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Uniaxial Compression Fractal Damage Constitutive Model of Rock Subjected to Freezing and Thawing

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Abstract

Mechanical properties of the rock in the cold regions are often affected by freeze-thaw cycles and loads. It is of great theoretical significance and engineering value to establish a uniaxial compression damage constitutive model of the rock under freeze-thaw cycles that can reflect the relationship between macroscopic and mesoscopic structural damage. In this paper, macroscopic and mesoscopic methods are combined with statistical methods to quantitatively analyze the damage degree of rock under freeze-thaw cycles and loads. Combined with the fractal features of the macroscopic image of the section, a fractal damage constitutive model considering the residual strength of rock is established. In addition, the model is subsequently verified by the experiment. The experiment shows that the mechanical properties of rocks subjected to freeze-thaw cycles and loads are determined by freeze-thaw damage variables, load damage variables, and their coupling effects. As the number of freeze-thaw cycles increases, the uniaxial compressive strength and elastic modulus of rocks decrease, and peak strain increases. By using the fractal dimension of the compression fracture surface as a bridge considering the residual strength of the rock, the constitutive model can better reflect the compaction stage, elastic deformation stage and plastic deformation stage of the uniaxial compression process of the freeze-thaw rocks.

Keywords

freeze-thaw cycles, residual strength, uniaxial compression, fractal, damage constitutive model

1 Introduction

Rock is a kind of heterogeneous material with many initial defects. In the water-enriched cold environment, this initial defect is easily affected by the loss effect induced by freeze-thaw cycles and loads, causing changes in the mechanical properties and failure laws of rocks. The damage change, the mechanical behavior and deformation characteristics of the rock under different freeze-thaw cycles have certain differences [1–4]. The research about constitutive model can accurately reflect the stress-strain relationship of the rock under the effect of freeze-thaw cycles, which can provide a theoretical reference for the analysis and prediction of the rock material strength, deformation and failure characteristics under the freezing and thawing environment. There is no doubt that it is of great theoretical significance and engineering value to construction in cold regions.

The study of physical and mechanical properties of frozen rock is an important part of rock mechanics under freezing and thawing conditions. It is also the base of multi-field coupling, phase transition process and other mechanisms research. Many scholars have achieved fruitful results in the basic physical and mechanical parameters of frozen rock [5–8]. With the application of damage mechanics, the exploration of the damage evolution characteristics of rock materials is developing rapidly. Damage variables are chosen to link irreversible structural changes of rock materials and their macroscopic response characteristics. The damage variable selected in the damage mechanics should not only be able to visually reflect the deterioration characteristics and deterioration degree of the material, but also be easily measured and analyzed in practical engineering applications. Therefore, Zhang and Yang [9], Liu et al. [10], Yang e al. [11], Lemaitre [12], Zhang et al. [13], Farhidzadeh et al. [14] described the damage of materials by using parameters such as elastic modulus, wave velocity, density, strain, CT number and fractal dimension. Chen et al. [15], Zhou et al. [16], Al-Omari et al. [17] and others conducted freeze-thaw

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experiments on different rocks and explored the effects of different freezing and thawing conditions on rock damage. Park et al. [18],Yang et al. [19], Gao et al. [20], and Zhou et al. [21] used CT scanning technology, nuclear magnetic resonance technology, scanning electron microscope and other detection methods to study the degradation mechanism of different rocks under freezing and thawing conditions, and analyzed the freeze-thaw failure mechanism of the rock. Huang et al. [22], Eslami et al. [23] studied the damage characteristics of rock under freezing and thawing cycles and different stress fields and obtained the evolution law of total damage of frozen-thawed rock. Moreover, a rock damage constitutive model under the coupling of freeze-thaw cycles and loads was established, which was based on the meso-damage theory and the macroscopic statistical damage theory. Liu et al. [24], Fang et al. [25] proposed a solution method for model parameters under different freeze-thaw cycles and established a damage constitutive model that can predict the stress-strain relationship of rocks under different freeze-thaw cycles. Although these damage constitutive models can well describe the main deformation characteristics of rock, they are all based on the damage theory of metal materials. It is believed that the root cause of rock damage is the constant creation of new voids inside the rock, and these voids do not have any bearing capacity. The rock no longer has any carrying capacity after complete damage. However, after the rock material is completely damaged, there is still a certain bearing capacity [26]. The method of determining the strength of the micro-element in the existing model cannot reflect the characteristics of the partial bearing capacity of the rock micro-elements in the rock after the damage. The existing models mostly use macroscopic parameters to describe the damage, and the relationship between the mesoscopic parameters and the macroscopic mechanical properties is not established. In addition, the form of damage at the mesoscopic level cannot be explored in essence. There are still some limitations and shortcomings.

