https://doi.org/10.3311/PPci.19695
Creative Commons Attribution | 809

Periodica Polytechnica Civil Engineering, 66(3), pp. 809–819, 2022

Optimal Number and Location of Sensors for Structural Damage Detection using the Theory of Geometrical Viewpoint and Parameter Subset Selection Method

Sahar Beygzadeh[1], Peyman Torkzadeh[1*], Eysa Salajegheh[1]

[1] Department of Civil Engineering, Faculty of Engineering, Shahid Bahonar University of Kerman, Pajoohesh Sq., 76169-14111 Kerman, Iran

[*] Corresponding author, e-mail: torkzadeh@uk.ac.ir

Received: 13 December 2021, Accepted: 27 March 2022, Published online: 04 May 2022

Abstract

The recorded responses at predefined sensor placements are used as input to solve an inverse structural damage detection problem. The error rate that noise causes from the recorded responses of the sensors is a significant issue in damage detection methods. Therefore, an optimal number and location of sensors is a goal to achieve the lowest error rate in structural damage detection. To overcome this problem, an algorithm (GVPSS) based on a Geometrical Viewpoint (GV) of optimal sensor placement and Parameter Subset Selection (PSS) method is proposed. The goal of the GVPSS algorithm is to minimize the effect of noise on damage detection problem. Therefore, the fitness function based on error in damage detection is minimized by GVPSS. In this method, the degrees of freedom are arranged to place sensors using a fitness function based on GV theory. Then, the optimal number and location of sensors are found on these arranged the degrees of freedom using the objective function. The efficiency of the proposed method is studied in a 52-bar dome structure under static and dynamic loadings. In the examples, damages are detected in two states: 1) using responses recorded at all DOFs, 2) using responses recorded at the optimal number and location of sensors obtained by GVPSS. The results showed that the damage detection error in state 2 is approximately equal to the error in state 1. Therefore, the GVPSS have the high performance to find the optimal number and location of sensors for structural damage detection.

Keywords

damage detection, optimal number and location of sensors, geometrical viewpoint, parameter subset selection, sensitivity matrix

1 Introduction

Detection of damages is necessary for monitoring of structural health and preventing the structural failures. Ghiasi et al. [1] proposed a novel strategy using least square support vector machines based on a new combinational kernel for structural damage detection. Homaei et al. [2] proposed a direct damage detection method using multiple damage localization index based on mode shapes criterion. A two-stage method was presented for structural damage detection using an optimized artificial neural network. In the first stage, location of damages was detected using curvature-moment and curvature-moment derivative concepts. In the second stage, Bat optimization algorithm was engaged by an artificial neural network as a surrogate of FE model to assess the severity of damages [3]. Ghiasi et al. [1] proposed a new strategy for structural damage detection using least square support vector machines based on a new combinational kernel. Kaveh and Dadras [4] enhanced and applied a developed optimization algorithm for damage detection problem. Dinh-Cong et al. [5] presented a multi-stage optimization method for damage detection in plate-like structures. In this method, the objective function is established via flexibility change of the structure, which is minimized using a modified differential evolution algorithm. Khatir et al. [6] proposed a novel method for crack identification using vibration analysis based on model reduction. Kaveh and Zolghadr [7] formulated an approach for damage detection as an inverse optimization problem. In this approach, the amounts of damage to each element are considered as the optimization variables. The objective function is based on setting these variables such that the characteristics of the modal correspond to the experimentally measured characteristics of the actual damaged structure. Khatir et al. [8] presented a two-stage method for damage detection in beam-like structures. In this method,

Cite this article as: Beygzadeh, S., Torkzadeh, P., Salajegheh, E. "Optimal Number and Location of Sensors for Structural Damage Detection using the Theory of Geometrical Viewpoint and Parameter Subset Selection Method", Periodica Polytechnica Civil Engineering, 66(3), pp. 809–819, 2022. https://doi.org/10.3311/PPci.19695

a new damage index is proposed to locate the damaged elements. Krishnanunni et al. [9] used vibration data and static displacement measurements to detect structural damages. In this study, an objective function is proposed using the sensitivity equation that was minimized by the cuckoo search algorithm. Ghannadi et al. [10] studied the efficiency of grey wolf optimization algorithm for damage detection. In this study, the residual force vector based on expended mode shapes was considered as an objective function. Mishra et al. [11] proposed ant lion optimizer to detect structural damage. The objective function is based on vibration data, such as natural frequencies and mode shapes. Baneen and Kausar [12] presented a baseline-free approach to improve damage detection accuracy using the modal strain energy method. Alexandrino et al. [13] considered an inverse problem of damage detection as a robust optimization problem. In this study, the robust optimum value was obtained by solving a multiobjective problem. Blachowski [14] proposed a comprehensive approach for damage detection in spatial truss structures. In this method, first, sensitivity of assumed modal characteristics is calculated. After that, natural frequency sensitivity is used to determine hardly identifiable structural parameters and mode shape sensitivity is applied to select damage-sensitive locations of sensors. Next, two sparsity constrained optimization algorithms are tested towards efficient identification of applied damage scenarios.

