Comparison of Different Empirical Correlations to Estimate Permeability Coefficient of Quaternary Danube Soils

Miklós Pap[1]*, András Mahler[1]

[1] Department of Engineering Geology and Geotechnics, Faculty of Civil Engineering, Budapest University of Technology and Economics, H-1111 Budapest, Műegyetem rkp. 3., Hungary
* Corresponding author, e-mail: pap.miklos@epito.bme.hu

Abstract

Permeability coefficient is the most significant soil parameter in seepage calculations. It has been recognized that permeability of granular soils is strongly related to the grain size, thus numerous empirical correlations have been developed to estimate permeability using its grain size characteristics. In this study the empirical correlations proposed by Hazen (1911), Carrier (2003) and Chapuis (2004) are evaluated and compared to laboratory measurement results. Quaternary Danube soils are very typical in the Carpathian basin, thus their permeability is an important question in many geotechnical applications.

Keywords

permeability coefficient, falling head test, grain size distribution, empirical equations, prediction

1 Introduction

The permeability coefficient (k) of soils is the most significant parameter in groundwater seepage calculations. Determination of a reliable permeability coefficient is inevitable to reliably calculate, model and evaluate seepage in porous medium. There are several techniques such as in-situ tests, laboratory measurements and empirical correlations to assess the value of permeability. In case of in-situ tests the accurate determination of k is limited by the uncertainties in geometry of the investigated soil layer and in hydraulic boundary conditions. Besides these facts it must be also noted that the installation costs of the wells are very high. Furthermore, it is difficult to obtain representative samples for the laboratory tests; the tested specimens are rather limited in size, thus those may not properly represent the whole layer on site. Because of these facts the use of empirical or semi-empirical equations is needed and it also enables fast and cost-efficient estimation of permeability. Quaternary Danube deposits with varied composition are very typical in the Carpathian basin, thus their permeability is an important question in many geotechnical applications [11].

The permeability for a single fluid flow can be predicted using empirical relationships, capillary models, statistical models and hydraulic radius theories. Many empirical or semi-empirical equations are published in the literature to estimate the permeability of porous material [1–5]. There are simple models that use some characteristic pore diameter, while more sophisticated formulas take into account other factors (e.g. void ratio, viscosity etc.) too, which have large effect on soil properties [9]. The most complex models contain numerous parameters as such as the size of the pores, their tortuosity and their connectivity to consider the relationships between the flowrate and the porous space.

The model developed by Hazen [8] has been used for a century. This empirical equation uses the effective diameter for predicting the permeability of saturated loose sand. The formula is the following:

$$k = C_H \cdot D_{10} \tag{1}$$

where k is the permeability in cm/s, C_H is the Hazen empirical coefficient and D_{10} is the diameter at which 10% of the sample's mass is comprised of particles with a diameter less than this value (mm). The value of C_H is between 0.0 and 1.5, but usually assumed to be equal to 1.0. In this recent study $C_H = 1.0$ is used consequently. The formula's applicability is generally limited to the range of $d = 0.01 - 0.30$ cm [3].

A frequently cited equation is the Kozeny-Carman formula, which was proposed by Kozeny [10] and later modified by Carman [1, 2]. This relation describes the

permeability coefficient as a function of the void ratio, the specific surface and a factor to take into account the shape and tortuosity of channels:

$$k = \frac{\gamma}{\mu \cdot C \cdot S^2} \cdot \frac{e^3}{1+e} \qquad (2)$$

where k is the permeability in cm/s, γ is the unit weight of permeant, μ is the viscosity of permeant, C is the Kozeny-Carman empirical coefficient, S is the specific surface area per unit volume of particles (1/cm) and e is the void ratio.