In this paper, the characteristics of a certain bearing capacity after the damage of micro-members inside the rock are considered. According to the damage mechanics and fractal theory, the meso-damage characteristics of the rock are related to the macro-mechanical characteristics, and the uniaxial compression damages constitutive model considering the residual strength characteristics of the rock are established. The rationality of the uniaxial compression fractal damage constitutive model of frozen-thawed rock is verified by uniaxial compression test. Research results provide a good theoretical basis for the study of the mechanical properties and deformation characteristic of the rock under the freezing and thawing cycles.

2 Uniaxial compression fractal damage constitutive model for rock

2.1 Evolution equation of rock damage based on Weibull statistical distribution

Under external load, the mesoscopic defects inside the rock continue to expand and accumulate, the strength of the rock decreases, the stiffness deteriorates, and the rock continues to damage until failure. Therefore, we can consider that irreversible changes in the mesostructure cause damage to the rock. The distribution of mesoscopic defects inside the rock and the evolution of the damage are random. It is believed that the rock is composed of micro-elements from the mesoscopic perspective. The micro-element is the decisive factor of rock deformation and failure, the mesoscopic parameters are controlled by the failure of micro-elements. Macroscopic mechanical parameters are controlled by the mesoscopic parameters. In other words, macroscopic non-linear deformation of rock is formed by the continuous failure of micro-elements. If the rock is discrete into a myriad of micro-elements, the mechanical properties of each micro-element affect the macro-mechanical properties of the rock. Furthermore, micro-element damage is probabilistic. The number of destroyed micro-elements can be quantified by the theory of statistical damage mechanics. The degree of damage inside the rock can be characterized by the micro-element damage cumulative effect.

Assuming, that the strength of the rock micro-element obeys the Weibull statistical distribution, the damage probability density function of the micro- element is:

$$f(F)=\frac{\beta}{\eta}\left(\frac{F}{\eta}\right)^{\beta-1}\exp\left[-\left(\frac{F}{\eta}\right)^{\beta}\right], \tag{1}$$

where F is the micro-element strength variable; β and η are Weibull statistical distribution parameters.

The damage variable D_p is defined as the ratio of the number of micro-elements $n(F)$ that have been destroyed in the rock to the number N of total micro-elements at present [26]. The damage variable D_p is the macroscopic expression of the cumulative effect of micro-element damage. D_p can be written as:

$$D_p=\frac{n(F)}{N}. \tag{2}$$

Assuming, that the micro-element strength obeys the maximum tensile strain strength criterion, it can be expressed as:

$$F = F(\varepsilon) = \varepsilon, \tag{3}$$

where ε is the maximum strain.

During the continuous action of the load, as the strain level increases, the number of damaged micro-elements increases. Within any strain level interval $[\varepsilon, \varepsilon + d\varepsilon]$, the number of broken micro-elements is $Nf(\varepsilon)d\varepsilon$. When loaded to a certain strain level ε, the number of destroyed micro-elements can be computed as:

$$n(\varepsilon) = \int_0^\varepsilon Nf(x)dx = N\left[1 - \exp\left(-\left(\frac{\varepsilon}{\eta}\right)^\beta\right)\right]. \tag{4}$$

Substituting the Eq. (4) into the Eq. (2), the equation of rock damage evolution under axial load can be obtained as follows:

$$D_p = \frac{n(\varepsilon)}{N} = 1 - \exp\left[-\left(\frac{\varepsilon}{\eta}\right)^\beta\right]. \tag{5}$$

2.2 Establishment of constitutive model of rock uniaxial compression fractal damage

When the rock is subjected to load, the structural changes caused by the mesoscopic defect expansion inside the rock have affected the macroscopic mechanical properties of the rock before its bearing capacity is lost. According to the strain equivalence principle [12], the damage constitutive relation of rock can be expressed as:

$$\sigma = \sigma'(1 - D) = E(1 - D)\varepsilon, \tag{6}$$

where σ is the nominal stress; σ' is the effective stress; E is the elastic modulus of the non-destructive rock; D is the damage variable and ε is the strain.