Structural damage detection using recorded responses of the sensors has received a lot of attention in the recent years; since it is important in the accuracy of damage detection [15]. The suggested methods are based on the minimization of the difference between the measured and analytical static responses of structures. The suggested methods are based on the minimization of the difference between the measured and analytical static responses of structures. The quality of recorded responses depends on the number and location of sensors in structures [16]. On the other hand, the high cost of data acquisition systems and accessibility constraint in many cases lead to the distribution of a limited number of sensors on a structure [17]. So, determination of the optimal number and location of sensors is significant in structural damage detection process. Shahbaznia et al. [18] proposed a time-domain efficient response sensitivity-based modal update procedure for the identification of railway bridge damage subject to unknown moving loads. Talebpour et al. [19] proposed the simultaneous use of mathematical and statistical methods to reduce the search space. To this aim, a two-step damage detection method was proposed. In the first step, the structural elements were initially divided into different groups using the k-means method. Subsequently, the possibly damaged elements of each group were identified. In the second step, the items selected in the first step were placed in a new set and a process was applied to identify their respective location and severity of damage.

Many researchers have studied the optimal number and location of sensors problem for identification of damage detection during the past few years. The primary evaluation criteria and main sensor placement methods was discussed by Yi and Li [20]. Dinh-Cong et al. [21] proposed an efficient approach for optimal sensor placement and damage detection in laminated composite structures. In this approach, first, a reduced order model for OSP is developed using an iterated improved reduced system (IIRS). The OSP is then formulated as an optimization problem and solved using Jaya algorithm. Next, the approach uses the measured incomplete modal data from OSP for damage detection and the damage is detected again by Jaya algorithm. Li et al. [22] proposed a novel method called dual-structure coding and mutation particle swarm optimization (DSC-MPSO) algorithm for optimal sensor placement. This approach first selected the main modes of contribution using the cumulative effective modal mass participation factor. Then, a novel method was used that combines dual-structure coding with the mutation operator to obtain the optimal sensor location. Hou et al. [23] presented a criterion for optimizing the sensor quantity and location for stay cable damage identification. In this study, the random elimination algorithm and the heuristic random elimination algorithm were utilized to solve the sensor location optimization problem. Beygzadeh et al. [24] proposed an improved genetic algorithm for OSP in space structures damage detection based on a geometrical viewpoint. Wu et al. [25] presented the development and application of optimal sensor placement methods, which have been integrated into a versatile software tool to maximize sensor convergence capacity for structural performance evaluation. Sensor placement methods are optimized using a parallel optimization framework based on the competent genetic algorithm, generating a number of characteristics that enrich the flexibility of the application. The developed methods and software tool provide a genetic approach for multi-type sensor placement. Chisari et al. [26] proposed a novel method for OSP based on definition of the representativeness of the data with respect to the global displacement field. This method employed an optimization procedure based on genetic algorithms. Sadat Shokouhi and Vosoughifar [27] proposed a novel numerical method for optimal sensor placement, which was called transformed time-history to frequency domain (TTFD) algorithm. This method uses nonlinear time-history analysis results as an exact seismic response despite the common OSP algorithms utilize the eigenvalue responses. A review of optimization of sensor placement for structural health monitoring was presented by Ostachowicz et al. [28]. Gomes and Pereira [29] detected the damages by solving an inverse problem. In this study, a fuselage model of an E190 aircraft is considered and the firefly algorithm (FA) is applied to solve the inverse problem. Obtaining the modal response at all points in a large-scale structure is prohibitive. Therefore, the Fisher Information Matrix (FIM) is performed for OSP.

In this study, an algorithm called GVPSS is proposed to find the optimal number and location of sensors for structural damage detection. In this numerical method, the Geometrical Viewpoint (GV) of OSP [24] is combined with the Parameter Subset Selection (PSS) method [30] for structural damage detection. The PSS method employs the recorded responses of sensors for structural damage detection. Due to the noise of the existing responses, the damage is not detected accurately. In the GVPSS method, the optimal number and location of sensors is obtained based on the reduction of the error rate in damage detection. Therefore, the degrees of freedom (DOFs) are arranged to place the sensors using a fitness function based on GV theory. The optimal number and location of sensors are then attempted on these arranged DOFs. The objective function is established based on error in damage detection using PSS that should be minimized. The responses recorded at predefined locations in the structure are input data for damage detection. Damages are detected in two states: 1) using responses recorded in all DOFs, 2) using responses recorded in the optimal number and location of sensors obtained by GVPSS. Damage detection error is minimized by using responses recorded in all DOFs of the structure. When the damage detection error in state 2 is approximately equal to the error in state 1, the number of sensors is optimum.

This article is organized as follows: the geometrical viewpoint as a theory for optimal sensor placement is described in Section 2. The parameter subset selection method for damage detection is discussed in Section 3. The proposed algorithm to find the optimal number and location of sensors (GVPSS) is presented in Section 4. Numerical results are obtained and discussed in Section 5 and conclusions are presented in Section 6.