The Kozeny-Carman equation is approximately valid for sands and is not valid for clays. In practice, the relation is not frequently used, because it is difficult to determine the soil specific surface that can be either measured or estimated [7]. For practical use, Carrier [3] modified the Eq. (2) by applying for calculation of the specific surface the effective diameter that can be determined using the grain size distribution curve. The finally equation describe the permeability as follows:

$$k = 1.99 \cdot 10^4 \cdot \left(\frac{100\%}{\sum \frac{f_i}{D_{li}^{0.404} \cdot D_{si}^{0.595}}} \right)^2 \cdot \left(\frac{1}{SF} \right)^2 \cdot \frac{e^3}{1+e} \qquad (3)$$

where k is the permeability in cm/s, f_i is the fraction of particles between two sieve seizes (%), D_{li} is the diameter seize of the larger sieve (cm), D_{si} is the diameter seize of the smaller sieve (cm), e is the void ratio and SF is the shape factor. The magnitude of SF may vary from between 6 and 8, depending on the angularity of the soil particles. In his recent study $SF = 7$ is used for all soil types. This formula can be applied in silts, sands, and even gravelly sands [3, 7].

More recently, Chapuis [4] proposed an empirical relationship for the permeability coefficient. This equation is valid for natural, uniform sand and gravel to estimate the permeability coefficient that is in the range of 10^{-1}–10^{-3} cm/s. This can be extended to natural, silty sands without plasticity. It is not valid for crushed materials or silty soils with some plasticity [4, 7]. The equation uses the effective diameter and the void ratio:

$$k = 2.4622 \left[D_{10}^2 \cdot \frac{e^3}{1+e} \right]^{0.7825} \qquad (4)$$

where k is the permeability in cm/s, D_{10} is the effective size (mm) and e is the void ratio.

The scope of the study was to compare the empirical equations developed by Chapuis (2004), Carrier (2003) and Hazen (1911) for prediction of the permeability coefficient. Furthermore, the aim was to evaluate the applicability of
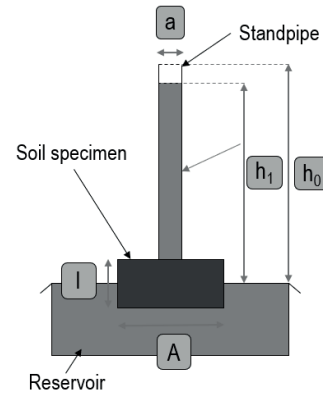

Fig. 1 Falling head test device

the recent formulas for different soil types (e.g. silty sand, gravelly sand, sandy clayey silt) by comparing the predicted values of the permeability to the coefficients given by the laboratory measurements.

2 Materials and methods
2.1 Laboratory permeability test
In this study, the falling head permeability test based on the Hungarian standard MSZE CEN ISO/TS 17892-11:2010 was used to measure the permeability coefficient of soil samples. The test was carried out in a falling head permeability device. The permeability test involves seepage through a soil sample connected to a standpipe which provides the water head and allows measuring the volume of water flowing through the soil sample. The water starts to flow through the sample until the water in the standpipe reaches a given lower limit. The time required for the water in the standpipe to drop to the lower level is measured. Fig. 1 presents a schematic view of the measuring method.

Based on the measurement results the permeability coefficient can be calculated with the following formula:

$$k = \frac{a \cdot l}{A \cdot \Delta t} \cdot \ln \frac{h_0}{h_1} \qquad (5)$$

where k is the permeability coefficient in m/s, a is the cross section of the standpipe (m²), A is the cross section of the soil sample (m²), Δt is the measured time for the water column decreasing (s), h_0 is the initial water head (m), h_1 is the water head related to the recorded time (m).

2.2 Grain size distribution
The classification of Quaternary Danube soils was performed according to the Hungarian standard MSZ EN ISO 17892-4:2017. The test consists of shaking the soil sample through a set of sieves that have progressively smaller
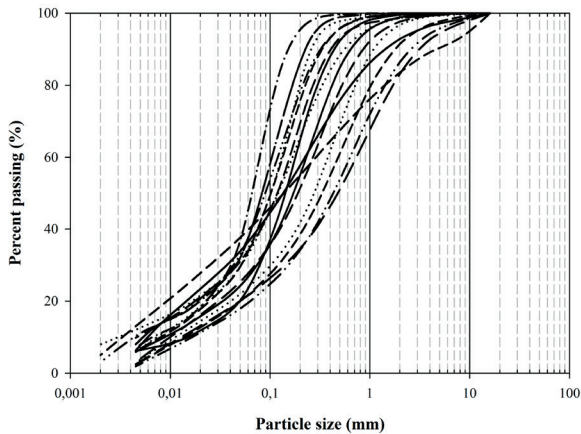
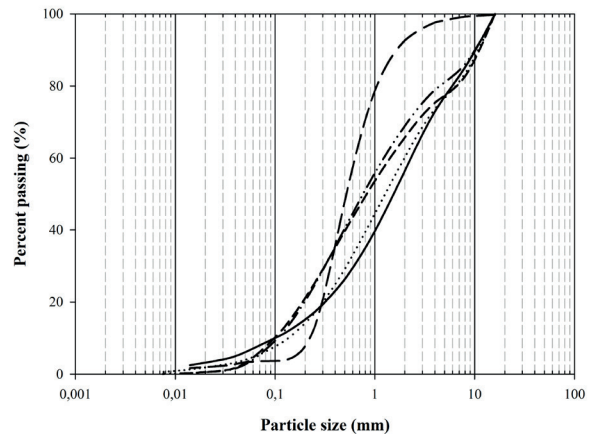**Fig. 2** Grain size distribution curves of the sandy silt

