

Reliability Based Design Optimization of Reinforced Concrete Frames Using Genetic Algorithm

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Abstract

This paper introduces a new framework for reliability based design optimization (RBDO) of the reinforced concrete (RC) frames. This framework is constructed based on the genetic algorithm (GA) and finite element reliability analysis (FERA) to optimize the frame weight by selecting appropriate sections for structural elements under deterministic and probabilistic constraints. Modulus of elasticity of the concrete and steel bar, dead load, live load, and earthquake equivalent load are considered as random variables. Deterministic constraints include the code design requirements that must be satisfied for all the frame elements according to the nominal values of the aforementioned random variables. On the other hand, this framework provides the minimum required reliability index as the probabilistic constraint. The first-order reliability method (FORM) using the Newton-type recursive relationship will be used to compute the reliability index. The maximum inter-story drift is considered as an engineering demand parameter to define the limit-state function in FORM analysis. To implement the proposed framework, a mid-rise five-story RC frame is selected as an example. Based on the analysis results, increasing the minimum reliability index from 6 to 7 causes an 11 % increase in the weight of the selected RC frame as an objective function. So, we can obtain a trade-off between the optimized frame weight and the required reliability index utilizing the developed framework. Furthermore, the high values of the reliability index for the frame demonstrate the conservative nature of code requirements for interstory drift limitations based on the linear static analysis method.

Keywords

reinforced concrete frame, reliability based design optimization, first-order reliability method, genetic algorithm

1 Introduction

In the majority of structural optimization problems in reinforced concrete (RC) framed structures, it is mainly focused to optimize the objective function according to deterministic constraints. In this way, Kaveh and Sabzi [1] optimized the design of RC frames using a big bang-big crunch algorithm. Esfandiari et al. [2] optimized the seismic design of RC frames subjected to time-history loadings using an algorithm combining multi-criterion decision-making and particle swarm optimization. The considered deterministic constraints in these problems usually consist of code requirements such as lateral displacement limitation. In this procedure, uncertainties in the structural analysis will be ignored that can lead to unsafe design. These uncertainties in RC framed structures come from improper design and construction practices and other sources like time-dependent deterioration induced by corrosion [3]. To solve this problem and to design a safe structure, structural reliability

analysis should be performed during the optimization process so that various sources of uncertainties [4] in the design process can be properly accounted for. This approach is called reliability based design optimization (RBDO) that has been emerged as a new optimization procedure for various structural systems [5, 6].

Reliability analysis in RBDO can be performed by various methods. Gholizadeh et al. [7] proposed a neural network-based method for reliability assessment of optimally seismic designed moment frames. Grubišić et al. [8] conducted reliability analysis of the RC frame by finite element method considering implicit limit state functions.

In RBDO of RC framed structures, in addition to the deterministic constraints in the optimization practice, probabilistic constraints must also be examined in which failure probability or corresponding structural reliability index of the structure is determined. Consequently, it can be argued

that the frame is designed to meet a specific reliability index uniformly, assuring its performance over the structure service life under the involved uncertainties. In this way, Aoues and Chateaneuf [9] examined the RBDO of RC frames by adaptive target safety. Khatibinia et al. [10] introduced a discrete gravitational search algorithm and a meta-modelling framework for RBDO of RC structures including soil-structure interaction. Shayanfar et al. [11] estimated the corrosion occurrence in RC structures using reliability based particle swarm optimization. Léger et al. [12] performed an RBDO of RC structures with elastomeric isolators using adaptive sparse polynomial chaos expansion. Zou et al. [13] formulated a reliability-based performance design optimization for seismic retrofit of RC buildings with fiber-reinforced polymer composites.

Genetic algorithm (GA) is a very powerful optimization method used in structural optimization problems [14–16]. The design of RC frames has been optimized using a GA considering the deterministic constraints [17, 18]. Shahraki and Noorossana [19] demonstrated that multi-objective GA can be used to solve discrete RBDO problems. There is a limited practical application of GA in the structural RBDO problems. In this regard, Biabani Hamedani and Kalatjari [20] performed structural system reliability-based optimization of truss structures using a GA through the branch and bound method. Shayanfar et al. [21] developed a genetic algorithm based method for RBDO of structures with discrete and continuous design variables using OpenSees and TCL. However, no research has been conducted on the RBDO of the RC frames using a GA.

This paper presents a new framework for RBDO of the RC frames employing finite element reliability analysis (FERA) and genetic algorithm (GA) to optimize the frame weight incorporating the deterministic and probabilistic constraints. Deterministic constraints include the code design requirements while probabilistic one provides the minimum required reliability index. The first-order reliability method (FORM) using the Newton-type recursive relationship is used to compute the reliability index or corresponding probability of failure. The maximum inter-story drift is considered as an engineering demand parameter to define the limit-state function in FORM analysis. The linear static analysis is used to calculate the structural response during the optimization process. To evaluate the plastic performance of the frame, reliability based limit and shakedown analyses [22–24] can be utilized. However, this research is limited to the elastic performance of the framed structure.

The remainder of this paper is prepared as follows:

In Section 2 an RBDO problem of RC frames is defined. The GA technique is then presented in Section 3 for solving the RBDO of RC frames. An illustrated example is introduced in Section 4. Analyses results are presented in Section 5. The final section concludes the paper.