Since most of the rock samples in reality cannot be said to be completely dense and lossless, the elastic modulus of the non-destructive rock is difficult to obtain. Zhang et al. [13] generalized strain equivalence principle, considering the natural initial state of the rock as the first damage state, and the other state in which the positive or negative damage occurred as the second damage state. On the basis of the generalized strain equivalence principles, the damage constitutive relationship in the initial state can be written as:

$$\sigma = E_0(1 - D)\varepsilon, \tag{7}$$

where E_0 is the elastic modulus of rock in the natural initial state; D is the damage variable also based on the natural initial state. It should be noted that E in Eq. (6) is the elastic modulus of the non-destructive rock, and D is the damage variable in the non-destructive state.

It can be seen from Eq. (7) that the micro-elements which make up the rock cannot propagate stress after failure. Because the effective stress originally defined in the damage mechanics is introduced to study the damage of the tensile metal material, which assumes that the stress cannot be transferred through the injury site. In fact, during the compression process of the rock, part of the compressive and shear stresses can still be transmitted after the micro-element is destroyed. The rock damage constitutive model based on the theory of continuous damage mechanics does not take this feature into account. Therefore, the residual strength characteristics of the rock cannot be reflected in the stress-strain curve.

Yang et al. [26] and Xu and Wei [27] considered that the effects that damaged cross section transfers the partial compressive stress and shear stress are consistent. The introduction of the coefficient δ in the effective stress expression under compression can make the statistical constitutive model more reasonable. The damage constitutive model can better conform to the rock deformation and failure law, and can more reflect the stress-strain variation characteristics of the rock, which can be written as:

$$\sigma = E_0(1 - \delta D)\varepsilon, \tag{8}$$

where δ is the coefficient that varies between [0, 1].

The macroscopic failure characteristics of rocks have a close relationship with the degree of damage inside them, which is related to the structural changes caused by the development and aggregation of internal micro-fractures. Liu et al. [28] found that the damage zone of rock evolved in a self-similar manner. The characteristics and behavior of fractal damage can be found from the expansion and aggregation of micro-fractures to the evolution of material damage. On the basis of this, the self-similar fractal idea can be used to analyze the damage mechanics of the material. The fractal dimension of the rock fracture surface is homogenized, and the coefficient δ in the Eq. (8) is replaced by the value f after the treatment of fractal dimension. Let δ be equal to *f*, which is also equal to the quotient of the fractal dimension of main rock fracture in each rock divided by the maximum fractal dimension of main rock fractures in all rocks, including:

$$\sigma = E_0(1 - fD)\varepsilon. \tag{9}$$

Substituting the Eq. (5) into the Eq. (9), we can gain:

$$\begin{aligned}\sigma &= E_0\left\{1 - f + f\exp\left[-\left(\frac{\varepsilon}{\eta}\right)^{\beta}\right]\right\}\varepsilon \\ &= E_0\varepsilon - fE_0\varepsilon + fE_0\varepsilon\exp\left[-\left(\frac{\varepsilon}{\eta}\right)^{\beta}\right].\end{aligned} \tag{10}$$

The above formula is a uniaxial compression fractal damage constitutive model established by the Weibull statistical distribution combined with the fractal dimension of the main rock fracture.

3 Study on uniaxial compression fractal damage constitutive model of frozen-thawed rock

3.1 Establishment of a constitutive model for fractal damage of frozen-thawed rock under uniaxial compression

After undergoing multiple freeze-thaw cycles, damage occurred inside the rock and evolves randomly, which is macroscopically characterized by deterioration of rock mechanical properties. Therefore, the macroscopic mechanical response characteristics of the rock after freezing and thawing can be used to reflect the degree of internal damage of the rock. Considering that the definition of rock damage variable needs to be easily measured and analyzed under actual conditions, this paper selects the elastic modulus which is easy to measure during the test as the metric for measuring the internal damage of rock. The freeze-thaw damage variable of rock (D_n) (subscript n is the number of freeze-thaw cycles) can be expressed as:

$$D_n = 1 - \frac{E_n}{E_0}, \tag{11}$$

Where E_0 is the elastic modulus of rock before freezing and thawing, that is the initial elastic modulus under the natural state; E_n is the elastic modulus of rock after n cycles of freezing and thawing. Since the freeze-thaw cycle conditions remain unchanged during the test, the elastic modulus of the rock sample after freeze-thaw cycles is related to the number of freeze-thaw cycles, in other words, the freeze-thaw damage of the rock sample is related to the number of freeze-thaw cycles.

