Periodica Polytechnica Civil Engineering, 64(2), pp. 524–534, 2020

Influence of Rail Pad Stiffness and Axle Loads on Dynamic Responses of Train-track Interaction with Unsupported Sleepers

Jabbar Ali Zakeri[1]*, Morvarid Fattahi[2], Mehrdad Nouri[2] Fatemeh Janatabadi[2]

[1] The Center of Excellence in Railway Transportation, School of Railway Engineering, Iran University of Science and Technology, Narmak, Tehran 16844, Iran
[2] School of Railway Engineering, Iran University of Science and Technology, Narmak, Tehran 16844, Iran
* Corresponding author, e-mail: zakeri@iust.ac.ir

Received: 10 August 2019, Accepted: 28 February 2020, Published online: 07 April 2020

Abstract

Increasing the axle load causes track deterioration and permanent settlement of the ballast layer. In the ballasted railway tracks, due to the inevitability of unequal settlements in the ballast layer, part of the rail due to high flexural rigidity will become suspended, which causes the formation of the track with unsupported and partially supported sleepers. This situation increases rail vertical displacement and reactions on adjacent sleepers. Several models have been presented to study the effect of unsupported sleepers on dynamic responses of train-track interaction. In this paper, by applying mathematical model, unsupported and partially supported sleepers have been modeled and equations of motions for train-track interaction were written by assuming nonlinear behavior of rail and wheel contact. Following by solving the equations via numerical integration in the time domain, the effect of axle load and pad stiffness on rail vertical displacement were investigated. Results of the analysis suggested that through increasing the axle load rail displacement increased by 13 % in the unsupported sleeper and from 5 to 10 % in the partially supported sleeper. Also, by increasing the pad stiffness rail displacement decreased from 2 to 13 % in the unsupported sleeper and from 1 to 6 % in the partially supported sleeper.

Keywords

railway track dynamics, unsupported sleeper, partially supported sleeper, axle load, pad stiffness

1 Introduction

Railway transportation system is one of the most important and efficient modes of passenger and freight transport in the world. One way to improve its performance and efficiency is to increase the axle load, however, axle load increment can cause track distortion and permanent ballast settlement. Hence, particularly in cases where there are defects in track components, it is vital to investigate the effect of axle load increase on track structural function.

Train unavoidably regulates its behavior regarding the track condition and the dynamic interaction between the rolling stock and the track. Therefore, some defects in track structure might cause changes in this dynamic interaction and affect safety and ride comfort. Some of the defects in ballasted track occur as a result of unequal settlements in ballast layer, in which a part of the rail due to high rigidity detaches from the ballast and becomes suspended, this causes the formation of the unsupported and partially supported sleepers in the track structure. This occurrence also increases rail vertical displacement and responses on adjacent sleepers. Moreover, due to deterioration in some parts of the ballast, one or a number of the sleepers will hang from the rail. So whenever the train passes the detached sleeper and ballast, it forms an unsupported sleeper and if after a few rail displacements, the sleeper strikes the ballast, it results in the partially supported sleeper.

This article attempts to investigate the effects of axle load and pad stiffness increment on the rail's vertical displacement using a developed mathematical model and finite element methods, with regards to the effect of unsupported and partially supported sleepers. Also, the track structure has been modeled as a connected mass, spring, and damper system.

2 Literature review

In past years, numerous studies were conducted regarding railway track dynamic behavior under various conditions, as a result, numerous models have been presented.

Cite this article as: Zakeri, J. A., Fattahi, M., Nouri, M., Janatabadi, F. "Influence of Rail Pad Stiffness and Axle Loads on Dynamic Responses of Train-track Interaction with Unsupported Sleepers", Periodica Polytechnica Civil Engineering, 64(2), pp. 524–534, 2020. https://doi.org/10.3311/PPci.14826

Grassie and Cox [1] presented a laboratory numerical model by applying quasi-static calculations assumption. He investigated the effect of contact force at various speeds for the unsupported sleeper and the results were compared with fully supported sleeper. Nielsen and Igeland [2] investigated the dynamic behavior of track and the effect of speed on the unsupported sleeper. They discovered that at the speed of 150 km/h, the maximum calculated bending stress at rail-seat was increased 33 % compared with normal track. Ishida et al. built a model with unsupported sleeper and studied the track dynamic behavior and the effect of flexural fatigue on welded rails. They concluded that with two unsupported sleepers and 2 mm gap, the estimated life span of welded rail with fatigue is about half the normal [3]. Zakeri et al. [4] presented a model with 53 sleepers and two infinite boundary conditions by assuming one unsupported sleeper and nonlinear wheel-rail contact. They discovered that with roughness existence, high acceleration on sleepers and high force on rail-seats are created. Lundqvist and Dahlberg [5] presented a model regarding the effect of the unsupported sleeper on the dynamic behavior of track's components and also investigated the effect of the gap between sleeper and ballast on the interaction force increment. Kim et al. [6] studied the dynamic behavior of unsupported sleeper and the effect of axle load on high-speed railways. Also, Zhang et al. [7] analyzed the effect of railways defects and unsupported sleepers on created forces between rail and wheel. They used nonlinear springs and dampers to simulate the gap between sleeper and ballast. The results elucidated that this gap has a great effect on the force created between rail and wheel and also the created track settlement led to the axle load increase which enhances ballast's deterioration and settlement. Kaewunruen and Remennikov [8] conducted several laboratory studies on the effect of unsupported sleepers on modal analysis and consequences of vibration modes on the track. Zhu et al. [9] investigated the effect of vehicle speed, gap size and the number of unsupported sleepers by assuming a continuous track system model and an adaptive wheel-rail contact model. Rezaei and Dahlberg [10] by presenting their cross-section model of track, showed that the effect of pad stiffness is negligible on sleeper dominant frequency and the effective parameters are sleepers flexural stiffness, mass, and its distribution . Also, Zhu et al. [11] investigated track dynamic behavior with two unsupported sleepers and building a model in 1:5 scale. Zoller and Zobory [12, 13] studied the dynamic interaction of the railway track with varying stiffness in the Winkler foundation and also in the presence of inhomogeneous rail supporting parameters . Zakeri et al. [14] developed a mathematical model to investigate and compare the changes in the rail displacement while the spacing and the number of partially supported and unsupported sleepers change . Mosayebi et al. [15] using finite element method developed a pyramid model equation for three different stress conditions. The result of analyses presented the ratios of the railway track vertical displacement to the vehicle axle load for various foundation stiffness. Sadeghi et al. [16] investigated the effects of unsupported sleepers on the dynamic behavior of railway track using a numerical three-dimensional model and illustrated the effects of the various conditions on the rail seat load, sleeper bending moment and sleeper–ballast contact force at various train speeds. Dai et al. [17] analyzed dynamic responses of high-speed train-track and investigated various effective factors such as the train speed, the number of hanging sleepers and their pattern via calculation scheme in combination with the moving element method. Mosayebi et al. [18] investigated the effects of the unsupported sleepers on the dynamic behavior of the track via a finite element model of three vehicles, without bogies and with two and three-axle bogies and illustrated various regression equations concerning the train axle loads with rail bending moments, sleeper displacements, and support forces for tracks.

