Centrifuge Modelling and Finite Element Analysis of Laterally Loaded Single Piles in Sand with Focus on P–Y Curves

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Abstract
Centrifuge modelling and finite element analysis are powerful tools of research on the lateral pile/soil interaction. This paper aims at presenting the main results of experimental and numerical analysis of the pile response under monotonic lateral loading in sand. After description of the experimental devices, it focuses on the determination of the load-transfer P–Y curves for rigid and semi-rigid piles embedded in dry dense sand by using the experimental bending moment profiles obtained in centrifuge tests, as well as by a three-dimensional finite element models using ABAQUS Software. The elastic perfectly plastic Mohr-Coulomb constitutive model has been used to describe the soil response, and the surface-to-surface contact method of ABAQUS software has been used to take into account the nonlinear response at soil/pile interface. The analysis methodology has allowed to propose a hyperbolic function as a model to construct P–Y curves for rigid and semi-rigid piles embedded in dry dense sand, this model is governed by two main parameters, which are the initial subgrade reaction modulus, and the lateral soil resistance, the latter has been formulated in terms of Rankine’s passive earth pressure coefficient, the sand dry unit weight, and the pile diameter.

Keywords
pile, lateral load, centrifuge modelling, finite element modelling, P–Y curves, initial subgrade reaction modulus, lateral soil resistance

1 Introduction
Pile foundations are often laterally loaded due to many sources such as wind, traffic and earthquakes. Lateral loading of the piles occurs either in active form as concentrated loads at the pile head, or passively as distributed pressure acting by the soil along the pile shaft. Horizontal loading sometimes becomes as a key parameter for the design of pile foundations, such in case of monopile foundations of offshore wind turbines, which are subjected to very important lateral loads such as, wind and wave loads.

Several design methods have been proposed in the literature but most of them are based on simplistic mechanisms of pile/soil interaction. Although the full-scale lateral loading test is a pragmatic approach to predict the pile response, its relatively high cost eclipses its field of application to relatively important projects. However, physical and numerical modelling are an important alternative to such approaches and are widely used in engineering practice as well as in research.

The problem of laterally loaded piles has been analyzed in the literature using different approaches which may be subdivided into four main categories, namely: the elastic continuum models where the soil is represented by a linear elastic continuum [1, 2] which have the advantage of taking into account the continuity of the soil material, but they are limited to small deformations behavior.

The second category encompasses the numerical methods. In fact, important advances in the field of computer science these last decades allows developing powerful numerical methods which are widely used for the analysis of piles under lateral loads, such as the finite element method and the finite difference one [3, 5], these methods offer the advantage of taking into account the material nonlinearity of the soil/pile system, through increasingly developed constitutive models.

The third one contains the semi-empirical or empirical methods [6, 7], while the last category is called the subgrade reaction methods (called also the P–Y curves methods),
which are based on the model of beam on elastic foundations suggested first by Winkler [8]. First generation of the $P-Y$ curves method was developed by Matlock [9] and Reese et al. [10], and subsequently adopted by the API [11].

The pile is modelled by a linear elastic beam surrounded by a series of infinite independent nonlinear springs characterized by the subgrade reaction modulus $E_s$, as illustrated in Fig. 1. The pile bending is then governed by a fourth order nonlinear differential equation, the term $E_p I_p$ being the pile flexural stiffness:

$$E_p I_p \frac{d^4 Y}{dz^4} + E_s Y = 0.$$  \hspace{1cm} (1)

Most of the proposed methods for the construction of the $P-Y$ curves are empirical and based on lateral loading tests of full-scale piles [9–15], while the other ones are semi-empirical and based on the analogy of the soil reaction with the pressuremeter test [16–18].

Physical modelling using the geotechnical centrifuge is a powerful tool not only for the development of $P-Y$ curves but also for conducting parametric studies to evaluate the effect of different parameters on the response of piles under lateral loading [19–22]. Table 1 summarizes the main similarity scales used within the scope of the topic, the similarity scale $X^*$ of a parameter $X$ being the ratio of the $X_{\text{model}}$ to $X_{\text{prototype}}$. $L^*$ is then the dimension scale of the physical model.

The purpose of this paper is to present the main results of physical modelling in centrifuge, as well as numerical analysis based on 3D finite element model using ABAQUS Software, of a single pile response under monotonic lateral loading in sand. Focus is made on the determination of the $P-Y$ curves for two piles: semi-rigid pile denoted hereafter $P_1$ and having slenderness ratio of 10, and a rigid pile denoted $P_2$ and having slenderness ratio of 5.55, both of the piles being embedded in dry dense sand.

