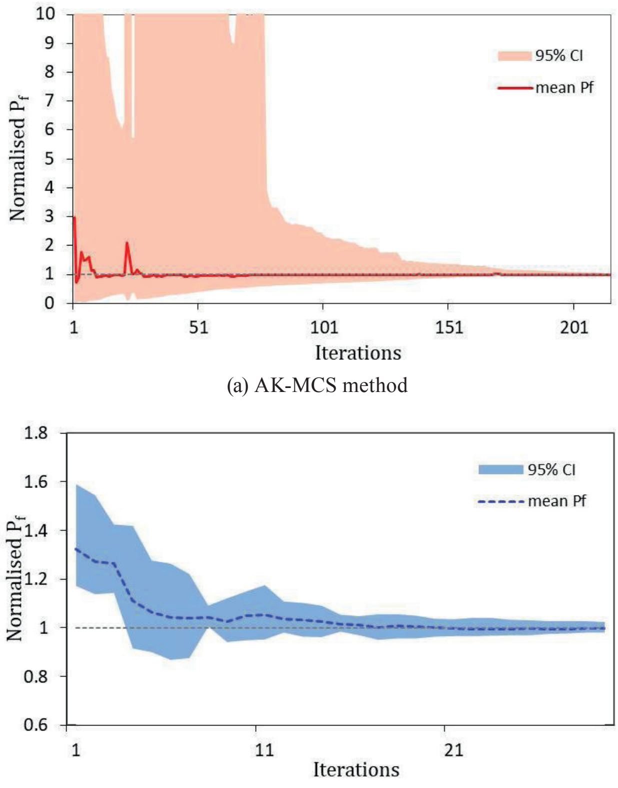


Fig. 2 Cantilever tube

Table 2 Results for Example 1: Cantilever tube problem

Method	N_{call}	P_f	$\in_{P_f}(\%)$
MCS	3×10^6	1.780×10^{-4}	-
AK-MCS	12 + 215 = 227	1.780×10^{-4}	0
Proposed method	12 + 29 = **41**	1.777×10^{-4}	0.1685



(a) AK-MCS method

(b) Proposed method

Fig. 3 Convergence histories of failure probabilities for the cantilever tube problem

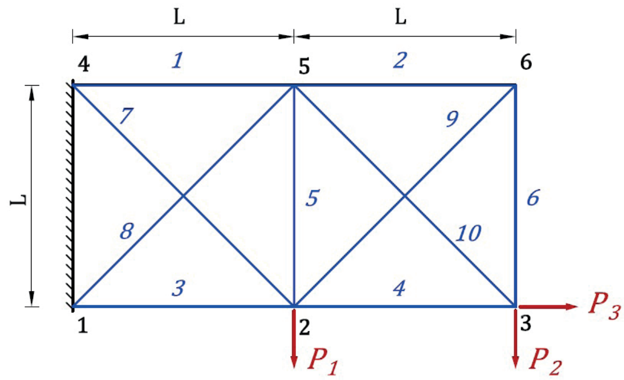


Fig. 4 10-bar truss structure

Convergence histories with two methods are compared in Fig. 3. As can be seen, the proposed entropy-based method stops with less iteration. The shading area shows 95 % of confidence interval (CI) for prediction based on the upper and lower bounds of failure ($\hat{P}_f^+$ and $\hat{P}_f^-$) using Eq. (13). It is seen, the shading area is greater with the AK-MCS method. So, more iteration is needed to satisfy the stopping condition. It should be noted that the results are normalized to the exact prediction (i.e., $P_{f,MC}$ in Eq. (28)).

Table 3 Example 2- Random variables of the 10-bar truss structure

Random variables	A_i	L	E	P_1	P_2	P_3
Mean	0.001 m^2	1 m	100G Pa	80 kN	10 kN	10 kN
Coefficient of variation	0.05	0.05	0.05	0.05	0.05	0.05

Table 4 Results for Example 2: the 10-bar truss structure

Method	N_{call}	P_f	$\in_{P_f}(\%)$
MCS	10^4	0.0694	-
AK-MCS	12 + 807 = 819	0.0695	0.1441
Proposed method	12 + 209 = **221**	0.0694	0