3 Train-Track Model

For dynamic analysis, the track was modeled as an Euler-Bernoulli beam and the components were assumed as a series of discrete masses, springs, and dampers. By assuming that the vehicle has two biaxial bogies, thus it will have 10 degrees of freedom in two-dimensional modeling. The total number of degrees of freedom are two rotational and translational degrees of the wagon, two rotational and translational degrees for each bogie, and a transition degree related to each wheel. The vehicle dynamic interaction model and its components are shown in Fig. 1.

4 Matrices formation and determining the equations

Utilizing matrices and vectors to analyze the dynamic behavior of the wheel and the rail, makes it possible to conduct meticulous qualitative and quantitative studies regarding the dynamic phenomena behavior between the train and track. To analyze wheel-rail dynamic interaction the fundamental step is the equation of motion configuration.

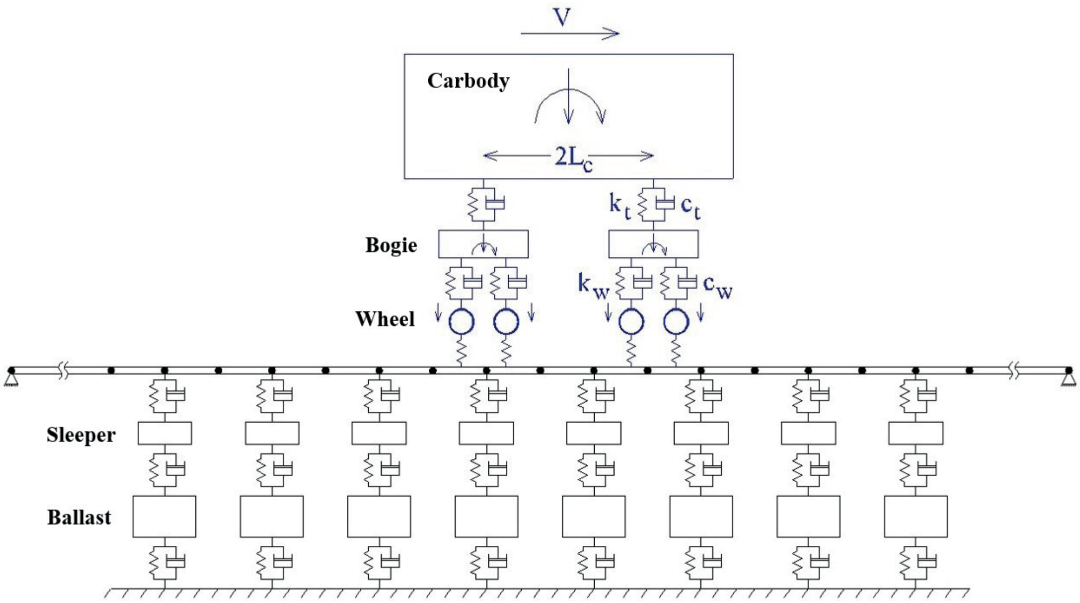

Fig. 1 Dynamic interaction of train-track model

The total equation of motion is shown in Eq. (1). Where Z_{total}, $\dot{Z}_{total}$ and $\ddot{Z}_{total}$ are respectively representing the displacement, velocity and acceleration vectors of the whole system and can be calculated via Eq. (2) to Eq. (6).

$$M_{total}\left\{\ddot{Z}_{total}\right\}+C_{total}\left\{\dot{Z}_{total}\right\}+K_{total}\left\{Z_{total}\right\}=\left\{F_{total}\right\} \quad (1)$$

$$\left\{Z_{total}\right\}=\left[\left\{Z_V\right\},\left\{Z_R\right\},\left\{Z_S\right\},\left\{Z_B\right\}\right]_{10+2NJ+2NS\times 1} \quad (2)$$

$$\left\{Z_V\right\}=\left[Z_c,\varphi_c,Z_{t1},\varphi_{t1},Z_{t2},\varphi_{t2},Z_{w1},Z_{w2},Z_{w3},Z_{w4}\right]_{10\times 1}^T \quad (3)$$

$$\left\{Z_R\right\}=\left[Z_1^r \quad \theta_1^r \quad Z_2^r \quad \theta_2^r \cdots Z_{2NJ}^r \quad \theta_{2NJ}^r\right]_{2NJ\times 1}^T \quad (4)$$

$$\left\{Z_S\right\}=\left[Z_1^S \quad Z_2^S \quad \cdots \quad Z_{NS}^S\right]_{NS\times 1}^T \quad (5)$$

$$\left\{Z_B\right\}=\left[Z_1^B \quad Z_2^B \quad \cdots \quad Z_{NS}^B\right]_{NS\times 1}^T \quad (6)$$

$[K]$ is the stiffness matrix that includes car-body and bogie, wheel, rail, sleeper and ballast matrices and also the interactions between them. $[M]$ is the mass matrix that includes car-body and bogie, wheel, rail, sleeper and ballast masses. $[C]$ is the damping matrix, which is similar to $[K]$ matrix, except that there is no interaction matrix between rail and wheel. The size of the matrix is the number of degree of freedom (NODF). Stiffness, mass and damping matrices are presented in Eq. (7), Eq. (8) and Eq. (9) respectively.