After a certain number of freeze-thaw cycles, the micro-fractures expand and produce. Under the load, the original micro-fractures and meso-defects in the rock will expand and connect. Freeze-thaw cycles and loads promote the expansion and connection of the initial damage with different mechanical mechanisms. Therefore, we still use the strain equivalence principle proposed by Zhang et al. [13] to treat the damage state of the rock after freezing and thawing as the first damage state, and the damage state of the frozen-thawed rock after the process of loading is regarded as the second damage state. The total damage effect can be measured through the total damage variable in the two-stage loading state deduced by Zhang et al. [13], including:

$$D_m = D_n + D_p - D_nD_p, \tag{12}$$

where D_m is the total damage variable of rock under freeze-thaw cycles and axial loads.

It can be seen from Eq. (12) that the total damage variable of rock under freeze-thaw cycles and axial loads is not simply superimposed by freeze-thaw damage and load damage, indicating the nonlinear characteristics of the two damage interactions. Under the axial compression load, the misalignment between grains inside the rock is reduced, and the friction is enhanced, which reduces the total damage and makes it appear as "$-D_nD_p$" in the damage variable.

Substituting the Eq. (5) and Eq. (11) into the Eq. (12), the total damage evolution equation of the uniaxially compressed rock after freeze-thaw cycles is:

$$D_m = 1 - \frac{E_n}{E_0}\exp\left[-\left(\frac{\varepsilon}{\eta}\right)^{\beta}\right]. \tag{13}$$

It can be seen from the discussion in this section that the damage section can still transmit part of the compressive stress. Therefore, substituting the Eq. (13) into the Eq. (9), the uniaxial compression fractal damage constitutive model of the rock subjected to freeze-thaw cycles can be obtained:

$$\sigma = E_0(1 - fD_m)\varepsilon = E_0\varepsilon - fE_0\varepsilon + fE_n\varepsilon\exp\left[-\left(\frac{\varepsilon}{\eta}\right)^{\beta}\right]. \tag{14}$$

It can be acknowledged from Eq. (14) that when only the damage caused by freeze-thaw cycles is considered, the strain level $\varepsilon = 0$, there is $D_m = D_n$; when only the damage caused by the load is considered, $E_n = E_0$, there is $D_m = D_p$.

Equation (14) shows that the stress-strain relationship of uniaxially compressed rock under freezing and thawing is related to elastic modulus, fractal dimension of fracture surface and statistical distribution parameters η and β. The distribution parameters η and β can be obtained from the mechanical characteristic parameters of frozen-thawed rocks through. Moreover, they play a decisive role in the nonlinear behavior of rock deformation.

3.2 Method for determining model parameters

There are two main methods for solving the model parameters η and β, including linear regression method and peak point method. The linear regression method calculates the model parameters by linearly processing the damage statistical constitutive equation and taking the data points on the full stress-strain curve of the rock uniaxial compression. The linear regression method is fully considering the full stress-strain curve of rock uniaxial compression. Still, it may simulate distortion of the characteristic points of the test curve, such as the peak point of the curve which represents the uniaxial compressive strength of the rock. The peak point method is based on the boundary conditions of the stress-strain curve at the peak point to solve the model parameters. The method is taking the highest point of the stress-strain curve into consideration, and it is clearer in physical meaning and easy to solve. Therefore, we choose the peak point method to solve the model parameters.

It can be known from the uniaxial compression stress-strain curve of sandstone that the curve should meet the following two boundary conditions at the peak point:

1. The slope of the curve at the peak point is 0.
2. The stress-strain relationship at the peak point should satisfy the established damage statistical constitutive equation.