2 Geometrical viewpoint theory for optimal sensor placement

Due to the damage, the responses recorded by the sensors change in the structures and these responses are used as input data in damage detection. Therefore, the damage detection problem is equivalent to a nonlinear system of equations, which can be expressed as [31]:

$$\boldsymbol{R}_d = \boldsymbol{R}(\boldsymbol{Z}) \quad \Rightarrow \quad \boldsymbol{Z} = ? \quad (0 \le z_i \le 1)\,, \tag{1}$$

where $\boldsymbol{Z} = (z_1, z_2, \ldots, z_{ne})^T$ is the damage vector and z_i is the damage ratio of the ith element where $z_i = 0$ and $z_i = 1$ indicate the intact and completely damaged states, respectively. ne is the number of structural elements, $\boldsymbol{R}_d = (r_{d,1}, r_{d,2}, \ldots, r_{d,m})^T$ is the vector of m responses of the existing damaged structure and $\boldsymbol{R}(\boldsymbol{Z}) = (r_1(\boldsymbol{Z}), r_2(\boldsymbol{Z}), \ldots, r_m(\boldsymbol{Z}))^T$ is the vector of m responses of a hypothetically damaged structure that can be evaluated from the analytical model. In this study, the damage is considered as a reduction in the modulus of elasticity of the elements. The structural response in the intact state is $\boldsymbol{R}(0) = \boldsymbol{R}_h$. The function $\Delta\boldsymbol{R}(\boldsymbol{Z})$ is defined as $\boldsymbol{R}(\boldsymbol{Z})$-$\boldsymbol{R}_h$.

Using the first order approximation, linearization of Eq. (1) can be estimated as follows:

$$\begin{aligned} &\boldsymbol{R}_d = \boldsymbol{R}_h + \frac{\partial \boldsymbol{R}}{\partial \boldsymbol{Z}}\Delta\boldsymbol{Z}, \qquad \boldsymbol{S} = \frac{\partial \boldsymbol{R}}{\partial \boldsymbol{Z}} \quad \Rightarrow \\ &\boldsymbol{R}_d - \boldsymbol{R}_h = \boldsymbol{S}\,\Delta\boldsymbol{Z} \quad \Rightarrow \quad \Delta\boldsymbol{R}_{m\times 1} = \boldsymbol{S}_{m\times ne}\boldsymbol{Z}_{ne\times 1}. \end{aligned} \tag{2}$$

The structural response in the damaged state is $\boldsymbol{R}(\boldsymbol{Z}_d) = \boldsymbol{R}_d$, where $\boldsymbol{Z}_d = (z_{d,1}, z_{d,2}, \ldots, z_{d,ne})^T$, which is the actual damage vector. $\Delta\boldsymbol{Z} = \boldsymbol{Z} - \boldsymbol{Z}0 = \boldsymbol{Z} - 0$ and $\boldsymbol{S}$ is the sensitivity matrix. Matrix $\boldsymbol{S}$ can be found using [32] in structures under external loadings and general excitation, respectively. The damage is detected by Eq. (3).

$$\boldsymbol{Z}_{ne\times 1} = \boldsymbol{S}^{+}_{ne\times m}\Delta\boldsymbol{R}_{m\times 1}\,, \tag{3}$$

where $\boldsymbol{S}^+$ is the pseudo-inverse of $\boldsymbol{S}$ and it can be found by Singular Value Decomposition (SVD) [33]. According to Eq. (3), the vectors $\boldsymbol{Z}$ and $\Delta\boldsymbol{R}$ are geometrically members of ne-dimensional and m-dimensional Euclidean space, which are called the damage space and the response change space, respectively. Eq. (3) is geometrically interpreted as the response change space maps approximately linear into the damage space by $\boldsymbol{S}^+$ matrix, according to Fig. 1.

The recorded responses from the sensors are noisy. According to Eq. (4), this noise is added to the damage detection equation.

$$\boldsymbol{S}\ \boldsymbol{Z} = \Delta\boldsymbol{R} + \vartheta \quad \Rightarrow \quad \boldsymbol{Z} = \boldsymbol{S}^{+}\Delta\boldsymbol{R} + \boldsymbol{S}^{+}\vartheta\,, \tag{4}$$