$$\left[K_{total}\right]=\begin{bmatrix} \left[carbody\,\&\,bogie\right]_{6\times 6} & C/W & 0 & 0 & 0 \\ W/C & \left[Wheel\right]_{4\times 4} & W/R & 0 & 0 \\ 0 & R/W & \left[Rail\right]_{2NJ\times 2NJ} & R/S & 0 \\ 0 & 0 & S/R & \left[Sleeper\right]_{NS\times NS} & S/B \\ 0 & 0 & 0 & B/S & \left[Ballast\right]_{NS\times NS} \end{bmatrix}_{TNODF\times TNODF} \quad (7)$$

$$\left[M_{total}\right]=\begin{bmatrix} \left[carbody\,\&\,bogie\right]_{6\times 6} & 0 & 0 & 0 & 0 \\ 0 & \left[Wheel\right]_{4\times 4} & 0 & 0 & 0 \\ 0 & 0 & \left[Rail\right]_{2NJ\times 2NJ} & 0 & 0 \\ 0 & 0 & 0 & \left[Sleeper\right]_{NS\times NS} & 0 \\ 0 & 0 & 0 & 0 & \left[Ballast\right]_{NS\times NS} \end{bmatrix}_{TNODF\times TNODF} \quad (8)$$

$$\left[C_{total}\right]=\begin{bmatrix}\left[carbody\,\&\,bogie\right]_{6\times 6} & C/W & 0 & 0 & 0\\ W/C & \left[Wheel\right]_{4\times 4} & 0 & 0 & 0\\ 0 & 0 & \left[Rail\right]_{2NJ\times 2NJ} & S/R & 0\\ 0 & 0 & S/R & \left[Sleeper\right]_{NS\times NS} & S/B\\ 0 & 0 & 0 & B/S & \left[Ballast\right]_{NS\times NS}\end{bmatrix}_{TNODF\times TNODF} \tag{9}$$

After modeling the track components, mass, stiffness, and damping matrices for each element of the rail are calculated as below:

The rail stiffness matrix:

$$K=\frac{EI}{L^3}\begin{bmatrix}12 & 6L & -12 & 6L\\ 6L & 4L^2 & -6L & 2L^2\\ -12 & -6L & 12 & -6L\\ 6L & 2L^2 & -6L & 4L^2\end{bmatrix} \tag{10}$$

The rail mass matrix:

$$m=\frac{mL}{420}\begin{bmatrix}156 & 22L & 54 & -13L\\ 22L & 4L^2 & 13L & -3L^2\\ 54 & 13L & 156 & -22L\\ -13L & -3L^2 & -22L & 4L^2\end{bmatrix} \tag{11}$$

The rail damping matrix is calculated based on the rail mass and stiffness matrix as follows:

$$C=\alpha M+\beta K. \tag{12}$$

In the above equations, E, I, L, m, α, and β are rail Young's modulus, rail inertia moment, length of the element, mass per unit rail length, and Rayleigh damping coefficients, respectively. Mass, stiffness, and damping of the rail are determined by assembling each element.

In Eq. (13), F_t is the total external forces applied on the system. Knowing that $R(x)$ is wavy roughness of the rail's surface and K_H is Hertzian spring stiffness, F_{wj} can be calculated using Eq. (14) to Eq. (20).

$$\begin{aligned}F_{total}^{T}=&[0\ 0\ 0\ 0\ 0\ 0-F_{w1}(t)-F_{w2}(t)\\ &-F_{w3}(t)-F_{w4}(t)]_{(10\times 10)},\end{aligned} \tag{13}$$

$$\begin{cases} f\ Z_w\geq Z_r+R(x)\rightarrow F_{wj}=K_H\left[Z_{wj}-Z_r\pm R(X)\right]\\ if\ Z_w\geq Z_r+R(x)\rightarrow F_{wj}=K_H=0\end{cases}, \tag{14}$$

$$Z_r=\Psi_1(x)Z_i^r+\Psi_2(x)\theta_i^r+\Psi_3(x)Z_{i+1}^r+\Psi_4(x)\theta_{i+1}^r, \tag{15}$$

$$\Psi_1(x)=1-3\left(\frac{x}{l}\right)^2+2\left(\frac{x}{l}\right)^3, \tag{16}$$

$$\Psi_2(x)=x-2\left(\frac{x^2}{l}\right)+\frac{x^3}{l^2}, \tag{17}$$

$$\Psi_3(x)=3\left(\frac{x}{l}\right)^2-2\left(\frac{x}{l}\right)^3, \tag{18}$$

$$\Psi_4(x)=\frac{x^3}{l^2}-\frac{x^2}{l}, \tag{19}$$

$$K_H=\frac{3}{2}C_H\times Z_r^{\frac{1}{2}}, \tag{20}$$

$$TNODF=10+(2\times NJ)+(2\times NS), \tag{21}$$

$$Time(s)=\frac{l+l_0}{V}, \tag{22}$$

where Z_r is the vertical displacement of rail between nodes i and $i+1$ and at a x distance from node i Z_i^r, is vertical displacement of rail at node I, Z_{i+1}^r, is vertical displacement of rail at node $i+1$, $\Psi_1(x)$ is shape function of vertical displacement of node i, $\Psi_2(x)$ is shape function of rotation of node i, $\Psi_3(x)$ is shape function of vertical displacement of node $i+1$, $\Psi_4(x)$ is shape function of rotation of node $i+1$, l is the distance between nodes i and $i+1$ (length of the element), θ_i^r is rotation of node I, θ_{i+1}^r is rotation of node $i+1$, and K_H is the nonlinear Hertzian stiffness. It should be noted that this model had been previously validated by Zakeri et al. [14].

5 The inputs

In this study, the model consists of 60 sleepers with the constant space of 60 cm, therefore the length of the model is 35.4 m. In this model, each joint is comprised between two sleepers so the number of joints is 119 (NJ = 119) and the number of supports is 60 (NS = 60). Hence, the total number of freedom is acquired using Eq. (21). Considering the last wheel crossing the track, the calculating time can be computed through Eq. (22), where $l0$ is the rigid wheelbase of the wagon and l is the length of the selected track.