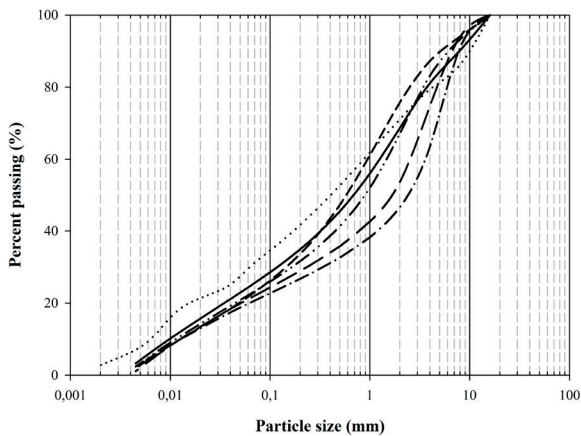**Fig. 4** Grain size distribution curves of the gravelly sand

Fig. 3 Grain size distribution curves of the sandy silty gravel
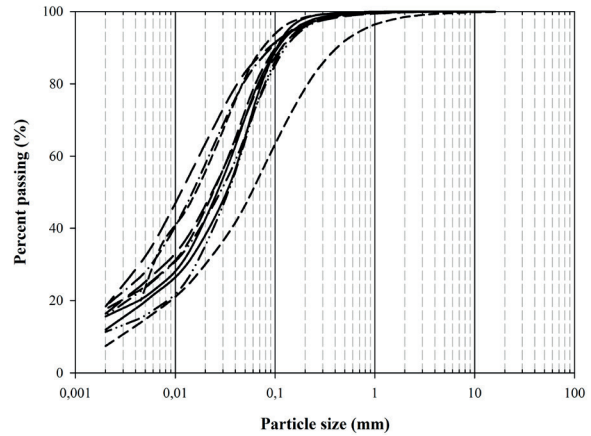

Fig. 5 Grain size distribution curves of the sandy clayey silt

openings. After the shaken, the mass of soil remained on each sieve is measured. The amount of the silt and clay particles is determined by hydrometer analysis [6].

More sophisticated semi-empirical equations take into account the void ratio too to estimate the permeability coefficient. To have a better understanding of the void ratio's effect some soil types were tested with specimens having various void ratios.

Based on the results of the grain size distribution and the hydraulic conductivity properties the soil samples were divided into four different categories. The first division is for the silty sand and sandy silt soils (Fig. 2). These samples include ca. 20–50% silt size (between 0.002 and 0.063 mm) fraction and the void ratio varied between 0.48 and 0.84.

The sandy silty gravel and gravelly silty sand soils belong to the second group (Fig. 3). These soil specimens include ca. 20–25% silt content and ca. 25–55% gravel size ($d > 2$mm) fraction. The void ratio of the sandy silty gravel and gravelly silty sand soils varied from 0.45 to 0.59.

The third section includes the gravelly sands and sandy gravels that consist ca. 40–60% gravel size fraction and only

ca. 1–5% fine particles (Fig. 4). The void ratios of the specimens vary over a narrow range: between 0.33 and 0.35.

The cohesive soils are in the fourth group (Fig. 5). The sandy clayey silt and sandy silty clay soils contain ca. 22–40% sand size fraction and ca. 10–20% clay size particle. The value of the void ratio for the cohesive soils is given between 0.53 and 0.75.