Table 5 Example 3- Random variables of the 23-bar truss structure

Variable	Distribution	Mean	Standard deviation
E_1, E_2, (Pa)	Lognormal	2.1×10^{11}	2.1×10^{10}
A_1(m^2)	Lognormal	2.0×10^{-3}	2.0×10^{-4}
A_2(m^2)	Lognormal	1.0×10^{-3}	1.0×10^{-4}
P_1, ..., P_6 (N)	Gumbel	5.0×10^4	7.5×10^3

Table 6 Results for Example 3: 23-bar truss

Method	N_{call}	P_f	$\in_{P_f}(\%)$
MCS	10^5	8.91×10^{-3}	-
AK-MCS	12+241=253	8.91×10^{-3}	0
Proposed method	12+135=**147**	8.91×10^{-3}	0

4.2 Example 2: Ten bar truss structure

The second example is a ten-bar truss structure taken from [24], which is illustrated in Fig. 4. The length of horizontal and vertical bars is denoted as L.

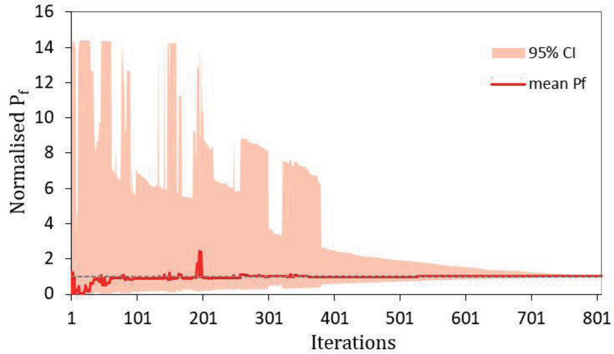
The section area and elastic modulus of bars are A_i(i = 1, 2,…, 10) and E, respectively. Moreover, P_1, P_2 and P_3 are the point loads. The fifteen random variables are independent with normal distribution, and their mean value and coefficient of variation are given in Table 3.

The deflection of the truss is a function of random variables, and can be computed by a finite-element model. The vertical displacement of the node 3 needs to be less than 0.0035 m. So, the performance function of the structure can be written as

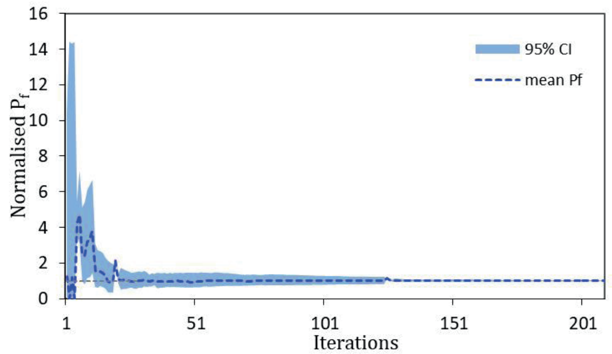
$$G = 0.0035 - \Delta_y \,, \tag{33}$$

where Δ_y is the vertical displacement of node 3.

The results of AK-MCS and proposed methods are given in Table 4. It is seen that N_{call} is 819 in the AK-MCS method, whereas it reduces to 221 by the proposed method. Fig. 5 compares convergence histories of the AK-MCS and proposed methods. As can be seen, the proposed entropy-based method converges faster than the original method. Moreover, the difference between the upper and lower bounds of failure probabilities is greater with the AK-MCS method. So, more iteration is needed to satisfy the stopping condition. This means that P_f converges in less iteration by the proposed method.



(a) AK-MCS method



(b) Proposed method

Fig. 5 Convergence histories of failure probabilities for 10-bar truss problem

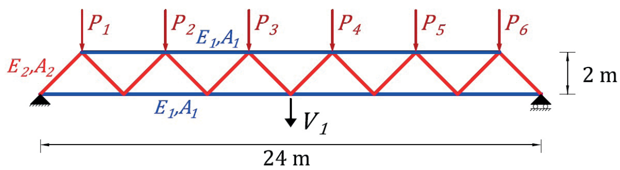


Fig. 6 Two-dimensional 23-bar truss

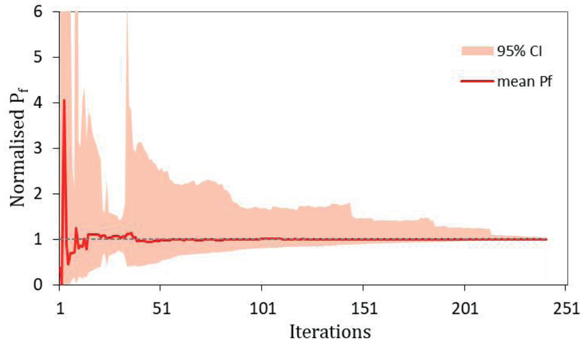
4.3 Example 3: Two-dimensional 23-bar truss

The last example deals with a two-dimensional 23-bar truss [25] as shown in Fig. 6. It consists of ten independent random variables (E_1, E_2, A_1, A_2, P_1, P_2, P_3, P_4, P_5, P_6).