To solve the differential equations, the Newmark-β method is used (Eqs. (23) and (24)), where the values for γ and β are considered to be 0.5 and 0.25 respectively [19]:

$$\dot{u}_{n+1} = \dot{u}_n + (1-\gamma) h\ddot{u}_n + \gamma h\ddot{u}_{n+1}, \tag{23}$$

$$u_{n+1} = u_n + h\dot{u}_n + \left(\frac{1}{2} - \beta\right) h^2\ddot{u}_n + \beta h^2\ddot{u}_{n+1}. \tag{24}$$

All the input values used in this study are shown in Table 1.

6 Implanting the effect of the partially supported sleeper in the equations

Due to various settlements on the railway track, though small, there are significant stresses and vertical displacements in the system [20]. Fig. 2 demonstrates the process of implementing the effect of settlements in the equations.

Table 1 Input values for each parameter

M_C	49500 kg	K_t	1.72 × 10⁶ N/m	I	3.22 × 10⁻⁵ N/m	M_b	700
J_C	1.7 × 10⁶ kg.m²	C_t	3 × 10⁵ N.S/m	ρ	60 kg/m³	M_S	160
M_t	10750 kg	L_C	8.5 m	A	1 m²	K_f	1.3 × 10⁸ N/m
J_t	9.6 × 10³ kg.m²	L_t	1.25 m	K_P	2.4 × 10⁸ N/m	C_f	6.23 × 10⁴ N.sec/m
M_W	2200 kg	K_H	108.7 × 10³ N/m	C_P	2.48 N.sec/m		
K_W	4.36 × 10⁶ N/m	C_H	1 × 10¹¹ N/m³ᐟ²	K_b	7 × 10⁷ N/m		
C_W	2.2 × 10⁵ N.sec/m	E	2.06 × 10¹¹ N/m²	C_b	1.8 × 10⁵ N.sec/m		

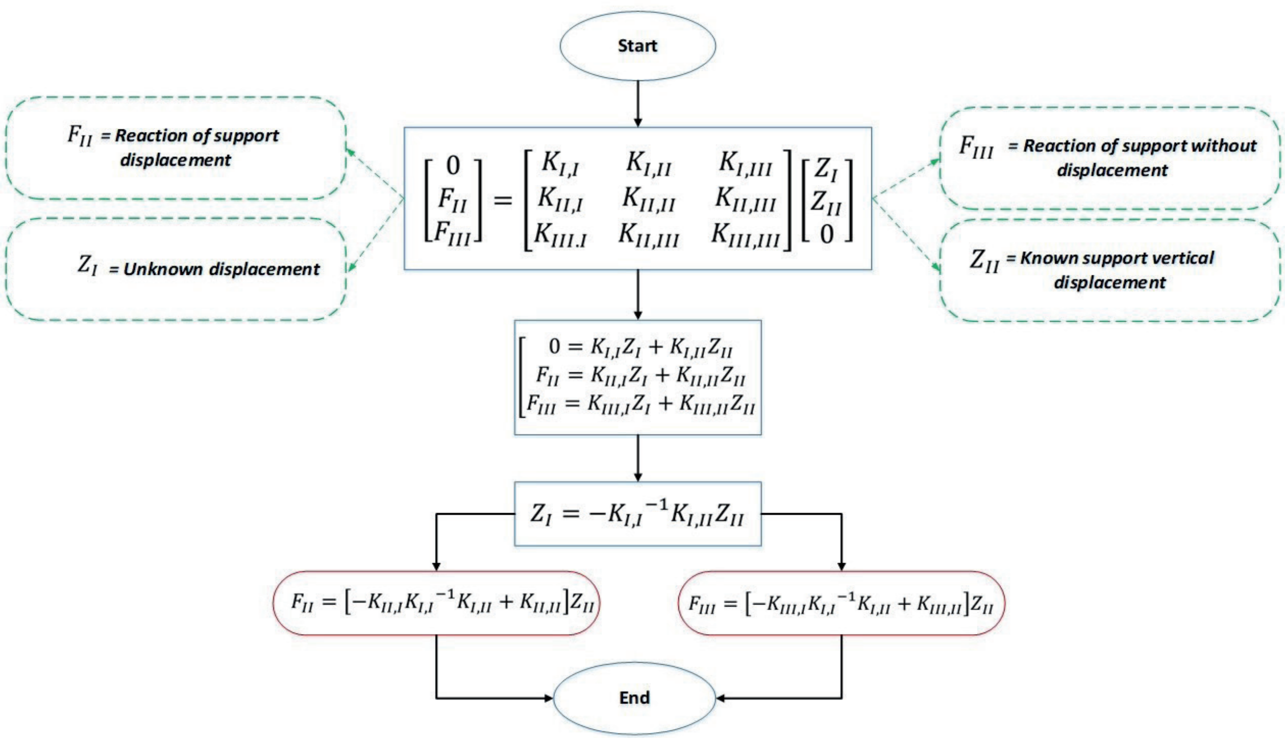

Fig. 2 Flowchart of inserting effect of settlement in the equations

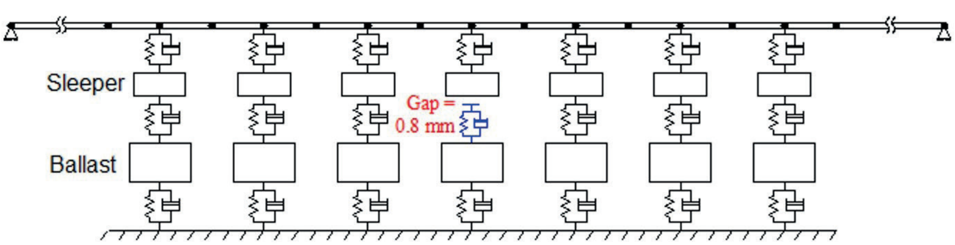

Fig. 3 Damping and stiffness model of partially supported sleeper

Throughout determining these equations, degrees of freedom for sleeper and ballast matrices were assumed to be known support settlements and a determined amount of vertical displacement was applied to them, ultimately, the unknown vertical displacements of the rail were obtained. Fig. 3 displays how the current model was developed. To determine rail displacement, the spring was modeled with a bilinear behavior as shown in Fig. 4.