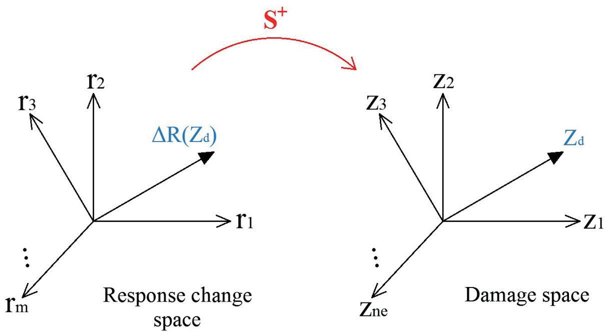


Fig. 1 Geometrical interpretation of damage detection equation

where ϑ is the random variable vector. The noise has a multivariate normal distribution which is indicated as, $\vartheta \sim \boldsymbol{N}(\boldsymbol{\mu},\sum)$ where $\boldsymbol{\mu}$ is the mean vector and $\sum$ is the covariance matrix. The noise can be considered additive or multiplicative. The covariance matrix is a diagonal matrix and a complete matrix in additive and multiplicative noise, respectively. The additive and multiplicative noises are geometrically interpreted as the spherical and the ellipsoid in Euclidean space, respectively. In this study, the noise is a normally distributed additive standard error with zero mean and unit standard deviation, $\vartheta \sim \boldsymbol{N}(\boldsymbol{0},\boldsymbol{I})$ [34]. According to Eq. (4), the additive noise in the response change space is transferred to the damage space by $\boldsymbol{S}^{+}$ matrix and it is changed to the multiplicative noise with covariance matrix $\boldsymbol{S}^{+}\boldsymbol{I}\boldsymbol{S}^{+T}$, this mapping is called direct mapping. In inverse mapping, the additive noise in the damage space is changed to multiplicative noise in the response change space by $\boldsymbol{S}$ matrix, according to Fig. 2.

It is supposed that $\boldsymbol{A}$ is 2 × 2 real-valued matrix. Geometrically, SVD is a decomposition of matrix A into one rotation, scaling and second rotation of the form:

$$\boldsymbol{A}=\boldsymbol{U}\sum\boldsymbol{V}^{T}=\boldsymbol{U}\begin{bmatrix}\sigma_1 & 0\\ 0 & \sigma_2\end{bmatrix}\boldsymbol{V}^{T}, \tag{5}$$

where the T denote transpose. Here, $\boldsymbol{V}^{T}$ is the first rotation, $\sum$ is the scaling matrix and $\boldsymbol{U}$ is the last rotation. This is shown in Fig. 3:

Therefore, the diameters of ellipsoid noises in the damage space and the responses change space can be created by SVD according to Eqs. (6) and (7) ($ne > m$):

$$\boldsymbol{S}^{+}_{ne\times m}=\boldsymbol{U}_{ne\times ne}\sum\nolimits_{ne\times m}\boldsymbol{V}^{T}_{m\times m}=\boldsymbol{U}\begin{bmatrix}\gamma_1 & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & \gamma_m\\ 0 & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & 0\end{bmatrix}\boldsymbol{V}^{T}, \tag{6}$$

$$\boldsymbol{S}_{m\times ne}=\boldsymbol{V}_{m\times m}\sum\nolimits_{m\times ne}\boldsymbol{U}^{T}_{ne\times ne}=\boldsymbol{V}\begin{bmatrix}\frac{1}{\gamma_1} & \cdots & 0 & 0 & \cdots & 0\\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\ 0 & \cdots & \frac{1}{\gamma_m} & 0 & \cdots & 0\end{bmatrix}\boldsymbol{U}^{T}. \tag{7}$$

According to geometrical viewpoint, if the diameters of the elliptical noise are large in the inverse mapping ($1/\gamma_i$) and then the diameters of the elliptical noise in the damage space in the direct mapping (γ_i) will become small and the damage detection will be accurate. Therefore, the elliptical noise in the response change space consisting of the DOFs corresponding to OSP is greater than that in other spaces. Then, the projection of ellipsoid noise on each side of this space is greater than that of other spaces. Therefore, the objective function for obtaining OSP is as follows:

$$f_i=\left\|proj_{e_i}\boldsymbol{S}_{m\times ne}\boldsymbol{I}_{ne\times ne}\boldsymbol{S}^{T}_{ne\times m}\right\|=\left\|\boldsymbol{e}_{i_{m\times 1}}\boldsymbol{e}^{+}_{i_{1\times m}}\boldsymbol{S}_{m\times ne}\boldsymbol{I}_{ne\times ne}\boldsymbol{S}^{T}_{ne\times m}\right\|, \tag{8}$$

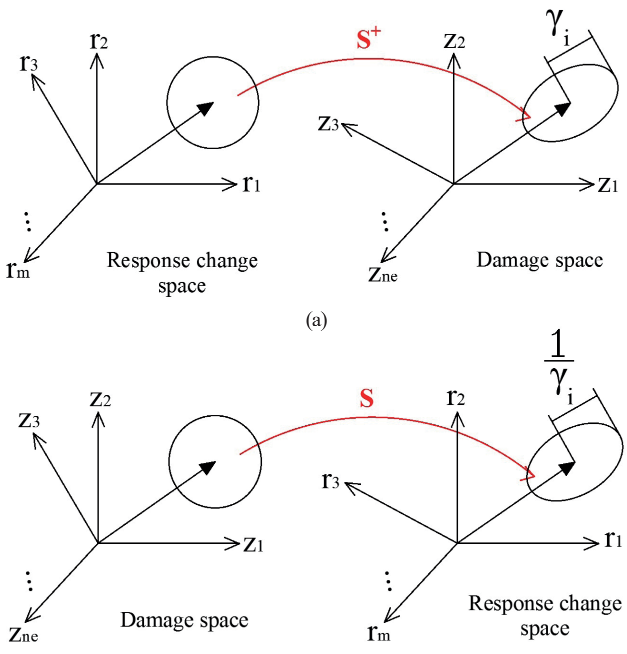


Fig. 2 The geometrical viewpoint; (a) Direct mapping, (b) Inverse mapping

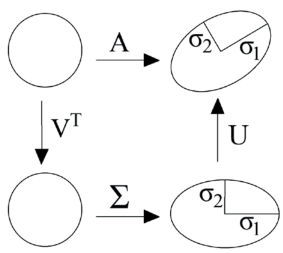


Fig. 3 Illustration of the singular value decomposition of a real 2 × 2 matrix $\boldsymbol{A}$