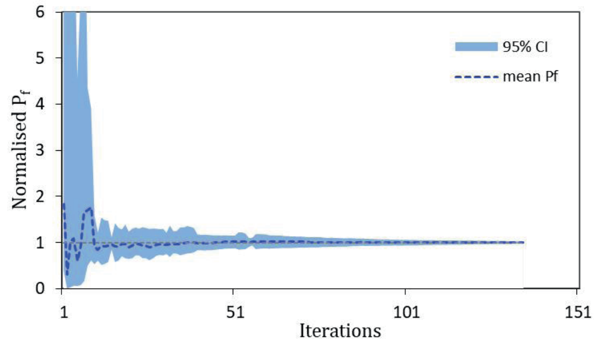
The distributional properties of these random variables are listed in Table 5. The performance function is defined based on the mid-span deflection as

$$G = 0.11 - V_1 \,, \tag{34}$$

where V_1 denotes the vertical displacement of the mid-span of the truss.



(a) AK-MCS method



(b) Proposed method

Fig. 7 Convergence histories of failure probabilities for the 23-bar truss problem

The obtained results are summarized in Table 6. The results show the proposed method predicts the probability of failure with N_{call} of 147, whereas the required N_{call} is equal to 253 with the AK-MCS method. Moreover, the error of prediction is equal to zero for both methods.

Comparing convergence histories of failure probability by the AK-MCS and proposed methods in Fig. 7 shows that the 95 % confident interval (CI) is narrower with the proposed method. So, the proposed method satisfies the stopping criterion (Eq. (11)) with less iteration.

5 Conclusions

An approach to combine multiple Kriging models has been proposed for reliability assessment of structures. The presented method is based on the maximum entropy of predictions which provided by multiple Kriging models. The efficiency of the proposed entropy-based method has been shown with three examples.

Although the proposed method can reduce the number of evaluations on the original performance function, it may require considerable computation cost for problems with a very low probability of failure and high dimension. So, the combination of the proposed method with variance reduction methods and dimension reduction approaches can be effective for such problems.

References

[1] Melchers, R. E., Beck, A. T. "Structural Reliability: Analysis and Prediction", 3rd ed., John Wiley & Sons, Hoboken, New Jersey, United States, 2018.

[2] Zaeimi, M., Ghoddosian, A. "Structural Reliability Assessment Based on the Improved Constrained Differential Evolution Algorithm", Periodica Polytechnica Civil Engineering, 62(2), pp. 494–507, 2018. https://doi.org/10.3311/PPci.11537

[3] Madsen, H. O., Krenk, S., Lind, N. C. "Methods of structural safety", Prentice-Hall, Upper Saddle River, New Jersey, United States, 1986.

[4] Rubinstein, R. Y., Kroese, D. P. "Simulation and the Monte Carlo Method", 2nd ed., John Wiley & Sons, Hoboken, New Jersey, United States, 2008.

[5] Ibrahim, Y. "Observations on applications of importance sampling in structural reliability analysis", Structural Safety, 9(4), pp. 269–281, 1991. https://doi.org/10.1016/0167-4730(91)90049-F

[6] Engelund, S., Rackwitz, R. "A benchmark study on importance sampling techniques in structural reliability", Structural Safety, 12(4), pp. 255–276, 1993. https://doi.org/10.1016/0167-4730(93)90056-7

[7] Au, S-K., Beck, J. L. "Estimation of small failure probabilities in high dimensions by subset simulation", Probabilistic Engineering Mechanics, 16(4), pp. 263–277, 2001. https://doi.org/10.1016/S0266-8920(01)00019-4

[8] Au, S. K., Ching, J., Beck, J. L. "Application of subset simulation methods to reliability benchmark problems", Structural Safety, 29(3), pp. 183–193, 2007. https://doi.org/10.1016/j.strusafe.2006.07.008