7 Results

To remove the boundary conditions, an adequate number of sleepers is required, which is considered to be 60 in this study [21]. The middle sleeper has been modeled as the suspended sleeper while to investigate its effects, ballast stiffness and damper under the examined sleeper were removed.

7.1 Effect of axle load with an unsupported sleeper

With a constant spacing of 60 cm between the sleepers and by various axle loads (12.5, 15, 17.5, 20, 22.5 and 25 kN) passing the track, for the 30th sleeper while it is fully supported and unsupported, the maximum vertical displacement is determined under the examined and adjacent sleepers and the results are shown in Fig. 5.

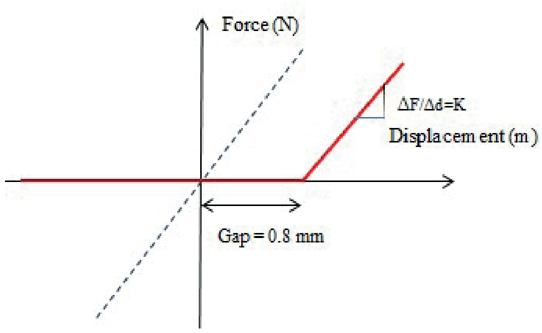

Fig. 4 The variation of stiffness in partially supported sleeper

By comparing the measured displacements, presented in Table 2, under the 30th sleeper, it was observed that the maximum vertical displacement of rail in the case of one unsupported sleeper is about 13 % more than a fully supported sleeper.

As can be seen in Fig. 6, the maximum vertical displacement under the first wheel was examined during the time that the train was passing, and the results indicate that the rail displacement is maximized over the unsupported sleeper and it increases with axle load increments, while there is no further displacement over the fully supported sleeper.

7.2 Effect of axle load in case of a partially supported sleeper

To analysis, the effects of different axle loads on the partially supported sleeper, the model presented in the previous section, was examined under various axle loads (12.5, 15, 17.5, 20, 22.5 and 25 kN), and the amount of displacements under each load is displayed in Fig. 7. As can be seen in Table 3, the maximum vertical displacement of rail over the partially supported sleeper is about 5–10 % more

Table 2 Outputs from unsupported and fully supported sleeper in various axle loads and their comparison

Axle load (kN)	Maximum rail displacement (mm)		Ratio of unsupported to fully supported sleeper
	Unsupported sleeper	Fully supported sleeper	
12.5	0.997	0.879	1.134
15	1.199	1.057	1.134
17.5	1.4	1.235	1.133
20	1.598	1.41	1.133
22.5	1.804	1.592	1.133
25	2.006	1.77	1.133

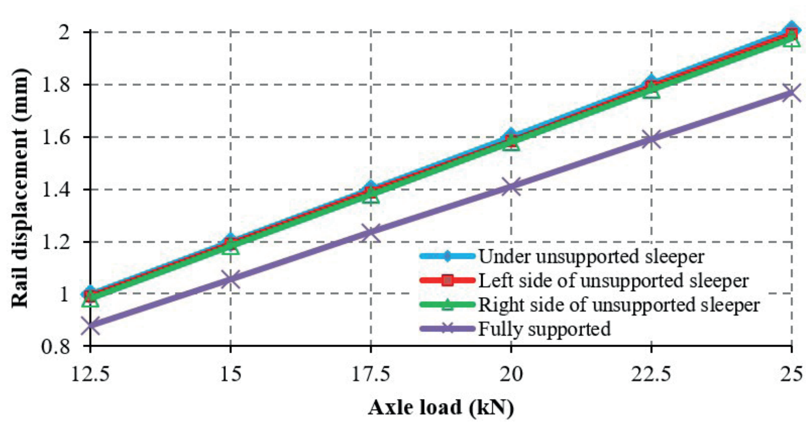

Fig. 5 Rail vertical displacement diagram in various axle loads under 30th sleeper and adjacent sleepers

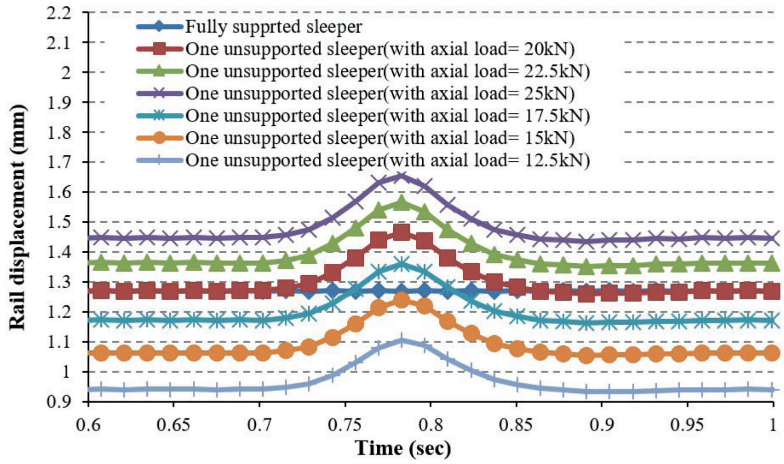

Fig. 6 Time history of rail displacement under the first wheel in various axle loads

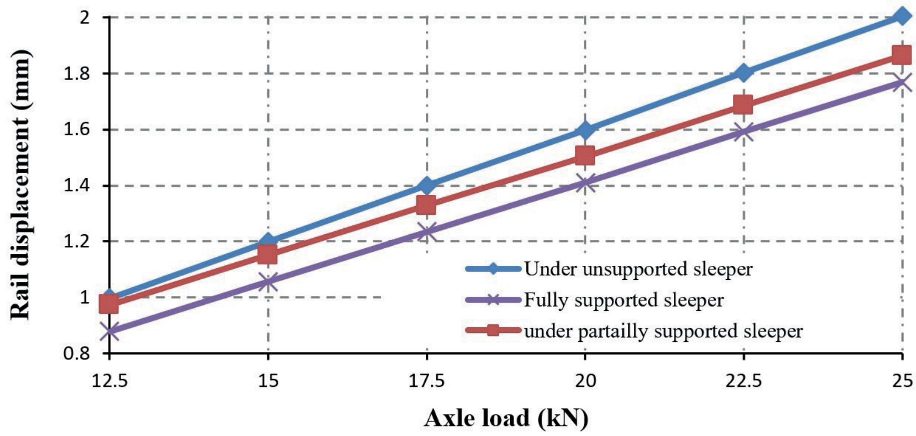

Fig. 7 Rail vertical displacement in various axle loads under 30th sleeper

than a fully supported sleeper. Also, maximum vertical rail displacement under the first wheel was investigated while the train was passing and the results are shown in Fig. 8.