From the results found, a model has been proposed for the construction of $P-Y$ curves for rigid, and semi-rigid piles embedded in dry dense sand, this model depends essentially on the initial subgrade reaction modulus, and the lateral soil resistance.

### 2 Centrifuge modelling

#### 2.1 Description of the centrifuge

Lateral loading tests on small scale models were carried out by Bouafia [23] in the geotechnical centrifuge of the IFSTTAR (Formerly LCPC) in France which is a powerful device of physical modelling characterized by a radius of 5.5 m, a maximum mass of rotating model of 2000 kg and a maximum centrifuge acceleration of 200 times the terrestrial gravity acceleration $g \ (g = 10 \text{ m/s}^2)$. Fig. 2 illustrates the geotechnical centrifuge of the IFSTTAR.
2.2 Description of the pile models

The pile models are tubes made of Duralumin AU-4G, these models have been instrumented by 12 equidistant pairs of strain gauges along two diametrically opposite axes, and two displacement transducers (LVDT) placed on the pile top, as depicted in Fig. 3. Table 2 summarizes the main properties of the two piles studied in geotechnical centrifuge.

The installation of pile models has been carried out at 1 g, i.e. before placing the container in the centrifuge and starting the centrifugation. The pile models have been installed into the sand by manual jacking at low rate. In order to get rough surfaces of the models, the sand was glued at their shafts.

Lateral loads were applied up to 200 kN for pile \( P_1 \) and up to 483.5 kN for pile \( P_2 \), by increments of 3 minutes of duration by a loading device, the latter is illustrated in Fig. 4, and schematized in Fig. 3. Where a beam swings around a pivot due to a moving mass, which generates tension in a metallic cable, the latter is attached to the pile head, which causes lateral displacement of the pile. Remote control of the mass movement allows assigning a well-controlled load increments program to the model, which is the main advantage of this experimental device [24].

Centrifuge acceleration corresponds to the inverse of the dimension scale \( L^* \), that is to say 17.85 times the terrestrial gravity acceleration for pile \( P_1 \), and 20 times for pile \( P_2 \).

2.3 Description of the soil model

The soil used is clean dry red quartzite poorly graded sand from Le-Rheu site (Rennes, France) classified as an SP sand according to the USCS system. Its dry unit weight \( \gamma_d \) is 16.5 kN/m\(^3\) corresponding to a density index \( I_d \) of 92 \% (very dense sand), and the measured internal friction angle by a direct shear test is 42°.

Static cone penetration tests (CPT) with a miniature penetrometer were carried out during the centrifugation, just before the start of the loading tests. The rod length of this penetrometer is 200 mm, its cone diameter is of 6 mm. Load cell and LVDT transducer are fixed on the cone in order to measure continuously the cone resistance \( q_c \) at various depths.

Fig. 5 represents a typical penetration resistance profile obtained at centrifuge acceleration of 20 g, \( g \) being the gravitational acceleration.

It is to be noticed that the cone resistance profiles are hyperbolic shaped, and may be fitted by the following equation:
where \( E_c \) and \( q_{cu} \) are respectively called the shallow cone stiffness and the deep cone resistance.

In geotechnical literature, there are several correlations between the sand elastic modulus \( E \) and the cone resistance \( q_c \) and the ratio \( E/q_c \) is ranging between 2.5 and 3.5 [25]. Elastic sand modulus \( E \) was estimated from the cone resistance \( q_c \) by multiplying it by an average factor of 3 as shown in the Eq. (3) and was then assumed increasing with depth following a hyperbolic function.

\[
E = 3q_c,
\]

The results presented hereafter correspond to the prototype scale of a rough pile installed by jacking into a very dense dry sand.

### 2.4 Construction of \( P-Y \) curves

Measurements obtained for each lateral load increment are the top lateral displacements, and the axial deformations \( \varepsilon(z) \) along the pile shaft by means of strain gauges, which allow calculating the experimental bending moment \( M(z) \) as follows:

\[
M(z) = 2E_p I_p \varepsilon(z)/B.
\]

The \( P-Y \) curves describe the relationship between the soil lateral reaction \( P \) and the pile lateral displacement \( Y \). In most cases, they are derived from the experimental bending moment profile \( M(z) \).

The deformations are measured at strain gauges levels, therefore the resulting experimental bending moment profile consist of discrete points that form a broken line, it is convenient to fit the latter by a continuous function.