It can be expressed as:

$$\frac{d\sigma}{d\varepsilon}\Big|_{\varepsilon=\varepsilon_c} = (1-f)E_0 + [1-\beta(\frac{\varepsilon_c}{\eta})^\beta]\times fE_n \exp\left[-(\frac{\varepsilon_c}{\eta})^\beta\right] = 0, \quad (15)$$

$$\sigma_c = (1-f)E_0\varepsilon_c + fE_n\varepsilon_c \exp\left[-(\frac{\varepsilon_c}{\eta})^\beta\right]. \quad (16)$$

The model parameters can be obtained by simultaneous Eqs. (15) and (16):

$$\eta = \frac{\varepsilon_c}{\left[\frac{1}{\beta}\frac{\sigma_c}{\sigma_c-(1-f)E_0\varepsilon_c}\right]^{\frac{1}{\beta}}}, \quad (17)$$

$$\beta = -\frac{\sigma_c}{\left[\sigma_c-(1-f)E_0\varepsilon_c\right]\ln\left[\frac{1}{f}\left(\frac{\sigma_c}{E_n\varepsilon_c}-(1-f)\frac{E_0}{E_n}\right)\right]}. \quad (18)$$

The analytical expressions about the model parameters η and β, the elastic moduli E_0 and E_n, the uniaxial compressive strength σ_c, the peak strains ε_c and fractal dimension coefficient f, are derived by Eq. (17) and Eq. (18). Among them, the model parameter β reflects the coupling relationship between plasticity and damage during rock failure.

4 Verification of uniaxial compression fractal damage constitutive model for freezing and thawing rocks

4.1 Obtainment of rock mechanical parameters

In order to verify the rationality of the uniaxial compression fractal damage constitutive model, the statistical parameters of the constitutive model need to be determined. In this paper, we consider that the rock is subjected to two-stage loads of freeze-thaw cycle and axial load. Firstly, the rock should be subjected to the freeze-thaw cycle test, and then the uniaxial compression test is carried out to obtain statistical parameters.

The rock used in the test is the Permian red sandstone from Wuhan, Hubei Province. It is mainly composed of quartz, feldspar, and calcite, and it also contains a small amount of hematite and mica. The density of red sandstone is about 2.3 g/cm^3. The specimens are standard cylinders with a diameter of 50 mm and a height of 100 mm. In order to ensure the uniformity of the rock sample, the rock samples with similar wave velocities are selected as the test rock samples after the wave velocity test. A total of 21 test samples whose wave velocities are about 3200 m/s were selected for this test, which can be divided into 7 groups of 3 pieces each (Fig. 1).

In this experiment, the rapid freezing and thawing model was used. The freeze-thaw cycle temperature of this test was set to - 20 °C~20 °C. The saturated rock samples were frozen in the high-low temperature alternating heat and humidity test chamber of the Rock Mechanics Laboratory in China University of Geosciences with the

Fig. 1 Standard rock sample after screening and processing

freezing rate 2 °C/min, and then melted in constant temperature water. Both the freezing time and the melting time were controlled to 4 hours and a complete freeze-thaw cycle was 8 hours. (Fig. 2) The number of freeze-thaw cycles was 0, 5, 10, 15, 20, 25, 30 times, respectively. The uniaxial compression test of rock samples that had undergone 0, 5, 10, 15, 20, 25, and 30 freeze-thaw cycles was carried out using the RMT-150C rock and concrete mechanics test machine of the Institute of Rock and Soil Mechanics of the Chinese Academy of Sciences. The axial displacement was used as the control amount, and the mechanical parameters were obtained at a loading rate of 0.005 mm/s. In order to better verify the rationality of the constitutive model in the following sections, Table 1 shows the mechanical parameters of the samples whose elastic modulus is closest to the average value of the elastic modulus in each group of samples.

It can be seen from Table 1 that as the number of freeze-thaw cycles increases, the uniaxial compressive strength and elastic modulus of the rock decrease, and the peak strain increases. The freeze-thaw cycle promotes the mechanical properties degradation of the rock in cold regions.

4.2 Calculation of rock main fracture surface fractal dimension

In order to obtain the parameter *f* related to the fractal dimension in the constitutive model, it is necessary to describe the fracture characteristics of the frozen-thawed sandstone by fractal dimension, thus quantitatively characterizing the complexity of the fracture morphology. The pixel-covering method based on the grayscale image is a widely used method for measuring the fractal dimension of fracture, so it was chosen to evaluate the complexity of fracture morphology.