7.3 Effect of pad stiffness with one unsupported sleeper

To analysis, the effects of different pad stiffness on the unsupported sleeper, the model presented in the previous section, was investigated using different pad stiffness (0.6, 15, 0.9, 1.2, 1.5, 1.8, 2.1 and 2.4 × 10^8 N/m) with regard to the range listed by Sun and Dhanasekar [22]. Fig. 9 presents the rail displacement related to each pad stiffness. As can be seen in Table 4, the vertical displacement of the rail is reduced by increasing the pad stiffness and with unsupported sleeper; these variations are in 2–13 % range. Also, Fig. 10 shows the vertical displacement with different pad stiffness under 30th sleeper and adjacent sleepers, while the train passes.

Table 3 Outputs of unsupported, partially supported and fully supported sleeper under various axle loads and their comparison

Axle load (kN)	Maximum rail displacement (mm)		
	Unsupported sleeper	Fully supported sleeper	Partially supported sleeper
12.5	0.997	0.879	0.973
15	1.199	1.057	1.151
17.5	1.4	1.235	1.324
20	1.598	1.41	1.504
22.5	1.804	1.592	1.686
25	2.006	1.77	1.864

7.4 Effect of pad stiffness with one partially supported sleeper

To analysis, the effects of different pad stiffness on the partially supported sleeper, the model presented in the previous sections, was examined using different pad stiffness (0.6, 15, 0.9, 1.2, 1.5, 1.8, 2.1 and 2.4 × 10^8 N/m), and the result of the analysis is displayed in Fig. 11.

As shown in Table 5 and Fig. 12, the vertical displacement of the rail is reduced while the pad stiffness increases and with partially supported sleeper; these variations are in the rage of 1–6 % [23].

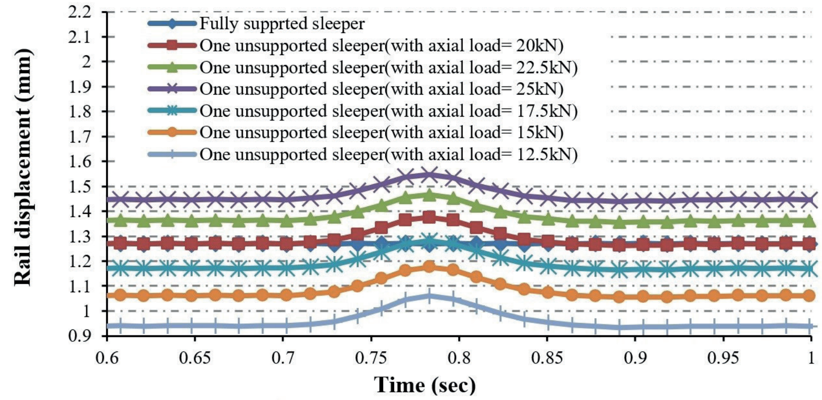

Fig. 8 Time history of rail displacement beneath the first wheel under various axle loads

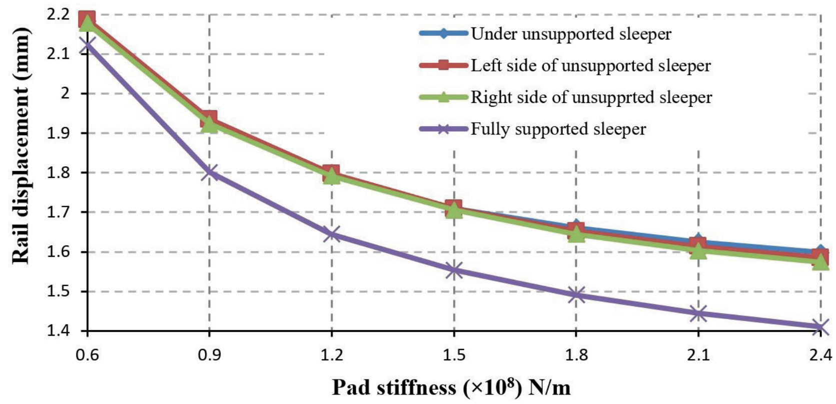

Fig. 9 Rail vertical displacement in various pad stiffness under 30th sleeper and adjacent sleepers

Table 4 Outputs of unsupported and fully supported sleeper in various pad stiffness and their comparison

Pad stiffness (N/m) × 10^8	Maximum rail displacement (mm)		Ratio of unsupported sleeper to fully supported sleeper
	Unsupported sleeper	Unsupported sleeper	
0.6	2.184	2.123	1.028
0.9	1.928	1.801	1.07
1.2	1.794	1.645	1.091
1.5	1.71	1.554	1.1
1.8	1.661	1.491	1.114
2.1	1.625	1.445	1.125
2.4	1.598	1.41	1.133

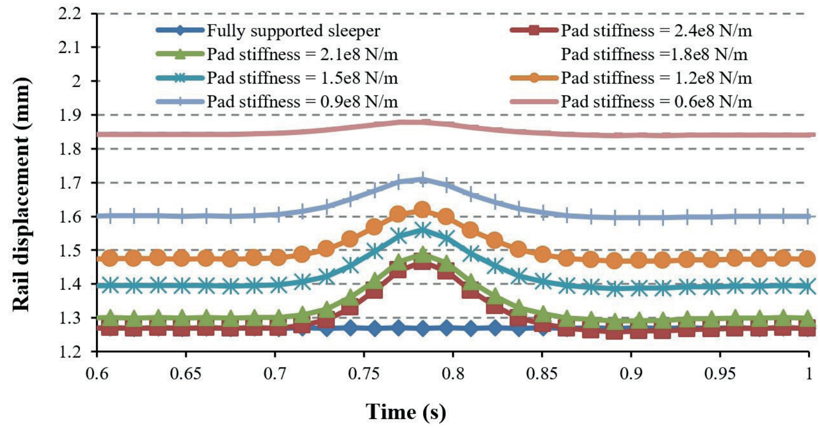

Fig. 10 Rail displacement under the first wheel in various pad stiffness in time history