Several methods have been used to fit experimental bending moment profiles, among these methods, we find smoothing by cubic and quintic splines [24, 26], polynomial fitting [27, 28], and the weighted residual method [27].

In this study, we fitted the experimental bending moment profiles by single term Fourier series using MATLAB software [29], with a fitting coefficient of 98 %, Eq. (5) describes Fourier series form used herein.

\[
M(z) = a_0 + a_1 \cos(\omega z) + b_1 \sin(\omega z),
\]

where \( a_0, a_1, b_1, \) and \( \omega \) are coefficients of the Fourier series.

The profile thus found is differentiated twice to obtain the soil lateral reaction \( P \) as given by Eq. (6) and integrated twice to obtain the pile lateral displacement \( Y \) as shown by Eq. (7). The integration constants are calculated from the displacement measurements on the pile head. Figs. 6 and 7 illustrate the soil lateral reaction \( P \), and the pile lateral displacement for pile \( P_1 \) and \( P_2 \), respectively.

\[
P = -\frac{d^2 M}{dz^2},
\]

\[
Y = \frac{1}{E_p I_p} \int \int M \, dz \, dz + Y_0 \varepsilon + Y_0
\]

Based on soil lateral reaction \( P \), and pile lateral displacement \( Y \), we plotted \( P-Y \) curves for piles \( P_1 \) and \( P_2 \) as shown in Fig. 8 and Fig. 9. We plotted these curves for the same depths where we plotted the ABAQUS FEM \( P-Y \) curves.
By observing these $P$-$Y$ curves, it can be noticed an inherent nonlinearity even at small displacements. Moreover, for a given load, the lateral soil stiffness which may be quantified by the secant slope of $P$-$Y$ curves increases with depth. It will be shown hereafter a linear variation of lateral soil stiffness versus depth which is typical of a Gibson's soil.

Given their shapes, the $P$-$Y$ curves may be well fitted by the following hyperbolic function:

$$P = \frac{Y}{\frac{1}{E_{ti}} + \frac{Y}{P_u}}, \quad (8)$$

where $E_{ti}$ is the initial subgrade reaction modulus, describing a linear relationship between the soil lateral reaction $P$ and the pile lateral displacement $Y$ for small deformations, as shown by Eq. (9).

$$E_{ti} = \frac{P}{Y}. \quad (9)$$

Fig. 10 shows that this modulus increases linearly with the depth $z$, with a gradient (slope) denoted $N_h$, as indicated by Eq. (10).

$$E_{ti} = N_h z. \quad (10)$$

Moreover, the values of this gradient are very close for both piles, in spite of the large differences in slenderness ratio ($D/B$), and in diameter as mentioned in Table 2. The average experimental gradient equals 9 MN/m$^3$.

$P_u$ is the lateral soil resistance corresponding to large pile displacements. It is the horizontal asymptote of the $P$-$Y$ curve, which corresponds to the maximum mobilized soil lateral reaction.
Mohr-Coulomb elastic-perfectly plastic constitutive model was used to describe the soil response and the representative parameters are summarized in Table 3. Pile material is however assumed to be linear elastic.

The surface-to-surface contact method of ABAQUS software has been used to take into account the nonlinear response at soil/pile interface.

The tubular pile are represented by 4-node, quadrilateral, stress-displacement shell elements with reduced integration and a large-strain formulation "S4R", while the soil is modelled by three-dimensional continuum stress/displacement, 8-node, reduced integration elements "C3D8R" [30].

The plane of symmetry is fixed against any displacements in the y direction, the back surface is fixed against displacements in the x and y directions, and of course the base of the model is fixed against displacements in all directions.

Considering the hyperbolic increase of the elastic modulus $E$ with the depth $z$, the sand material is defined in ABAQUS through the "user-defined field" option. First a "field variable" as depth $z$ is added through the Material Editor of ABAQUS/CAE, and then it was written a user-defined field USDFLD subroutine in FORTRAN Language within the Microsoft Visual Studio environment. This subroutine, defining the hyperbolic variation of elastic modulus $E$ with vertical coordinates, was incorporated to ABAQUS through the Job Editor, and finally the compilation and calculation were carried out with Intel FORTRAN Compiler.

The interfacial friction between the pile and the sand was taken into account according to the "contact pairs" (surface to surface) approach of ABAQUS using the "basic Coulomb friction model" which assumes that the friction coefficient $\mu$ is the same in all directions (isotropic friction). It was adopted a value of 1.0 for $\mu$, i.e. an interfacial friction angle $\delta$ equal to the sand internal friction angle $\phi$ due to the roughness of pile/soil interface.