2 Definition of RBDO problem of RC frames

In the RBDO problem of the RC frames, the objective is to find the sections (\mathbf{S}) for the structural members such that the frame weight (W) is optimized considering the involved deterministic and probabilistic constraints. Selected cross sectional areas are the discrete design variables extracted from a prepared section list (\mathbf{PSL}) where all of the existing sections in this list have satisfied the preliminary geometrical and reinforcement detailing requirements. Therefore, RBDO problem of RC frame structures can be formulated as follows:

$$\begin{aligned}
 & \text{find } \mathbf{S} \\
 & \min W(\mathbf{S}) \\
 & \text{s.t.} \begin{cases} S_i \in \mathbf{PSL} & i = 1, 2, \dots, n \\ M_j^{\max+}(\mathbf{S}, \mathbf{Nom}_X) \leq M_j^{r+}(\mathbf{S}) & j = 1, 2, \dots, n_b \\ |M_j^{\max-}(\mathbf{S}, \mathbf{Nom}_X)| \leq |M_j^{r-}(\mathbf{S})| & j = 1, 2, \dots, n_b \\ N_k^{\max+}(\mathbf{S}, \mathbf{Nom}_X) \leq N_k^{r+}(\mathbf{S}) & k = 1, 2, \dots, n_c \\ |N_k^{\max-}(\mathbf{S}, \mathbf{Nom}_X)| \leq |N_k^{r-}(\mathbf{S})| & k = 1, 2, \dots, n_c \\ p(g(\mathbf{S}, \mathbf{X}) \leq 0) \leq p_f^t \quad \text{where } p_f^t = \Phi(-\beta^t) \end{cases} \quad (1)
 \end{aligned}$$

Where S_i is the selected section for i th structural element in frame; n is the number of structural elements in frame; $M_j^{\max+}$ and $M_j^{\max-}$ are the maximum moment at the midspan and each end of j th beam in frame, respectively; M_j^{r+} and M_j^{r-} are the moment strength at the midspan and each end of j th beam in frame, respectively; \mathbf{Nom}_X is the nominal value vector of random variables; n_b is the number of beams in frame; $N_k^{\max+}$ and $N_k^{\max-}$ are the maximum compressive and tensile axial force of k th column in frame, respectively; N_k^{r+} and N_k^{r-} are the compressive and tensile axial strength of k th column in frame, respectively; n_c is the number of columns in frame; $p(g(\mathbf{S}, \mathbf{X}) \leq 0)$ represents the failure probability of the frame; \mathbf{X} is the vector of random variables; $g(\mathbf{S}, \mathbf{X}) \leq 0$ is the limit state function; p_f^t is the allowable probability of failure of the frame; $\Phi(\cdot)$ is the standard cumulative function of the normal distribution and β^t is the minimum required reliability index of the frame. A detailed presentation of the applied approach to solve the RBDO problem reported above can be found in following sections.

3 Solving RBDO problem of RC frames using GA

In present study, the RBDO problem as formulated in Eq. (1) will be solved using the genetic algorithm. GA is a heuristic method that uses the natural selection concept in the optimization process. Due to the involving discrete decision variables and also implicit form of the constraints, evolutionary algorithms like GA is more suitable than gradient-based methods to solve the introduced RBDO problem. The aim is to find the sections for structural members of RC frame for optimizing its weight considering the deterministic and probabilistic constraints. Due to variety of available RC sections according to their dimensions, bar sizes, and bar arrangements; set of selected sections (design solution) for structural members during the optimization process can be semi-infinite. This selection process entails high computational efforts. To mitigate this problem, a prepared section list (*PSL*) is constructed that consists of finite number of sections for structural members of the RC frame. In a *PSL*, all sections satisfy the preliminary code requirements include the minimum and maximum dimensions of the section, maximum aspect ratio, rebar sizes, the minimum and maximum spacing of the bars, the cover thickness, the minimum and maximum steel bar area, and etc. An identification number is assigned to each section in *PSL*. To consider the construction practices, the beam and column groups in the specific lines are selected in the frame. Therefore, the RBDO problem is solved by selecting the specific identification numbers from a *PSL* for each beam and column groups in the frame to optimize its weight considering the constraints. So, the number of dimensions (n_d) of design solution in the GA will be equal to sum of the number of beam groups (n_{BG}) and column groups (n_{CG}). The main steps of GA in this research can be listed as follows:

- Step 1: Select the number of dimensions (n_d), number of population (n_p), number of crossovers (n_{cr}), number of mutations (n_{mu}), and number of iterations (n_{iter}).
- Step 2: Generate the initial population (individuals). The number of positions in each individual (chromosome) is equal to the number of dimensions of the problem (n_d).
- Step 3: Evaluate the fitness of generated individuals. To handle the constraints, the constrained optimization problem in Eq. (1) is transformed into the unconstrained problem using penalty function as Eq. (2) to penalize infeasible design solutions by increasing their total weight (W_{tot}) as objective function which decreases their fitness values.

$$W_{tot}(\mathbf{S}, \boldsymbol{\kappa}, \boldsymbol{\psi}) = W(\mathbf{S}) + \kappa_\beta \cdot \psi_\beta^2 + \sum_{j=1}^{n_b} \kappa_b \cdot \psi_{bj}^2 + \sum_{k=1}^{n_c} \kappa_c \cdot \psi_{ck}^2, \quad (2)$$

where $\boldsymbol{\kappa}$ is the vector of constant coefficients consists of κ_β , κ_b and κ_c for reliability index of the frame, beam moment and column axial force constraints, respectively; $\boldsymbol{\psi}$ is the vector of penalty terms consists of ψ_β , ψ_{bi} and ψ_{ck} for reliability index of the frame, j th beam moment and k th column axial force, respectively. Other involved parameters were introduced in Eq. (1). It must be noted that κ_β , κ_b and κ_c are selected large enough to penalize the infeasible design solutions. To compute ψ_{bi} and ψ_{ck} terms, refer to Section 3.1 for deterministic constraints and to compute ψ_β term refer to Section 3.2 for probabilistic constraint. Therefore, a fitness value of each individual will be inversely proportional to its total weight.