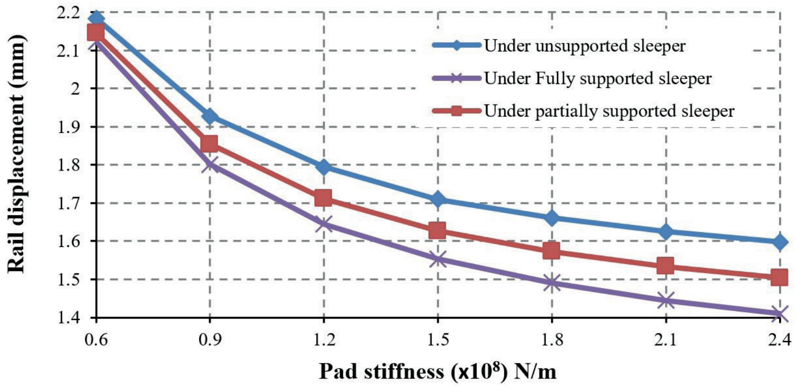

Fig. 11 Rail vertical displacement in various pad stiffness under 30th sleeper

Table 5 Outputs of unsupported, partially supported and fully supported sleepers under various pad stiffness and their comparison

Pad stiffness (N/m) ×10^8	Maximum rail displacement (mm)		
	Unsupported sleeper	Fully supported sleeper	Partially supported sleeper
0.6	2.184	2.123	2.145
0.9	1.928	1.801	1.854
1.2	1.794	1.645	1.711
1.5	1.71	1.554	1.627
1.8	1.661	1.491	1.573
2.1	1.625	1.445	1.534
2.4	1.598	1.4099	1.504

8 Conclusions

In this paper, the effect of partially and unsupported sleepers on the rail's vertical displacement was investigated through a numerical study and under various conditions. Railway track model was selected as an Euler-Bernoulli beam on discrete elastic supports which was modeled as a system with connected mass, spring, and damper. The vehicle was assumed to have one car-body, two bogies with two degrees of freedom for each one of them, and was modeled with four wheels. Considering that the length of the track model was 35.4 m, the model was examined under four conditions: a) Various axle loads with an unsupported sleeper, b) Various axle loads with one partially supported sleeper, c) Various pad stiffness with an unsupported sleeper, d) Various pad stiffness with one partially supported sleeper, and their results were compared to each other and also with the results of a fully supported sleeper. In partially supported sleepers the track stiffness behavior was bilinear, which means that whenever the sleeper is unsupported, the only system strengthening with that, is the rail. In other words, the vertical stiffness decreases, so when the sleeper strikes the ballast, the stiffness significantly increases.

Results of the analysis suggested that rail displacement increases while axle load increases and this process will elevate with unsupported sleeper. Rail displacement increased by 13 % through changing axle load from 12.5 kN to 25 kN in the unsupported sleeper and also it increased from 5 to 10 % in the partially supported sleeper. Moreover, it was seen that rail displacement decreased from 2 to 13 % over changing pad stiffness from 0.6 N/m to 2.4N/m ×10^8 in the unsupported sleeper and it decreased from 1 to 6 % in the partially supported sleeper.

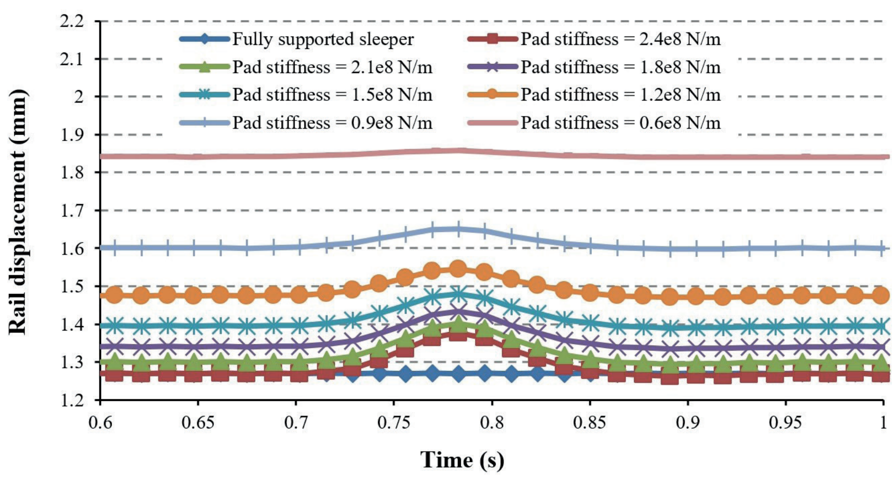

Fig. 12 Rail displacement under the first wheel with various pad stiffness in time history

Nomenclature

L_c	Half of distance between bogie centers
L_t	Half of distance between centers of wheel-axles in one bogie
E	Young's modulus
I	Moment of inertia
ρ	Mass density
A	Area section
M_w	Wheel-set mass
K_w	Primary vertical stiffness
C_w	Vertical primary damping
K_t	Secondary vertical stiffness
C_t	Vertical secondary damping
K_f	Formation stiffness in the ith support
C_f	Formation damping in the ith support
M_c	Car body mass
J_c	Polar moment of inertia of car body
M_t	Bogie mass
J_t	Polar moment of inertia of bogie
K_P	Rail pad stiffness
C_P	Rail pad damping
K_b	Ballast stiffness
C_b	Ballast damping
M_b	Ballast mass
M_s	Sleeper mass
K_H	Linearized Hertzian stiffness
C_H	Linearized Hertzian damping