### Table 3 Parameters of Mohr-Coulomb law for sand

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Sand for $P_1$</th>
<th>Sand for $P_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry unit weight $\gamma_d$ (kN/m$^3$)</td>
<td>16.5</td>
<td>16.5</td>
</tr>
<tr>
<td>Internal friction angle $\phi$ (deg)</td>
<td>42</td>
<td>42</td>
</tr>
<tr>
<td>Dilation angle $\psi$ (deg)</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>Cohesion $C$ (kPa)</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Poisson's ratio $\nu_s$</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Shallow cone stiffness $E_c$ (MN/m$^3$)</td>
<td>5.07</td>
<td>5.40</td>
</tr>
<tr>
<td>Deep cone resistance $q_{cu}$ (MPa)</td>
<td>20.88</td>
<td>22.38</td>
</tr>
</tbody>
</table>
The analysis is performed in two steps; the first is a "geostatic" step that is used to reproduce the initial stress condition in the soil by applying its dry self-weight as "body force", the second step is a "general static" step in which the lateral load $H$ is applied to the pile head as a "concentrated load".

4 Validation of Finite Element Model

The comparison is focused on the load-displacement curves at ground surface and the bending moment profiles along the pile shaft. Fig. 13 shows the load-displacement curves of Pile $P_1$ and Pile $P_2$, obtained from centrifuged model tests and the three-dimensional finite element models. It may be noticed that load-displacement curves derived from physical centrifuge modelling are fairly close to those developed from numerical finite element modelling, especially for very small displacements.

Fig. 14 illustrates the bending moment profiles, where it can be seen that differences between experimental and numerical profiles are very small. Based on these observations, especially those for bending moment profiles, we consider that the results are sufficiently coherent to say that the ABAQUS finite element models are valid.

5 Determination of $P-Y$ curves

From the "job results" option of ABAQUS, the lateral displacement $Y$ and the rotation $Y'$ along the pile shaft were directly collected through the "ODB field output" option, while the shear force $T$ and bending moment $M$ were obtained by specifying all "elements and nodes" cross-sections of the pile using the "Free Body Cut" option, then extracted by means of the "Free Body" output option. The first derivative of the shear force $T$ yields the lateral reaction $P$ of the soil as indicated by the Eq. (11).

Knowing the profiles of $P$ and $Y$, it is possible to construct the $P-Y$ curves derived from the three-dimensional finite element models.

$$P = -\frac{dT}{dz}$$  \hspace{1cm} (11)

Fig. 15(b) shows the lateral displacement profile of the pile $P_1$, exhibiting some flexibility expressed by a certain curvature which characterizes the semi-rigid pile deformability. The center of rotation (point of zero displacement) is located at about 0.75 $D$. Fig. 16(b) shows the lateral displacement of the pile $P_2$, where the deformed shape is a rather straight line which characterizes the rigid pile deformability, the center of rotation being located at about 0.8 $D$.

Figs. 15(a) and 16(a) illustrate the profiles of soil lateral reaction for piles $P_1$ and $P_2$, respectively, where it can be seen that the center of pressure (point of zero soil reaction) coincide with the center of rotation, confirming the fundamental postulate of Winkler according to which $P$ is proportional to $Y$ and have therefore both the same sign.
Figs. 17 and 18 show $P-Y$ curves for pile $P_1$ and $P_2$ respectively. Following the same methodology of interpretation of the experimental $P-Y$ curves described in Section 2.4, the initial subgrade reaction modulus $E_{ti}$ and the lateral soil resistance $P_u$ was derived.

In Fig. 19 are illustrated the profiles of $E_{ti}$ for piles $P_1$ and $P_2$ which exhibit a remarkable linear variation along the pile which is typical to Gibson's soil model characterized by a linear stiffness profile. From Fig. 19 and Fig. 10, it may be noticed that experimental and the FEM gradients $N_i$ are comparatively close, moreover the average numerical gradient is equal to 9 MN/m³, i.e. equal to the average experimental gradient.
The calculated values of the gradient $N_h$ are within the range of values proposed by Terzaghi [31] for the same sand density as shown in the Table 4, then the method of the latter can be used to calculate the gradient $N_h$, and thus to estimate the initial subgrade reaction modulus $E_{ti}$.