where *proj* is the projection of the ellipsoid in the response change space into each side of this space. $\boldsymbol{e}_i$ is the vector with the $m \times 1$ size in which only ith component of this vector is unity and the others are zero. f in the degrees of freedom corresponding to the proper locations for sensor placing is greater than other DOFs. The selected DOFs which are more suitable for sensor placement, have a higher objective function value than the other DOFs. Therefore, the objective function values for DOFs are sorted in a decreasing order and DOF corresponding to each value is the optimal placement, respectively.

3 Parameter subset selection method for damage detection

The subset selection method can be solved by a complete search of all possible subsets. This subset of the parameters that minimizes a function is based on the norm of the residuals in Eq. (3). Among the columns of $\boldsymbol{S}$, we search for the single column that best represents the vector $\Delta\boldsymbol{R}$. The selected parameter is the one that minimizes the residual according to Eq. (9).

$$q = \left\|\Delta\boldsymbol{R} - \boldsymbol{s}_i\beta_i\right\|^2 , \tag{9}$$

where $\boldsymbol{s}_i$ is ith column of $\boldsymbol{S} = [\boldsymbol{s}_1, \boldsymbol{s}_2, \ldots, \boldsymbol{s}_i, \ldots, \boldsymbol{s}_{ne}]$ and β_i is the least squares estimation of the ith parameter as follows:

$$\beta_i = \frac{\boldsymbol{s}_i^T \Delta\boldsymbol{R}}{\boldsymbol{s}_i^T \boldsymbol{s}_i}. \tag{10}$$

Then, the combination of two columns of $\boldsymbol{S}$ that constitutes the best sub-basis for the representation of $\Delta\boldsymbol{R}$ is determined. Let i_1 and $\boldsymbol{s}_{i1}$ represent the first selected parameter and the corresponding column of matrix $\boldsymbol{S}$, respectively. The optimum value for this parameter is:

$$\beta_{i1} = \frac{\boldsymbol{s}_{i1}^T \Delta\boldsymbol{R}}{\boldsymbol{s}_{i1}^T \boldsymbol{s}_{i1}}. \tag{11}$$

The columns of $\boldsymbol{S}$ and the vector $\Delta\boldsymbol{R}$ are replaced with $\boldsymbol{s}_i \rightarrow \boldsymbol{s}_i - \boldsymbol{s}_{i1}\alpha_1$ and $\Delta\boldsymbol{R} \rightarrow \Delta\boldsymbol{R} - \boldsymbol{s}_{i1}\alpha_1$, respectively, where $\alpha_i = \frac{\boldsymbol{s}_{i1}^T \Delta\boldsymbol{R}}{\boldsymbol{s}_{i1}^T \boldsymbol{s}_{i1}}$.

The procedure is repeated on this reduced problem to find the parameter β_i, for $i \neq i_1$, that gives the smallest residual [30].

4 The proposed algorithm for optimal location and number of sensors

The quality of a damage detection process is highly dependent on the quality and quantity of the recorded data, which further depends on the location and number of sensors in the structures. Therefore, it is important to find the optimum number of sensors in structures and the optimum location of these sensors. When the location and number of sensors are optimum in structure, damage detection will be exact.

A new algorithm called GVPSS, which combines the geometrical viewpoint (GV) for OSP with the Parameter Subset Selection method (PSS) is proposed to find the optimal locations and number of sensors in structural damage detection. This algorithm has two steps. In the first step, GV obtains the proper DOFs to place the sensors, and finally, in the second step, the optimal number and locations of the sensors are found during the damage detection process.

Finally, if damage detection using the data recorded from sensors is more accurate, the number of sensors is optimum.

The error in damage detection will be minimized when the data recorded on all DOFs of the structure is used. On the other hand, it is not economic to use the data recorded on all DOFs of the structure. To overcome this problem, the GVPSS algorithm is obtained with the optimal number and location of sensors in which the damage detection error using this incomplete measurement is approximately equal to the damage detection error using complete measurements in the structure. Therefore, the objective function is suggested as:

$$p = abs(\varepsilon_{comp} - \varepsilon_{incomp}) , \tag{12}$$

where ε_{comp} and ε_{incomp} are damage detection errors using complete and incomplete data, respectively, obtained by Eq. (13). $abs(.)$ represents the absolute value.

$$\varepsilon = \sum (abs(\breve{z} - z)) , \tag{13}$$

where $\breve{z}$ and $\boldsymbol{z}$ are the damage vector calculated using the parameter subset selection method and the hypothetically damage vector, respectively.