References

[1] Grassie, S. L., Cox, S. J. "The Dynamic Response of Railway Track with Unsupported Sleepers", Proceedings of Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering, 199(2), pp. 123–136, 1985.
https://doi.org/10.1243/PIME_PROC_1985_199_149_01

[2] Nielsen, J. C. O., Igeland, A. "Vertical dynamic interaction between train and track-influence of wheel and track imperfections", Journal of Sound and Vibration, 187(5), pp. 825–839, 1995.
https://doi.org/10.1006/jsvi.1995.0566

[3] Ishida, M., Moto, T., Kono, A., Jin, Y. "Influence of Loose Sleeper on Track Dynamics and Bending Fatigue of Rail Welds", Quarterly Report of RTRI, 40(2), pp. 80–85, 1999.
https://doi.org/10.2219/rtriqr.40.80

[4] Zakeri, J. A., Xia, H., Fan, J. J. "Effects of unsupported sleeper on dynamic responses of railway track", Journal of Northern Jiao tong University, 24(1), pp. 50–55, 2000.

[5] Lundqvist, A., Dahlberg, T. "Load impact on railway track due to unsupported sleepers", Proceedings of the Institution of Mechanical Engineers Part F: Journal of Rail and Rapid Transit, 219(2), pp. 67–77, 2005.
https://doi.org/10.1243/095440905X8790

[6] Kim, D. S., Kim, S. D., Lee, J. "Easy Detection and Dynamic Behavior Unsupported Sleepers in High Speed Ballasted Track", presented at Proceeding of the 7th World Congress on Railway Research, Montreal Canada, June, 4–8, 2006. [CD-Rom]

[7] Zhang, S., Xiao, X., Wen, Z., Jin, X. "Effect of unsupported sleepers on wheel/rail normal load", Soil Dynamics and Earthquake Engineering, 28(8), pp. 662–673, 2008.
https://doi.org/10.1016/j.soildyn.2007.08.006

[8] Kaewunruen, S., Remennikov, A. M. "Influence of Voids and Pockets on Vibration Characteristics of Railway Concrete Sleepers", presented at Proceedings of 2007 SEM Annual Conference and Exposition on Experimental and Applied Mechanics, Springfield, MA, USA, June, 4–6, 2007. [CD-Rom]

[9] Zhu, J. J., Ahmed, A. K. W., Rakheja, S., Khajepour, A. "Development of a vehicle-track model assembly and numerical method for simulation of wheel-rail dynamic interaction due to unsupported sleepers", Vehicle System Dynamics, 48(12), pp. 1535–1552, 2010.
https://doi.org/10.1080/00423110903540751

[10] Rezaei, E., Dahlberg, T. "Dynamic behavior of in situ partially supported concrete railway sleeper", Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit, 225(5), pp. 501–508, 2011.
https://doi.org/10.1177/2041301710392492

[11] Zhu, J. Y., Thompson, D. J., Jones, C. J. C. "On the effect of unsupported sleepers on the dynamic behavior of a railway track", Vehicle System Dynamics, 49(9), pp. 1389–1408, 2011.
https://doi.org/10.1080/00423114.2010.524303

[12] Zoller, V., Zobory, I. "On dynamics of the track/vehicle system in presence of inhomogeneous rail supporting parameters", Periodica Polytechnica Transportation Engineering, 39(2), pp. 83–85, 2011.
https://doi.org/10.3311/pp.tr.2011-2.06

[13] Zoller, V., Zobory, I. "Railway track dynamics with periodically varying stiffness and damping in the Winkler foundation", Periodica Polytechnica Transportation Engineering, 39(2), pp. 87–90, 2011.
https://doi.org/10.3311/pp.tr.2011-2.07

[14] Zakeri, J. A., Fattahi, M., Ghanimoghadam, M. M. "Influence of unsupported and partially supported sleepers on dynamic responses of train–track interaction", Journal of Mechanical Science and Technology, 29, pp. 2289–2295, 2015.
https://doi.org/10.1007/s12206-015-0521-3

[15] Mosayebi, S. A., Zakeri, J. A., Esmaeili, M. "Some Aspects of Support Stiffness Effects on Dynamic Ballasted Railway Tracks", Periodica Polytechnica Civil Engineering, (60)3, pp. 427–436, 2016.
https://doi.org/10.3311/PPci.7933

[16] Sadeghi, J., Zakeri, J.-A., Tolou Kian, A. R. "Effect of unsupported sleepers on rail track dynamic behavior", Proceedings of the Institution of Civil Engineers and Transport, 171(5), pp. 286–298, 2018.
https://doi.org/10.1680/jtran.16.00161

[17] Dai, J., Ang, K. K., Jiang, D. "Moving element analysis of high-speed rail system accounting for hanging sleepers", presented at MATEC Web Conference, International Conference on Engineering Vibration (ICoEV 2017), Sofia, Bulgaria, Sept. 4–7, 2017. https://doi.org/10.1051/matecconf/201814805007

[18] Mosayebi, S.-A., Zakeri, J. A., Esmaeili, M. "Effects of train bogie patterns on the mechanical performance of ballasted railway tracks with unsupported sleepers", Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit, 232(1), pp. 238–248, 2016. https://doi.org/10.1177/0954409716664932

[19] Krenk, S. "Non-linear Modeling and Analysis of Solids and Structures", Cambridge University Press, Cambridge, UK, 2009. https://doi.org/10.1017/CBO9780511812163

[20] Kardestuncer, H. "Elementary matrix analysis of structures", McGraw-Hill Companies, New York, NY, USA, 1974.

[21] Zakeri, J. A. "Computer simulation for dynamics of railway track structures", PhD Thesis, Northern Jiatong University, 2000.

[22] Sun, Y. Q., Dhanasekar, M. "A dynamic model for the vertical interaction of the rail track and wagon system", International Journal of Solids and Structures, 39(5), pp. 1337–1359, 2002. https://doi.org/10.1016/S0020-7683(01)00224-4

[23] Fattahi, M. "Dynamic Analysis of Track and Train with Unsupported Sleepers", MSc Thesis, School of Railway Engineering, Iran University of Science and Technology, 2014.