- Step 4: Perform the single-point crossover operation.
- Step 5: Perform the mutation operation.
- Step 6: Reproduce the next generation. Generated individuals in steps 2, 4 and 5 are mixed to construct the total population. These individuals compete with each other to survive in the next generation. To this end, the individuals are sorted corresponding to their fitness values.
- Step 7: Repeat the above mentioned steps until n_{iter} is achieved.

3.1 Deterministic constraints

The deterministic constraints in the RBDO problem as formulated in Eq. (1) are related to beam moments and column axial forces. These constraints were fixed in a way that the maximum moment at the midspan and each end of the j th beam ($M_j^{\max+}$, $M_j^{\max-}$) do not exceed the moment strength at the midspan and each end of the j th beam (M_j^{r+} , M_j^{r-}), respectively; and as well the maximum compressive and tensile axial forces of the k th column ($N_k^{\max+}$, $N_k^{\max-}$) do not exceed the compressive and tensile axial strength of the k th column (N_k^{r+} , N_k^{r-}), respectively. To compute the $M_j^{\max+}$, $M_j^{\max-}$, $N_k^{\max+}$ and $N_k^{\max-}$, the finite element (FE) analysis of the RC frame is conducted using OpenSees software under the code specified load combinations. On the other hand, to calculate the M_j^{r+} , M_j^{r-} , N_k^{r+} and N_k^{r-} , a simplified approach is used based on the provisions specified in Iranian design code for RC structures [25]. In this simplified approach, reinforcing bars is used only in two sides (top and bottom) of the RC section with symmetric layout. Furthermore, the specified yielding strength of tensile (f_y) and compressive (f_y') bars is assumed to be same. Also, effect of the compressive bars in reduction of the compression zone in RC section is neglected. In the adopted limit state design method, ultimate compressive strain of the concrete in extreme compressive fiber (ϵ_{cu}) is considered

equal to 0.0035. The selected RC sections for beams are designed to yield the tensile bars but not to compressive bars. Based on these assumptions, the absolute value of moment strength at the midspan (M^{r+}) and each end (M^{r-}) of rectangular RC beam will be same and can be calculated as follows:

$$|M^{r\pm}| = 0.85\phi_c f_c ab(d - \frac{a}{2}) + \phi_s A'_s f'_s (d - d'), \quad (3)$$

where ϕ_c and ϕ_s are the strength reduction factors for the concrete and steel, respectively; f_c is the specified compressive strength of the concrete; d and d' are the distance from the extreme compressive fiber to the tensile and compressive bars, respectively; b is the section width; A'_s is the total area of compressive reinforcement; the depth of the compressive stress block (a) can be derived by solving the following equation:

$$a^2 + \frac{\phi_s (700A'_s - A_s f_y)}{\phi_c (0.85 f_c b)} a - \frac{\phi_s (700\beta_1 A'_s d')}{\phi_c (0.85 f_c b)} = 0, \quad (4)$$

where A_s is the total area of tensile reinforcement; β_1 is equal to $0.97 - 0.0025 f_c$; and stress of the compressive bars (f'_s) is evaluated as $700(a - \beta_1 d')/a$. So, the penalty term ψ_{bj} for the j th beam in Eq. (2) can be computed as follows:

$$\psi_{bj} = \begin{cases} 0 & \text{if } \max(M_j^{\max+}, |M_j^{\max-}|) \leq |M_j^{r\pm}| \\ \frac{\max(M_j^{\max+}, |M_j^{\max-}|) - |M_j^{r\pm}|}{|M_j^{r\pm}|} & \text{otherwise} \end{cases}, \quad (5)$$

where $\max(M_j^{\max+}, |M_j^{\max-}|)$ denotes the maximum absolute value of maximum moment at the midspan and each end of the j th beam.

In the case of compressive axial force and moment interaction, if the axial load eccentricity (e) of the column is greater than the eccentricity in balanced condition (e_b), the compressive axial strength of the column (N^{r+}) can be calculated as follows (tension-controlled region):

$$N^{r+} = 0.85\phi_c f_c bd \left[\left(1 - \frac{e'}{d}\right) + \sqrt{\left(1 - \frac{e'}{d}\right)^2 + 2\rho m \left(1 - \frac{d'}{d}\right)} \right] \quad (6)$$

where ρ is the ratio of tension reinforcement equals to A_s/bd ; e' is equal to $e + d - h/2$; e is equal to M/P ; M and P are the applied moment and axial load on the column under the selected load combinations; e_b is equal to $(0.2 + 0.77\rho_g m)h$; ρ_g is the ratio of total reinforcement in section; m is equal to $\phi_s f_y / (0.85\phi_c f_c)$ and h is the section height. It should be noted that compressive force in concrete is approximately assumed as $0.85\phi_c f_c ab$.

Furthermore, if e is less than e_b , the compressive axial strength of the column can be calculated as follows based on the Whitney simplified assumptions (compression-controlled region):

$$N^{r+} = \frac{\phi_c f_c bh}{\frac{2.87eh}{d^2} + 1.18} + \frac{\phi_s A_s f_y}{\frac{e}{d-d'} + 0.5}. \quad (7)$$

The obtained compressive axial strength of the columns based on the Eqs. (6) and (7) must be less than the maximum axial strength ($N_{r\max}$) according to [25]:

$$N_{r\max} = 0.85[\alpha_1 \phi_c f_c (A_g - A_{st}) + \phi_s f_y A_{st}], \quad (8)$$

in which $\alpha_1 = 0.85 - 0.0015 f_c$, A_g is the total area of the section and A_{st} is the total reinforcement area in a section.