All in all, 78 different soil samples were tested and compared in this study due to the different grain size distribution and the variation of the void ratio.

3 Results and discussion

The permeability coefficient of each specimen was calculated based on the grain size distribution curves and void ratios; the results were compared to the ones obtained by laboratory measurements. Fig. 6 to 8 show the measured and predicted k-values using equation developed by Chapuis (2004), Carrier (2003) and Hazen (1911). On these figures, the linear function represents the perfect estimation (i.e. the measured and predicted values are equal).
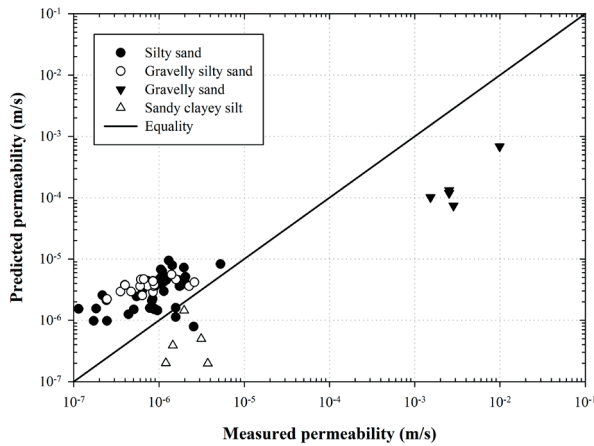
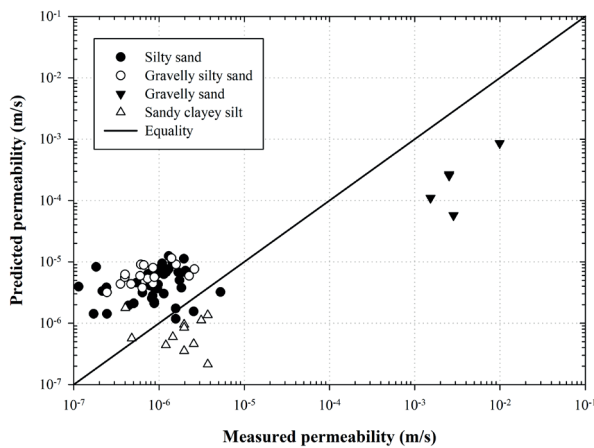**Fig. 6** Measured versus predicted k-value using Chapuis (2004) equation

Fig. 8 Measured versus predicted k-value using Hazen (1911) equation

Fig. 7 Measured versus predicted k-value using Charrier (2003) formula
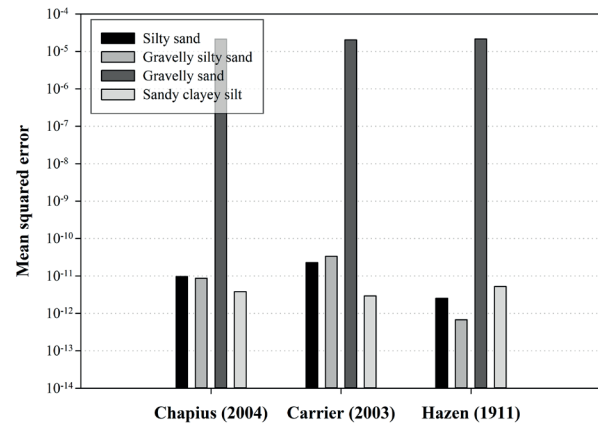
Fig. 9 Mean squared error of the three prediction methods

The formulas of Chapuis (2004) and Carrier (2003) provided very similar results for all soil types, while the correlation of Hazen (1911) resulted in significantly different values especially in the case of silty sands and gravelly silty sands. In general, the estimations of the Hazen formula seemed to give more realistic results for the two groups mentioned above.