[9] Cardoso, J. B., de Almeida, J. R., Dias, J. M., Coelho, P. G. "Structural reliability analysis using Monte Carlo simulation and neural networks", Advances in Engineering Software, 39(6), pp. 505–513, 2008. https://doi.org/10.1016/j.advengsoft.2007.03.015

[10] Hurtado, J. E., Alvarez, D. A. "Classification Approach for Reliability Analysis with Stochastic Finite-Element Modeling", Journal of Structural Engineering, 129(8), pp. 1141–1149, 2003. https://doi.org/10.1061/(ASCE)0733-9445(2003)129:8(1141)

[11] Gaspar, B., Teixeira, A. P., Soares, C. G. "Assessment of the efficiency of Kriging surrogate models for structural reliability analysis", Probabilistic Engineering Mechanics, 37, pp. 24–34, 2014. https://doi.org/10.1016/j.probengmech.2014.03.011

[12] Jones, D. R., Schonlau, M., Welch, W. J. "Efficient Global Optimization of Expensive Black-Box Functions", Journal of Global Optimization, 13(4), pp. 455–492, 1998. https://doi.org/10.1023/A:1008306431147

[13] Echard, B., Gayton, N., Lemaire, M. "AK-MCS: An active learning reliability method combining Kriging and Monte Carlo Simulation", Structural Safety, 33(2), pp. 145–154, 2011. https://doi.org/10.1016/j.strusafe.2011.01.002

[14] Lataniotis, C., Marelli, S., Sudret, B. "UQLab User Manual - Kriging, Gaussian Process Modelling", ETH Zurich, Zurich, Switzerland, Rep. UQLab v0.9-105, 2015.

[15] IMM Informatiocs and Mathematical Modelling, Nielsen, H. B., Lophaven, S. N., Søndergaard, J. "DACE - A Matlab Kriging Toolbox", [computer program] Available at: http://www2.imm.dtu.dk/pubdb/views/publication_details.php?id=1460 [Accessed: 07.03.2019]

[16] Sacks, J., Welch, W. J., Mitchell, T. J., Wynn, H. P. "Design and Experiments of Computer Experiments", Statistical Science, 4(4), pp. 409–123, 1989.
https://doi.org/10.1214/ss/1177012413

[17] Schöbi, R., Sudret, B., Marelli, S. "Rare Event Estimation Using Polynomial-Chaos Kriging", ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering, 3(2), 2017.
https://doi.org/10.1061/AJRUA6.0000870

[18] Vahedi, J., Ghasemi, M. R., Miri, M. "Structural reliability assessment using an enhanced adaptive Kriging method", Structural Engineering and Mechanics, 66(6), pp. 677–691, 2018.
https://doi.org/10.12989/sem.2018.66.6.677

[19] Borda, M. "Fundamentals in Information Theory and Coding", Springer, Berlin, Heidelberg, Germany, 2011.

[20] Settles, B. "Active Learning", Synthesis Lectures on Artificial Intelligence and Machine Learning, 6(1), pp. 1–114, 2012.
https://doi.org/10.2200/S00429ED1V01Y201207AIM018

[21] Vahedi, J., Ghasemi, M. R., Miri, M. "An adaptive divergence-based method for structural reliability analysis via multiple Kriging models", Applied Mathematical Modelling, 62, pp. 542–561, 2018.
https://doi.org/10.1016/j.apm.2018.06.015

[22] Du, X. "Interval Reliability Analysis", In: ASME 2007 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, 33rd Design Automation Conference, Las Vegas, Nevada, USA, 2007, pp. 1103–1109.
https://doi.org/10.1115/DETC2007-34582

[23] Xiao, N-C., Zuo, M. J., Zhou, C. "A new adaptive sequential sampling method to construct surrogate models for efficient reliability analysis", Reliability Engineering & System Safety, 169, pp. 330–338, 2018.
https://doi.org/10.1016/j.ress.2017.09.008

[24] Zhang, L., Lu, Z., Wang, P. "Efficient structural reliability analysis method based on advanced Kriging model", Applied Mathematical Modelling, 39(2), pp. 781–793, 2015.
https://doi.org/10.1016/j.apm.2014.07.008

[25] Lelièvre, N., Beaurepaire, P., Mattrand, C., Gayton, N. "AK-MCSi: A Kriging-based method to deal with small failure probabilities and time-consuming models", Structural Safety, 73, pp. 1–11, 2018.
https://doi.org/10.1016/j.strusafe.2018.01.002