In the case of tensile axial force and moment interaction, the tensile axial strength of the column (N^{r-}) can be calculated as follows:

$$|N^{r-}| = 1 / \left(\frac{1}{N_o} + \frac{e}{M_o} \right), \quad (9)$$

in which N_o is equal to $\phi_s A_{st} f_y$, and M_o is the pure bending moment in tension-controlled region. So, the penalty term ψ_{ck} for the k th column in Eq. (2) can be computed as Eqs. (10) and (11) in the case of compressive and tensile axial forces, respectively:

$$\psi_{ck} = \begin{cases} \frac{N_k^{\max+} - N_k^{r+}}{N_k^{r+}} & \text{if } N_k^{\max+} > N_k^{r+} \\ 0 & \text{if } N_k^{\max+} \leq N_k^{r+} \end{cases} \quad (10)$$

$$\psi_{ck} = \begin{cases} \frac{|N_k^{\max-}| - |N_k^{r-}|}{|N_k^{r-}|} & \text{if } |N_k^{\max-}| > |N_k^{r-}| \\ 0 & \text{if } |N_k^{\max-}| \leq |N_k^{r-}| \end{cases} \quad (11)$$

It must be noted that the mentioned procedure for calculating the axial strength of the columns has been validated by the P-M interaction diagram approach [26].

3.2 Probabilistic constraint

The probabilistic constraints in the RBDO problem as formulated in Eq. (1) are related to reliability index of the frame. This constraint was considered to provide the minimum allowable reliability index for the frame. To compute the reliability index of the RC frame or corresponding probability of the failure, the finite element reliability analysis (FERA) of the frame is done. In this research, the

FORM method 2 [27] will be utilized that Newton-type recursive formula is used in this method to find the design point without need to solve the limit state function.

The main steps of the FORM method 2 can be explained as follows:

- Step 1: Define the proper limit state function $g(\cdot)$.
- Step 2: Assume initial value of the design point \mathbf{X}^* .
- Step 3: Calculate the mean (μ_X^N) and standard deviation (σ_X^N) at the design point of the equivalent normal distribution for non-normal random variables. The coordinates of the design point in the equivalent standard normal space (\mathbf{X}'^*) are:

$$\mathbf{X}'^* = \frac{\mathbf{X}^* - \mu_X^N}{\sigma_X^N} \quad (12)$$

The FORM procedure can be used when all of the random variables are uncorrelated. If the random variables are correlated, these variables should be transformed into uncorrelated variables $\{\mathbf{Y}\}$. Because the joint PDF of basic random variables are not available in this study, Rosenblatt transformation [28] cannot be used. On the other hand, as similar to requirements of current study, when the joint PDF of basic random variables is unknown while the marginal PDFs and correlation matrix are known, the Nataf transformation [29] can be used to convert correlated variables to uncorrelated random variables. It has been concluded that the major obstacle for Nataf transformation is to evaluate the equivalent correlation matrix in standard normal space that should be computed by solving nonlinear integral equations which requires tedious calculations [27, 30]. Therefore, to avoid these complex calculations in the FORM analysis, a method proposed by Haldar and Mahedavan [27] has been used in this study to modify the original limit state function expressed in terms of correlated variables into a function of uncorrelated random variables. In this method, Eq. (13) is evaluated based on the obtained uncorrelated variables. This transformation can be performed according to following equation:

$$\{\mathbf{X}\} = [\sigma_X^N][\mathbf{T}]\{\mathbf{Y}\} + \{\mu_X^N\}, \quad (13)$$

where $\{\mathbf{X}\}$ is vector of the correlated random variables; $[\sigma_X^N]$ is the diagonal matrix containing the equivalent normal standard deviations; $\{\mu_X^N\}$ is vector of the equivalent normal means; σ_X^N and μ_X^N are calculated using two-parameter equivalent normal transformation suggested by Rackwitz and Fiessler [31]; $[\mathbf{T}]$ is an orthogonal

transformation matrix into uncorrelated reduced space including the eigenvectors of the correlation matrix $[\mathbf{C}']$ as Eqs. (14) and (15):

$$[\mathbf{C}'] = \begin{bmatrix} 1 & \rho_{X_1, X_2} & \dots & \rho_{X_1, X_n} \\ \rho_{X_2, X_1} & 1 & \dots & \rho_{X_2, X_n} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{X_n, X_1} & \rho_{X_n, X_2} & \dots & 1 \end{bmatrix}, \quad (14)$$

$$[\mathbf{T}] = \begin{bmatrix} \lambda_1^{(1)} & \lambda_1^{(2)} & \dots & \lambda_1^{(n)} \\ \lambda_2^{(1)} & \lambda_2^{(2)} & \dots & \lambda_2^{(n)} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_n^{(1)} & \lambda_n^{(2)} & \dots & \lambda_n^{(n)} \end{bmatrix}, \quad (15)$$

where ρ_{X_i, X_j} is the correlation coefficient of the X_i and X_j random variables; $\{\lambda^{(i)}\}$ is the eigenvector of the i th mode; $\lambda_1^{(i)}, \lambda_2^{(i)}, \dots, \lambda_n^{(i)}$ are the components of the i th eigenvector.

- Step 4: Calculate the partial derivative $\partial g / \partial \mathbf{X}$ at the design point \mathbf{X}^* . Finite difference method (FDM) is used to calculate the response sensitivities.

- Step 5: Calculate the partial derivative $\partial g / \partial \mathbf{X}'$ in the equivalent standard normal space as follows:

$$\frac{\partial g}{\partial \mathbf{X}'} = \frac{\partial g}{\partial \mathbf{X}} \frac{\partial \mathbf{X}}{\partial \mathbf{X}'} = \frac{\partial g}{\partial \mathbf{X}} \sigma_X^N \quad (16)$$

- Step 6: Calculate the new values for the design point in the equivalent standard normal space using the recursive formula as [32]:

$$\mathbf{X}'_{k+1} = \frac{1}{\|\nabla g(\mathbf{X}'_k)\|} [\nabla g(\mathbf{X}'_k)' \mathbf{X}'_k - g(\mathbf{X}'_k)] \nabla g(\mathbf{X}'_k), \quad (17)$$

where $\nabla g(\mathbf{X}'_k)$ is the gradient vector of the limit state function at \mathbf{X}'_k corresponding to k th iteration. Note that t represents the transpose of the vector.