In any step of this method, one sensor is added according to the DOFs sorted using the GV method. Then, the objective function according to Eq. (12) is obtained. When the value of the objective function is letter than α (α is the suggested maximum error, for example, 5%), the optimal number and location of the sensors are obtained.

The step-by-step summary of the GVPSS algorithm is in Algorithm 1.

5 Numerical results

In this section, the GVPSS algorithm is studied to obtain the optimal number and location of sensors for damage detection in a 52-bar dome structure under static and dynamic

Algorithm 1 GVPSS algorithm

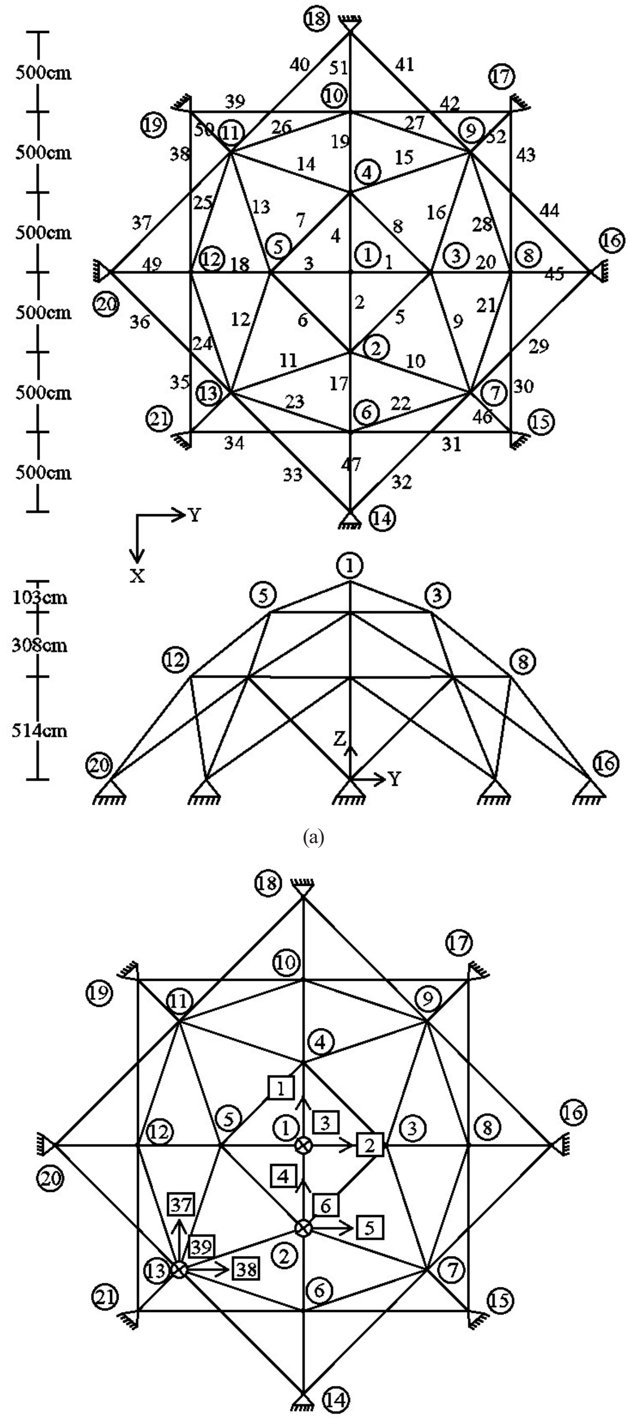
Step 1:
  Set i = 1, Q and G matrices with the number of DOFs × 1 size
  Repeat
    Compute $\boldsymbol{S}$ and $f_i = \left\| proj_{e_i}(\boldsymbol{S}_{m\times ne}\boldsymbol{I}_{ne\times ne}\boldsymbol{S}^T_{ne\times m}) \right\|$
    Increase i by one unit
  Until i = the number of DOFs
    Q_i = The values f_i arranged in a decreasing order
    G_i = The DOF corresponding to Q_i
Step 2:
  Set j = 1
  Repeat
    The sensor locations = G_1 to G_j DOFs
    Compute damage detection error (by parameter subset selection method) using complete and incomplete data (ε_{comp} and ε_{incomp})
    Compute objective function $p = abs(\varepsilon_{comp} - \varepsilon_{incomp})$
    Increase j one unit
  Until objective function < α (α is the suggested maximum error, for example, 5%)
The optimal number of sensors is j and the optimal sensor placement is DOFs of G_1 to G_j.

loadings. The type of sensors is displacement meter for static loading and acceleration meter for dynamic loading. In this example, the damages are considered as a reduction in the modulus of elasticity of an individual element. This space truss is shown in Fig. 4. All the cross-sectional areas of elements are 0.005 m^2. The material properties are taken from aluminum where the elastic modulus is 70 GPa and the density is 2770 kg/m^3. According to Fig. 4, the analytical model has 21 nodes, 52 elements and 39 active DOFs [31].