According to all empirical equations analyzed the permeability coefficient of the gravelly sand soils were underestimated by about an order of magnitude in all cases. The gravelly silty sands show a different picture. The permeability of these soils was overestimated by about a half order of magnitude when using the Chapuis (2004) and Carrier (2003) equations. The predicted and measured values were closer in the case of Hazen method, but it has to be noted that this formula resulted in very similar values (~10^{-6} m/s) for all specimens in this soils group (while the measured values varied over a significantly wider range). The estimations made for the silty sand provided the best fit. The correlations of Chapuis (2004) and Carrier (2003)
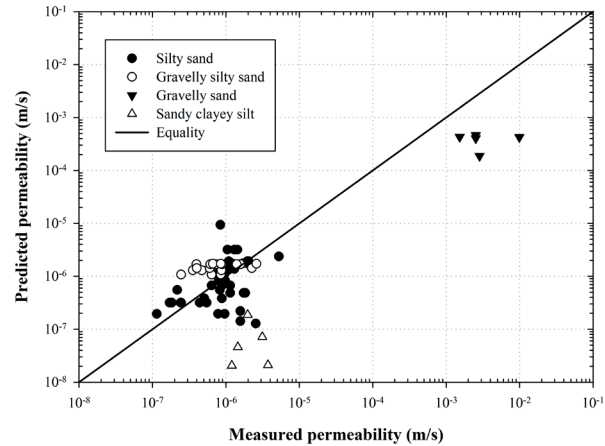
slightly overpredicted the permeability, but the deviation was quite consistent, thus the data points scatter in a narrow band. The Hazen (1911) equation did not result in a systematic deviation but the scatter of the data more significant. Based on the measurements data concerning to cohesive soils fit appropriate with equality function considering that applicability of these equations is limited smaller fine contents.

To be able to compare the different prediction methods considering the type of soils the mean squared error of the prediction and measured data was obtained. Values of the mean squared error are shown on Fig. 9.

Based on the mean squared error the applicability and reliability of the methods can be assessed. It can be stated that the hydraulic conductivity of the gravelly sands was estimated with large error by each formula, despite of the fact that Chapuis (2004) formula should be appropriate to estimate permeability of sand and gravel [4, 7]. It implies that the use different empirical constants may be useful for quaternary Danube sands and gravels. In the case of silty sand and gravelly silty sand all three equation show

similar errors, but the Hazen (1911) correlation gave the most accurate prediction. The permeability coefficient of the sandy clayey silt was estimated with similar accuracy by all three methods.

4 Conclusions

In this paper, empirical correlations for prediction the permeability coefficients of Quaternary Danube deposit were compared and evaluated. These formulas use different methodologies to calculate the permeability coefficient using the grain size distribution curve. According to the literature, Hazen (1911) formula is suitable only for soils having particle sizes in the range of 0.01–0.30 cm [3], Chapuis (2004) equation is capable to estimate the permeability coefficient of uniform sand and gravel in the range of 10^{-3}–10^{-5} m/s, while the Carrier (2003) formula is approximately valid for sands and is not valid for clays [3, 7]. The soils tested were divided to four groups: gravelly sand, gravelly silty sand, silty sand clayey silt. For each specimen the permeability coefficient was measured in the laboratory by the means falling head permeability test and was also estimated using the empirical correlations mentioned above. The estimated and measured values were compared and evaluated.

All three correlations provided estimation of comparable accuracy for all soil types in general, but some differences can be observed when investigating the soil types separately. The permeability coefficients of the gravelly sand specimens were underestimated consequently and significantly by all three methods indicating that different empirical constants may be necessary when estimating the permeability of Danube deposited gravelly sands. The permeability of gravelly silty sands was systematically overestimated by the Chapuis (2004) and Carrier (2003) correlation, but the Hazen (1911) equation resulted in estimations closer to the measured values. It must be also noted that while the measured permeability of these specimens varied over a range of about one order of magnitude the calculated values were almost the same. So the Hazen (1911) equation gave a good result for the average permeability of this soil group but couldn't really capture the effect of slight differences in the grain size distribution. In case of the silty sands the trend was similar: Chapuis (2004) and Carrier (2003) equation resulted in slight overestimation and the Hazen (1911) equation provided good results in average, but in the same time the data points show significant scatter. The permeability of the cohesive soils was systematically underestimated by all three methods.

As a conclusion it can be stated that all three methods predicts the permeability coefficient with similar reliability. The systematic deviations in case of some methods and soil types implies that different empirical constants may be necessary to estimate the permeability of Quaternary Danube soils in a more reliable way.

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