The SLR camera was used to observe the splitting fracture surface of the initial rock sample and the main failure surface of the rock sample after freezing and thawing. The observation model is shown in Fig. 3. A fixed shooting mode was set to avoid the strong light reflection and the frame error problems during the observation process. The main fracture surface images of each sandstone sample were obtained, as is shown in Table 2. After the observation, the images were processed. In order to reduce the influence of the observed noise on the observed image details, the images of the main failure surface were preprocessed by wavelet transform and Fourier transform. Then the average value of RGB in the picture is calculated as the grayscale value, and the true-color image is converted into the grayscale image. And the Otsu method was chosen to binarize the grayscale images. Firstly, the grayscale image is divided into the foreground area and the background area by setting a gray value. Then the inter-class variance of these two areas is calculated. When the two areas are divided under a certain gray value, the inter-class variance value is the largest, we used this gray value to convert a gray image into a binary image. Then we convert the binary image into the corresponding matrix. In the corresponding matrix, the value 1 represents white pixels and the value 0 represents black pixels, as is shown in Fig. 4. Subsequently, the box-counting dimension algorithm was used to calculate the binary image matrices, and the fractal dimension value of the main fracture surface

Table 1 Mechanical test results and fractal dimensions of samples subjected to different freeze-thaw cycles

Number of cycles (*n*)	E_n GPa	σ_c MPa	ε_c 10^{-3}	*Ds* *Ds*(max) = 2.2195
0	15.702	65.918	4.946	2.21920
5	13.599	56.909	5.086	2.21901
10	10.562	51.705	6.275	2.21872
15	9.953	44.835	5.852	2.21774
20	8.864	42.458	6.088	2.21856
25	8.543	41.781	6.255	2.21807
30	7.912	40.708	6.777	2.21563

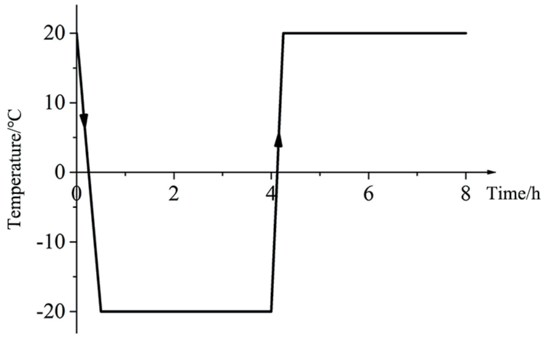


Fig. 2 Generalized temperature curve for each freeze-thaw cycle

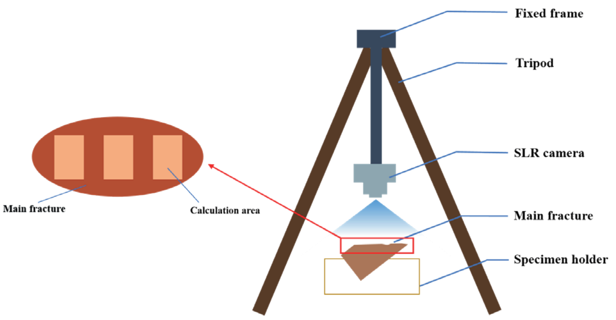
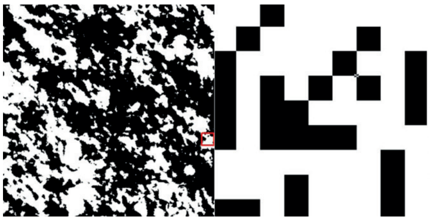


Fig. 3 observation model schematic diagram of the main fracture surface



$$\begin{matrix} 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \end{matrix}$$

Fig. 4 Binary images and corresponding matrices

under different freeze-thaw cycles can be obtained. The main steps of the algorithm are shown in Fig. 5. Noted that only one rock sample was selected to calculate the fractal dimension of the sandstone compressive section under the same freeze-thaw cycles (Table 1), to obtain the parameters required for the subsequent stress-strain curve comparison.

4.3 Comparative verification of stress-strain curves

According to the mechanical test results and the fractal dimension observations of sandstone samples in Table 1, the definition of fractal dimension and Eqs. (14), (17) and (18) can be used to obtain uniaxial stress-strain theoretical curves for sandstones with different freeze-thaw cycles. The comparison between the theoretical curve obtained from the uniaxial compression fractal damage constitutive model of frozen-thawed rock and the experimental curve is shown in Fig. 6. The initial elastic modulus E_0 of the sandstone is the average of the elastic modulus of the three rock samples