- Step 7: Calculate the reliability index of the frame (β) as distance to the obtained design point from the origin:

$$\beta = \sqrt{\sum_{i=1}^n (X'_i)^2} \quad (18)$$

- Step 8: Calculate the new values for the design point in the original space as:

$$\mathbf{X}^* = \mu_X^N + \sigma_X^N \mathbf{X}'^* \quad (19)$$

Then, value of the limit state function is computed for this new design point. The aforementioned steps will be repeated until convergence criteria in steps (7) and (8) are satisfied. This procedure is developed in MATLAB

software calling OpenSees software to finite element analysis of the frame. So, the penalty term ψ_β for the frame in Eq. (2) can be computed as follows:

$$\psi_\beta = \begin{cases} \frac{\beta - \beta'}{\beta'} & \text{if } \beta < \beta' \\ 0 & \text{if } \beta \geq \beta' \end{cases} \quad (20)$$

The mentioned method to evaluate the probabilistic constraint in the RBDO problem is named reliability index approach (RIA). Performance measure approach (PMA) is identified as another method to examine the probabilistic constraint. The main difference between the RIA and the PMA is the type of optimization problem which is solved in each case. It was concluded that PMA is inherently robust for RBDO and is more efficient in evaluating the inactive probabilistic constraint. In contrast, RIA may diverged and yield singularity when uniform or Gumble random variables were employed, though it is more efficient in evaluating the violated probabilistic constraint [33–35]. One of the most important goals of this research is to monitor the deterministic and probabilistic constraints during the optimization process, and to determine which of these constraints is more critical in the RBDO problem. However, it is not possible to directly monitor the reliability index during the optimization process in PMA approach, because during iterations the reliability index is kept fixed at β' [33]. Therefore, the RIA method is used in this study. It will be discussed more about this issue later in Section 5.

4 An illustrated example

A five-story intermediate RC frame as illustrated in Fig. 1 is utilized to implement the proposed framework. The story height and bay width of the frame are 3 and 4 m, respectively. As presented in Fig. 1, four column group (C1, C2, C3 and C4) and three beam group (B1, B2 and B3) are used to consider the construction practices.

Therefore based on the mentioned definitions, n , n_b , n_c , n_{BG} , n_{CG} and n_d will be equal to 35, 15, 20, 3, 4 and 7, respectively. During the optimization process, the sections for structural elements are extracted from prepared section list (PSL) consists of finite number of prequalified sections. This list for beams (17 sections) and columns (37 sections) are listed in Table 1.

Modulus of elasticity of the concrete (E_c), modulus of elasticity of the steel (E_s), dead load (D), live load (L) and earthquake load (E) are considered as random variables (\mathbf{X}). In deterministic constraints, the nominal value of these

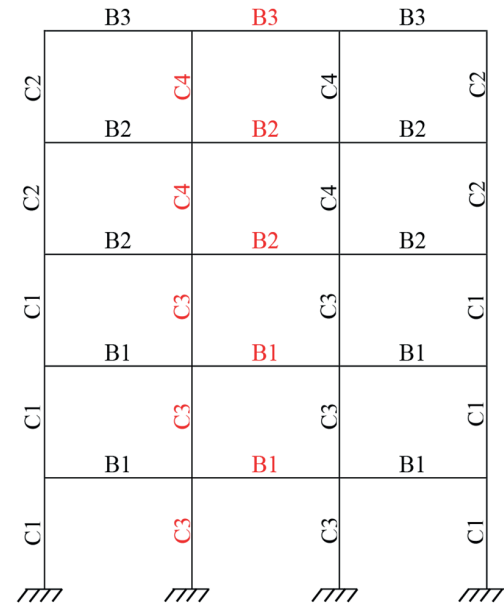


Fig. 1 The selected RC frame for RBDO problem

Table 1 Prepared section list (PSL)

Section number	Width (mm)	Depth (mm)	Bars* (top and bottom)
Beams			
#1	300	300	3 Φ 16
#2	300	300	3 Φ 18
#16	400	400	5 Φ 22
#17	400	400	4 Φ 25
Columns			
#1	300	300	3 Φ 16
#2	300	300	3 Φ 18
#36	450	450	4 Φ 30
#37	450	450	5 Φ 28

* Φ represents the bar diameter in mm

random variables will be employed to determine the structure response using the developed model in OpenSees (Please see Section 4.1).

The specified compressive strength of the concrete and yield strength of the steel bars are 25 and 400 MPa, respectively. The concrete cover thickness is equal to 50 mm. The strength reduction factors for the concrete (ϕ_c) and steel (ϕ_s) are considered 0.65 and 0.85, respectively. Earthquake load (base shear) is calculated by using equivalent static method and distributed as joint load in frame height. According to [36], the earthquake coefficient is equal to 0.1554. In the probabilistic constraint as defined in Eq. (1), the maximum inter-story drift (θ_{max}) is employed as engineering demand parameter. So, the limit state

function $g(\cdot) = \theta^a - \theta^{\max}$ is utilized in reliability analysis of the frame in which θ^a is the code-defined allowable drift. According to [36], θ^a is equal to 0.0045 for intermediate RC frames. Table 2 represents the characteristics of the aforementioned random variables.