5.1 Dome under static loading

The structure is subjected to static loading according to Fig. 5. The displacement in all DOFs is obtained in healthy and damaged structures. Then, the values of f_i are calculated according to Section 2. Table 1 demonstrates the results of DOFs corresponding to f_i arranged in decreasing order. In this table, for example, positions 2 and 17 indicate that the sensors should be placed on the degrees of freedom of the 1th node in the X-direction and the 6th node in the Y-direction.

The displacements of the DOFs corresponding to sensor placements are recorded for damage detection. Three damaged scenarios are considered as shown in Table 2 and Fig. 6. The results of the GVPSS algorithm are presented for all damaged scenarios in Fig. 7. According to this figure, for example, the value of objective function using the recorded responses of 23, 26 and 36 DOFs (3 sensors) is 0.4012 in scenario 1. The results show that the optimum number of sensors is 14. The obtained value of objective function using the recorded responses from 14 sensors is letter than α (α = 0.05).



(a)

(b)

Fig. 4 (a) 52-bar space truss, (b) Active DOFs

Fig. 8 demonstrates the results of damage detection using incomplete and complete data. The results show that the error rate for damage detection using recorded responses from 14 sensors is almost equal to that for damage detection using recorded responses from all DOFs. Therefore, the number and locations obtained from the sensors are optimum. The optimal sensor locations are 3, 6, 9, 12, 15, 19, 23, 24, 25, 31, 34, 35, 36 and 37 DOFs shown in Fig. 9.

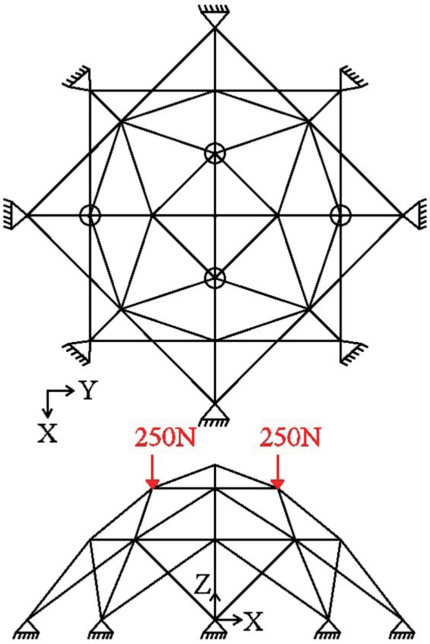


Fig. 5 Dome under static loading

Table 1 DOFs corresponding to the optimal placement for sensors

Optimal placement for sensors							
24	36	23	35	12	6	3	9
15	19	37	31	25	34	22	16
28	10	4	13	7	1	21	39
33	27	8	14	11	5	2	20
38	32	26	18	30	17	29	

Table 2 Damaged scenarios (Element number- Damage severity %)

Scenario 1	Scenario 2	Scenario 3
Element 15- 25%	Element 1- 20%	Element 1- 10%
Element 22- 25%	Element 3- 20%	Element 3- 10%
-	Element 15- 30%	Element 12- 10%
-	-	Element 23- 15%

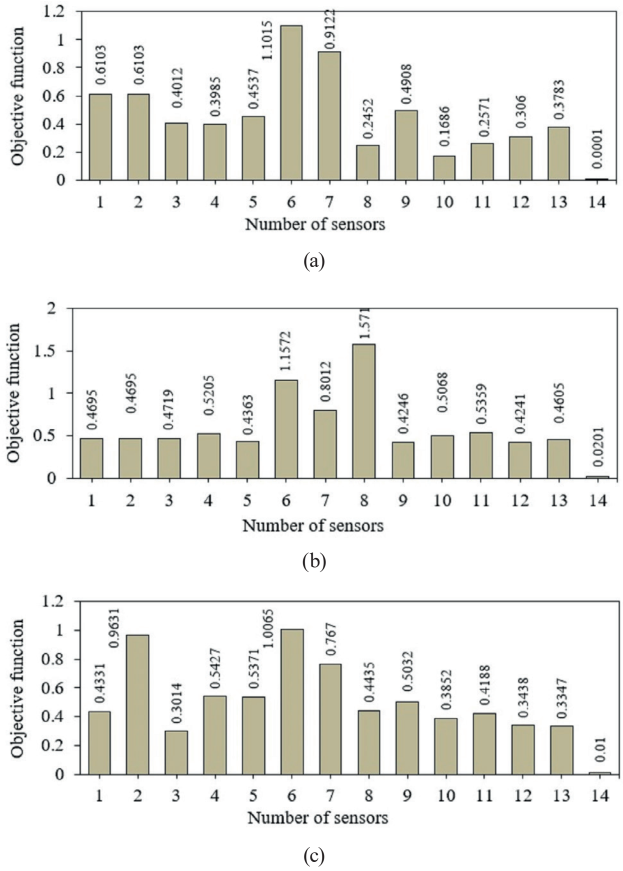


Fig. 7 Objective function value; (a) Scenario 1, (b) Scenario 2, (c) Scenario 3

5.2 Dome under dynamic loading

In this Section, the dome is under the excitation that is applied vertically at node 2. This force is a triangular impulsive force with a peak value of 320.4N and continues for 0.005s according to Fig. 10. The acceleration at DOF corresponding to sensor location is measured with a duration of 0.25 s. In GV theory, the sensor placements and the projection of ellipsoid noise are obtained by the algorithm