Figs. 20 and 21 illustrate the profiles of lateral soil resistance $P_u$ for pile $P_1$ and $P_2$, respectively, given their triphasic forms, we fitted them by piecewise linear functions with three segments PWL3 using OriginPro software [32]. Subsequently, each segment was formulated in terms of the Rankine’s passive earth pressure coefficient $K_p$, the dry unite weight $\gamma_d$, and the pile diameter $B$ as indicated by Eq. (13).

$$K_p = \tan^2(45 + \phi / 2), \tag{12}$$

$$P_u = aK_p^2 \gamma_d B(z + b), \tag{13}$$

where $a$ and $b$ are the coefficients of lateral soil resistance $P_u$, these coefficients are summarized in Table 5. It is assumed that the formulae obtained for the pile $P_1$ are valid for the semi-rigid piles, and those suggested for the pile $P_2$ are valid for rigid piles. Fig. 22 illustrates the typical $P-Y$ curve proposed herein.

### Table 4 Gradient $N_h$ according to Terzaghi (1955)

<table>
<thead>
<tr>
<th>Dry unite weight $\gamma_d$ (kN/m$^3$)</th>
<th>$N_h$ (MN/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>13–15</td>
<td>0.96–3.30</td>
</tr>
<tr>
<td>15–17</td>
<td>3.30–12.60</td>
</tr>
<tr>
<td>17–19</td>
<td>12.60–28.10</td>
</tr>
</tbody>
</table>

### Table 5 Coefficients of lateral soil resistance

<table>
<thead>
<tr>
<th>Type of pile</th>
<th>Pile intervals</th>
<th>$P_u$ coefficients $a$</th>
<th>$b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semi-rigid piles</td>
<td>0 - 0.25D</td>
<td>1.90</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>0.25D - 0.87D</td>
<td>-0.96</td>
<td>-3.70</td>
</tr>
<tr>
<td></td>
<td>0.87D - D</td>
<td>-5.00</td>
<td>-4.22</td>
</tr>
<tr>
<td>Rigid piles</td>
<td>0 - 0.35D</td>
<td>1.84</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>0.35D - 0.90D</td>
<td>-1.60</td>
<td>-3.75</td>
</tr>
<tr>
<td></td>
<td>0.90D - D</td>
<td>-4.60</td>
<td>-4.20</td>
</tr>
</tbody>
</table>

6 Conclusions

In this paper were presented and analyzed the main results of experimental and numerical modelling of the load-deflection behavior of two piles under monotonic lateral loading in dense sand. Brief description of the experimental devices used in centrifuge tests was followed by the presentation of the methodology of construction of the
experimental \( P-Y \) curves and the determination of initial subgrade reaction modulus \( E_{gr} \), as well as the lateral soil resistance \( P_s \).

It was found that \( E_{gr} \) is almost independent of the slenderness ratio \( D/B \), and also of pile diameter \( B \). Moreover, normalized lateral soil resistance was found decreasing with depth and vanishing at about 0.7 of the pile embedded length \( D \).

The second part of the paper was devoted to a three-dimensional finite element modelling using ABAQUS Software. The nonlinear response of the sand was described by elastic perfectly plastic Mohr-Coulomb constitutive model, where the elastic soil modulus was estimated by correlation with the static cone penetration resistance. Soil modulus was assumed increasing with depth following a hyperbolic function and was implemented in ABAQUS through the USDFLD user subroutine. The FEM model was validated by comparing the results with those of the centrifuged model which showed quite good coherence in terms of load-deflection curves, as well as the bending moment profiles.

Pile lateral displacements and soil lateral reaction along the pile shaft were derived through the "free body" option of ABAQUS, and the \( P-Y \) curves were constructed for each given depth \( z \). The parameters of the \( P-Y \) curves were obtained by hyperbolic fitting of the \( P-Y \) curves. It was found that both experimental and numerical \( E_{gr} \) moduli have a linear variation with depth, confirming the Gibson’s soil model for homogeneous cohesionless soils. Terzaghi method [31] may be used to estimate the initial subgrade reaction modulus \( E_{gr} \).

Analysis of the lateral soil resistance \( P_s \) led to a preliminary recommendation to estimate it for semi-rigid, and rigid piles embedded in dense sand, we formulated the lateral soil resistance \( P_s \) through a trilinear function, which depends on the Rankine’s passive earth pressure coefficient \( K_s \), the sand dry unit weight \( \gamma_{dr} \), and the pile diameter \( B \).

Subsequent work will focus on a thorough detailed comparative study with current design methods available in literature.

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