The nominal value of the resistance parameters is k_R standard deviation below the mean value, and nominal value of single load parameters is k_S standard deviation above the mean value. It must be noted that for all the random variables we have $k_R = k_S = 5$. The correlation coefficient between dead load and earthquake load is considered equal to 0.8.

4.1 RC frame modelling

A two-dimensional finite element model of the RC frame is constructed using the OpenSees platform. All the base nodes of the foundation are modeled as an ideal fixed bearing. The uniaxial Material Elastic is used to model the concrete and steel bar materials. To consider the reduced stiffness of RC members due to cracking, the moment of inertia of the cross section for beams and columns is reduced by 0.35 and 0.7, respectively. In order to consider the geometric nonlinearities in the RC frame analysis, geomTransf P-Delta transformation has been utilized. Force-based elements are the most common type of the element formulations that are extensively adopted in the analysis of framed structures. So, a nonlinearBeamColumn element with five integration points is used to model the RC columns and beams whose cross sections are discretized into a number of fibers including cover concrete, steel bar, and core concrete. Based on the sensitivity analysis to achieve suitable accuracy, number of fibers in the cover and core concrete patches in the two sides of the rectangular RC sections is considered as equal to 10.

A linear static analysis procedure was adopted in this study to calculate the structural response during the optimization process. Dead loads, live loads, and distributed lateral earthquake load in the frame height are applied to the structure with linear plain pattern. A load control with

a Newton-Raphson solution algorithm is used to run the linear analysis. A plain constraints are utilized to handle the boundary conditions. To provide the mapping between the degrees of freedom at the nodes and the equation numbers, plain numberer is employed. In order to store and solve the system of equations in the analysis, BandGeneral system is used. Finally, to ensure the numerical convergence of the solution over the maximum 10 iterations, the norm of displacement increment (NormDispIncr test) is checked with a defined tolerance of 10^{-8} .

5 Results

The parameters of the genetic algorithm must be selected according to the sensitivity analysis results. Also with a genetic algorithm, different results may be obtained based on the random seeds used. Therefore, 20 different runs are conducted for each selected set to examine the variability of the results with different random seeds. Results indicate the ability of the GA with $n_p \geq 30$ to estimate the optimum value with negligible variability. Based on the results, $n_p = 30$, $n_{cr} = 18$ and $n_{mu} = 5$ will be utilized in the following sections. Also based on the convergence considerations, n_{iter} is set to 150.

For comparison purposes, two levels of β^t equal to 6 and 7 will be used in this example. Table 3 shows the identification numbers of optimal design solutions for both cases of β^t equal to 6 and 7. Corresponding characteristics of the obtained optimal sections have been reported in Table 4.

The convergence history of the best and mean weight as an objective function in both cases has been shown in Fig. 2. The difference between best and mean objective values at initial generations is related to penalize the infeasible individuals by the penalty function.

The convergence is achieved after 53 and 68 iterations for β^t equals to 6 and 7, respectively. It is obvious that optimized frame weight will increase as the minimum required reliability index for frame increases. The optimized frame weights are 32394.66 and 36003.78 kg for β^t equals to 6 and 7, respectively.

In deterministic constraints, it is expected that the maximum moment of beams ($M^{\max\pm}$) and maximum axial force of columns ($N^{\max\pm}$) will not be significantly affected

Table 2 Characteristics of the random variables

Random variable	Distribution	Nominal value	Coefficient of variation	Unit
E_c	Lognormal	25×10^9	0.1	N/m ²
E_s	Lognormal	200×10^9	0.05	N/m ²
D	Lognormal	18000	0.15	N/m
L	Lognormal	6000	0.4	N/m
E	Lognormal	220181	0.5	N

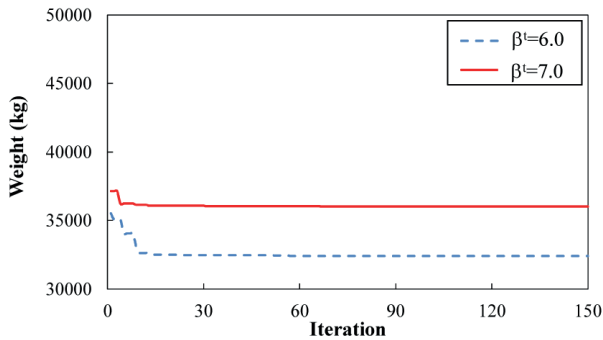
Table 3 Optimal design solutions

β^t	Column groups				Beam groups		
	C1	C2	C3	C4	B1	B2	B3
6	#21	#18	#26	#19	#13	#10	#2
7	#31	#18	#32	#25	#14	#12	#2

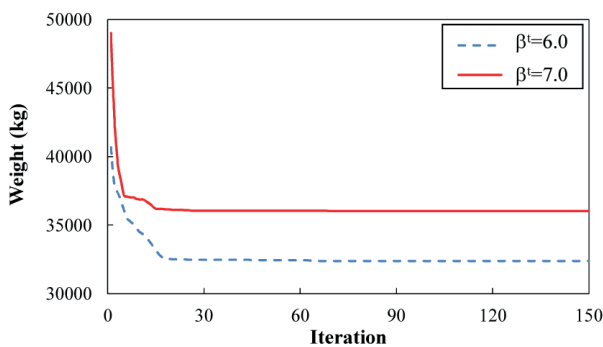
Table 4 Optimal design sections

Section number	Width (mm)	Depth (mm)	Bars* (top and bottom)
Beams			
#2	300	300	3 Φ 18
#10	300	350	4 Φ 20
#12	350	350	4 Φ 22
#13	350	400	3 Φ 25
#14	350	400	4 Φ 22
Columns			
#18	400	400	3 Φ 20
#19	400	400	4 Φ 18
#21	400	400	4 Φ 20
#25	400	400	4 Φ 25
#26	450	450	3 Φ 22
#31	450	450	5 Φ 22
#32	450	450	4 Φ 25