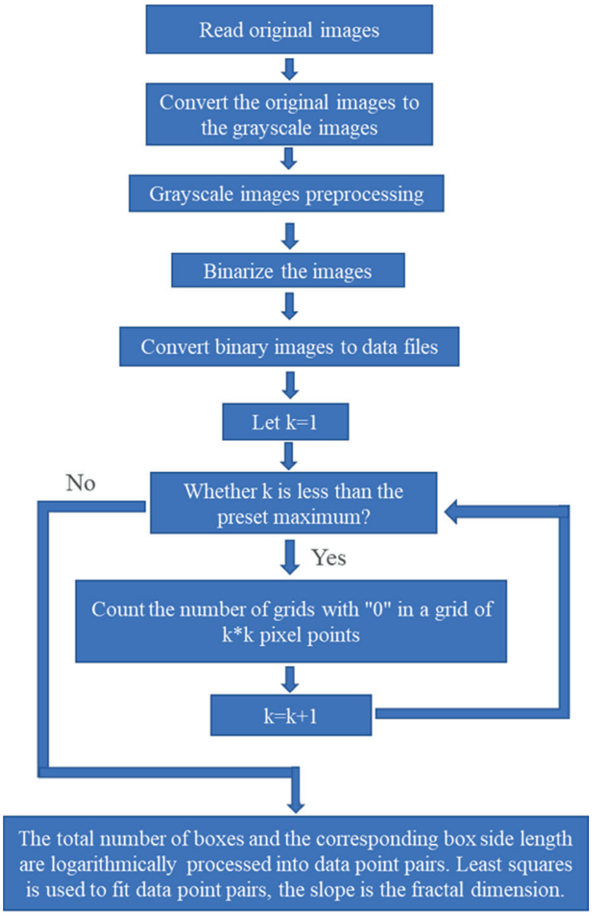
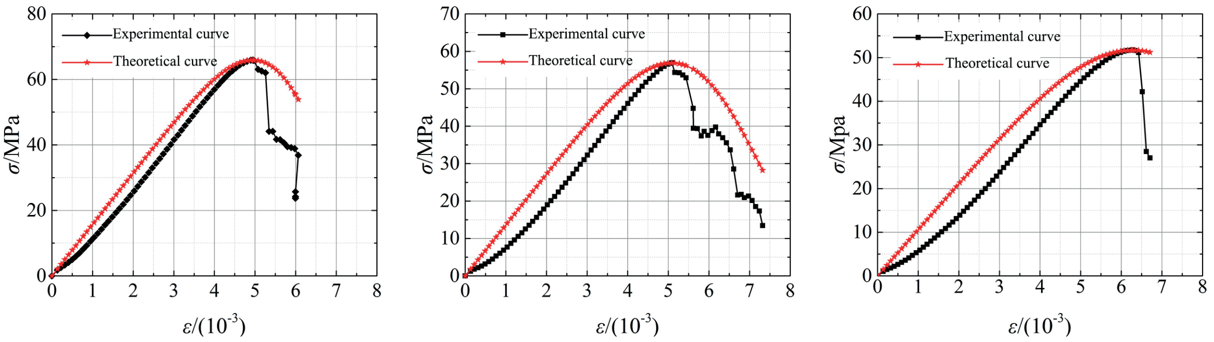


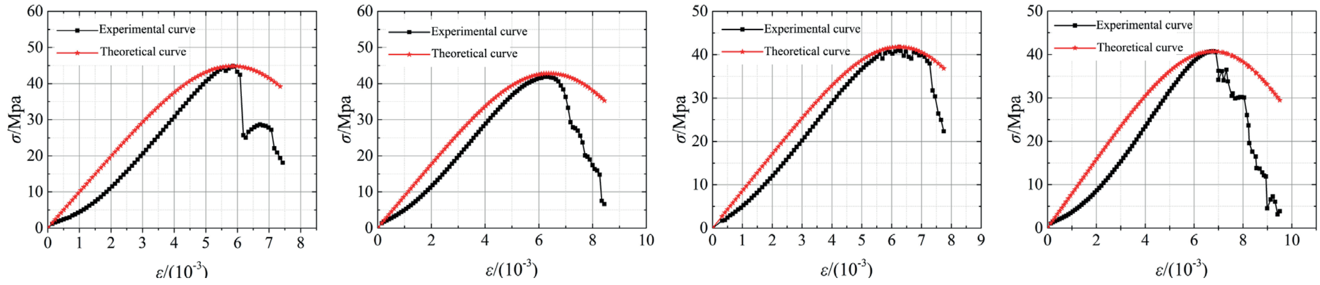
Fig. 5 Algorithm flow chart of pixel-covering method