* Φ represents the bar diameter in mm



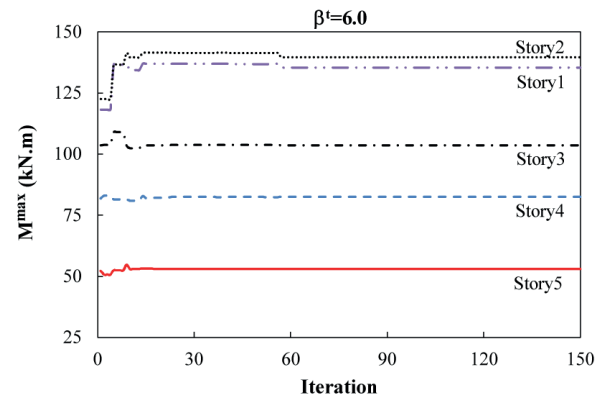
(a)



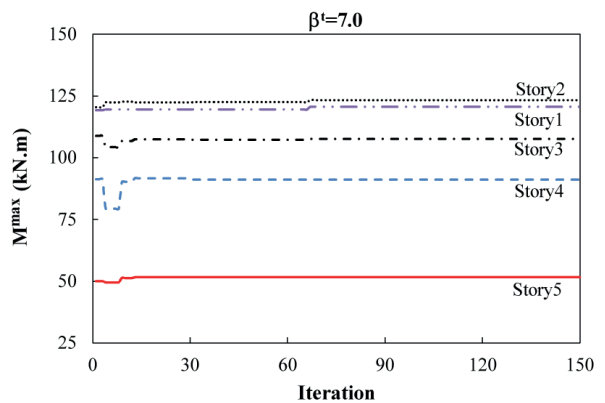
(b)

Fig. 2 The convergence history of (a) best and (b) mean objective

by section changes in the optimization process. However, these section changes have more effects on the maximum moment of beams than the maximum axial force of columns. Fig. 3 depicts the history of the maximum absolute value of maximum moment at the midspan and each end of the beams $\max(M_j^{\max+}, |M_j^{\max-}|)$ in different stories for both values of β^t . For brevity, this parameter is shown



(a)



(b)

Fig. 3 The history of maximum moment of beams in different stories (a) $\beta^t = 6$ and (b) $\beta^t = 7$

as M^{\max} in Fig. 3. For the introduced RC frame, no column under the defined load combinations undergoes the tensile axial force. Therefore, in this case study, interaction between the compressive axial force and moment has been occurred for all of the columns. Fig. 4 depicts the history of maximum compressive axial force of columns in different stories for both values of β^t . It should be noted that these results belong to the members specified with red color in Fig. 1. As shown in Figs. 3 and 4, the obtained results confirm the predicted trends.

On the other hand, it is expected that section changes during the optimization have a considerable effect on the moment strength of beams (M^{\pm}) and axial strength of columns (N^{\pm}). Fig. 5 illustrates the history of the maximum moment and moment strength of the beam in the third story for both values of β^t . Also, the history of maximum axial force and axial strength of the column in the third story for both cases of β^t has been shown in Fig. 6. As shown in Figs. 5 and 6, the obtained results confirm the expected trends. Surely, the frame design will be conservative for a higher value of the minimum required reliability index in the optimization process. As shown in

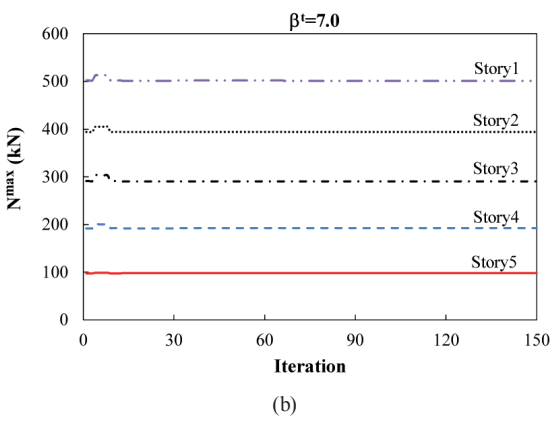
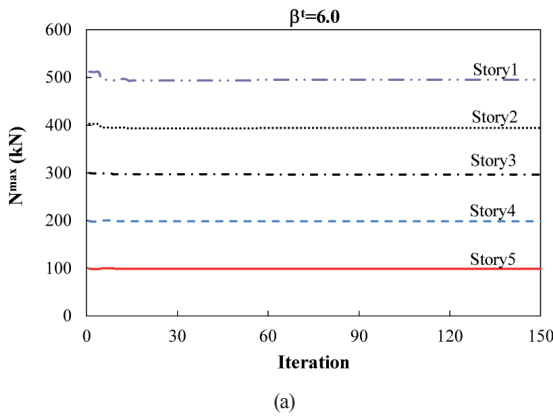


Fig. 4 The history of maximum axial force of columns in different stories (a) $\beta^t = 6$ and (b) $\beta^t = 7$

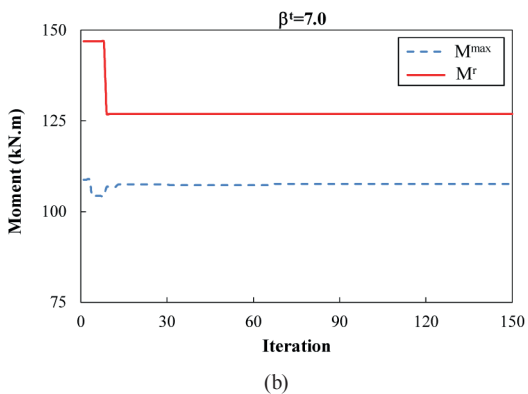
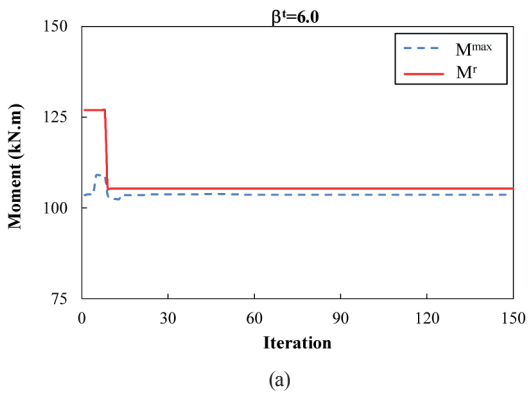


Fig. 5 The history of maximum moment and moment strength of third story beam (a) $\beta^t = 6$ and (b) $\beta^t = 7$

Fig. 5(b), the optimized maximum moment is significantly less than the moment strength for $\beta^t = 7$, but these two values are slightly different for $\beta^t = 6$. A similar trend is seen in Fig. 6(b) for the third story column as an instance.

In probabilistic constraints, the aim is to provide the minimum required reliability index (β^t) for the frame. The history of reliability index (β) during the optimization process is illustrated for both values of β^t in Fig. 7. It is worth to note that simultaneous satisfaction of deterministic and probabilistic constraints in the case of $\beta^t = 6$ is not possible for available discrete design solutions. As shown in Fig. 7, results indicate that the optimized

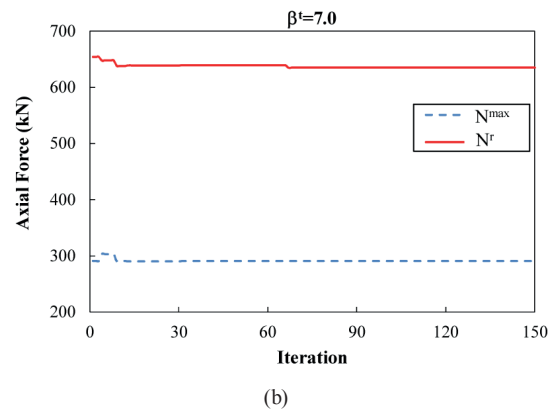
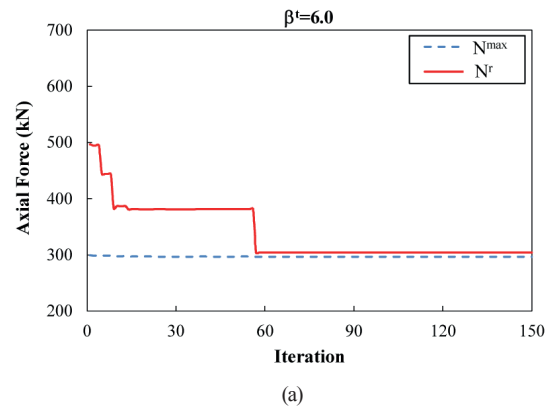


Fig. 6 The history of maximum axial force and axial strength of third story column (a) $\beta^t = 6$ and (b) $\beta^t = 7$

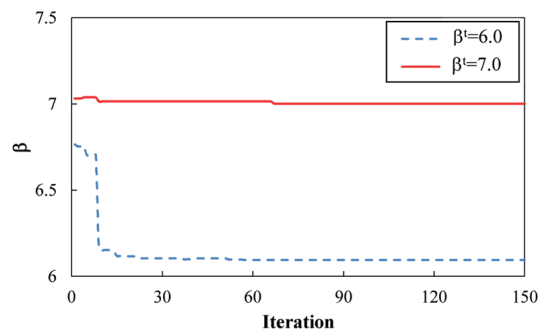


Fig. 7 The reliability index history for both values of β^t

minimum reliability index for satisfying all of the constraints is equal to 6.09. On the other hand, for the case of $\beta' = 7$, the probabilistic constraint is more critical than deterministic ones. Therefore, providing the $\beta' = 7$ will lead to satisfying the deterministic constraints as well.

In the introduced case study in this paper, all of the random variables have a lognormal distribution and no singularity and divergence are happened during the optimization process. Hence, RIA is a robust method for solving the introduced RBDO problem. In this case study, due to the obtained high values of reliability index, PMA is more efficient in evaluating the inactive probabilistic constraint. However, as a limitation of the current study, using different approaches for solving the RBDO problem of the RC frame is not investigated in this paper.

6 Conclusions

This paper presents a new framework for reliability based design optimization (RBDO) of reinforced concrete (RC) frames using a genetic algorithm (GA) and finite element reliability analysis (FERA). Results indicate the ability of the proposed GA-FERA method to solve the RBDO problem of the studied RC frame. Although this method is applied only to the optimization of the mid-rise intermediate

RC frame according to Iranian design codes, the proposed framework can also be applied to other types of RC frames utilizing other code provisions. The obtained high values of reliability index for the frame demonstrate the conservative nature of code requirements for interstory drift limitations based on the linear static analysis method. According to the proposed method, the relationship between the objective function and the minimum required reliability index can be studied. For example, the results of present research demonstrate that increasing the minimum reliability index from 6 to 7 resulted in an 11 % increase in the weight of the structure as an objective function. So, we can obtain a trade-off between the frame weight and minimum required reliability index. It is worth to note that comparison of different approaches to evaluate the probabilistic constraint is not a main goal of the present research. This study can be extended in the future by following aspects:

- Using nonlinear analysis procedures to estimate the realistic lateral capacity of RC frames considering the material and geometric nonlinearities.
- Investigating the effectiveness of the RIA and PMA approaches to evaluate the probabilistic constraint in the RBDO problem of RC frame in the cases of linear and nonlinear analyses.

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