(a) 0 freeze-thaw cycle

(b) 5 freeze-thaw cycles

(c) 10 freeze-thaw cycles



(d) 15 freeze-thaw cycles

(e) 20 freeze-thaw cycles

(f) 25 freeze-thaw cycles

(g) 30 freeze-thaw cycles

Fig. 6 Stress-strain test curves and model curves of sandstone under different freeze-thaw cycles

Table 2 Images of main fracture surface of rock under different freeze-thaw cycles

(a) 0 cycle	 (a1)	 (a2)	 (a3)
(b) 5 cycles	 (b1)	 (b2)	 (b3)
(c) 10 cycles	 (c1)	 (c2)	 (c3)
(d) 15 cycles	 (d1)	 (d2)	 (d3)
(e) 20 cycles	 (e1)	 (e2)	 (e3)
(f) 25 cycles	 (f1)	 (f2)	 (f3)
(g) 30 cycles	 (g1)	 (g2)	 (g3)

which have never undergone freeze-thaw cycles, the value of E_0 is 16.582 GPa. The experimental data of freeze-thaw rock samples used for model verification are taken as specific values. In addition, in order to make the image clearer, the stress-strain curve of one rock sample in each freeze-thaw group is selected for comparison verification.

Comparing the sandstone stress-strain test curves and model curves of different freeze-thaw cycles in Fig.6, the following conclusions can be drawn:

1. At the initial stage of applying the load, the theoretical model curve of the unfrozen rock sample ($n = 0$) has a high degree of coincidence with the test curve, which can better describe the nonlinear deformation characteristics of the initial compaction stage of the rock. As the number of freeze-thaw cycles increases, the theoretical curve deviates to the outside of the test curve, but the deviation is small.
2. In the elastic stage, the theoretical curve of the model approximates a straight line, which is consistent with the law. The elastic modulus of rock is a certain value, which remains consistent with the experimental curve.
3. For the plastic stage, the theoretical model curve can also be well simulated. The simulation results at the peak point are consistent with the experimental results, accurately reflecting the strength characteristics of the frozen-thawed rocks.
4. In the post-failure stage, the theoretical model curve shows the plastic characteristics of the rock, which indicates that the rock still has a certain bearing capacity after the failure. The test curve shows the characteristics of gradually decreasing brittleness and increasing plasticity. There is a certain difference between simulation results and test results, and the theoretical value of the residual strength is slightly higher than the experimental value.

In summary, the uniaxial compression fractal damage constitutive model can well simulate the stress-strain relationship of rock under different freeze-thaw cycles. The similarity between the model prediction curve and the experimental results is good, indicating the model is reasonable.

5 Conclusions

Based on the damage mechanics theory and Weibull statistical distribution function, considering the characteristics of partial bearing capacity after micro-elements destroyed, uniaxial compression fractal damage constitutive model for frozen-thawed sandstone is established. The rationality of the model is verified based on the test results. The following notes are drawn:

1. On the basis of the combination of macroscopic and mesoscopic methods and statistical methods, the cumulative effect of micro-elements damage and the degree of elastic modulus degradation are used as damage metrics for sandstone under uniaxial compression and freeze-thaw cycles. The damage evolution equation contains strain parameters and Weibull function parameters. The freeze-thaw damage evolution equation only contains sandstone elastic modulus. The parameters mentioned above can be solved by easily measurable test parameters.
2. According to the generalized strain equivalence principle and considering the characteristics of partial bearing capacity after micro-elements destroyed, the constitutive model of uniaxial compression fractal damage of frozen-thawed sandstone can be established. The constitutive model we established reflects the characteristics of damage evolution for the rock under freezing and thawing cycles and under load conditions. The fractal characteristics of compressive fracture surface are used as bridges to link the macroscopic failure characteristics of sandstone with the meso-damage failure mechanism. The parameters in this constitutive model are easy to solve and have strong applicability.
3. Through comparison with the experiment results, the theoretical stress-strain curve of rock under different freeze-thaw cycles described by the model is in good agreement with the experimental curve. The theoretical curve can better reflect the compaction stage, elastic deformation stage and plastic deformation stage of the uniaxial compression process for frozen-thawed rock. The simulation results of the theoretical curve at the peak point under different freeze-thaw cycles are consistent with the experimental results, accurately reflecting the strength degradation law of frozen-thawed rock.

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