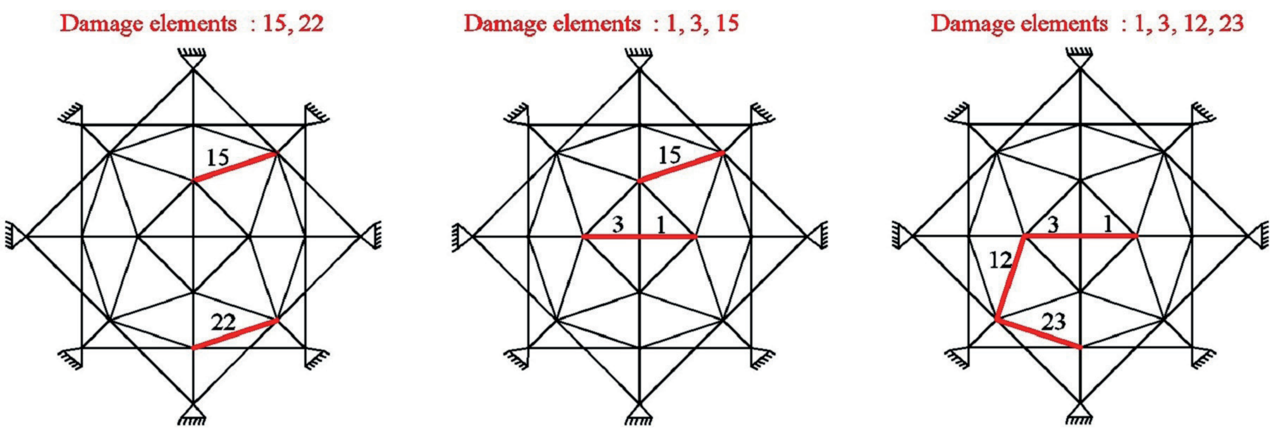
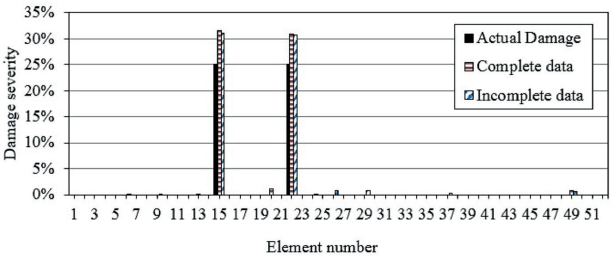
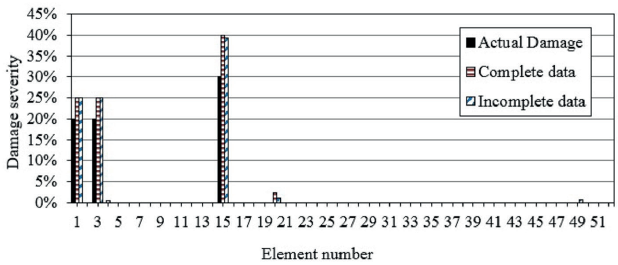


Fig. 6 Damage scenario for structure (a) Scenario 1, (b) Scenario 2, (c) Scenario 3



(a)



(b)

Fig. 8 Damage detection; (a) Scenario 1, (b) Scenario 2, (c) Scenario 3

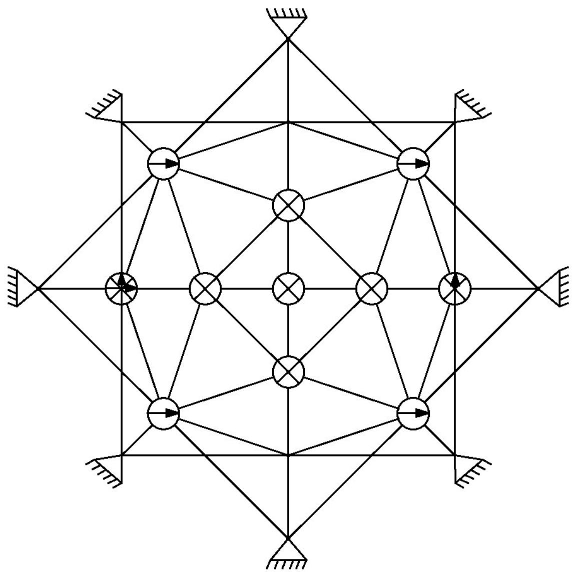


Fig. 9 Optimal location and number of sensors under static loading (→, ↑ and × represents horizontal, vertical and downward directions, respectively)

every 0.005 seconds. Therefore, the sensor placements with the maximum fitness function value are optimum, respectively. The results of DOFs according to the geometrical viewpoint are demonstrated in Table 3.

Damages are detected using the recorded acceleration at DOFs corresponding to the sensor placements. The different damaged scenarios are shown in Table 4 and Fig. 11. The results of objective function in the GVPSS algorithm are presented according to Fig. 12. In all three scenarios, the obtained objective function value using the recorded responses from the first 9 DOFs according to Table 3 is

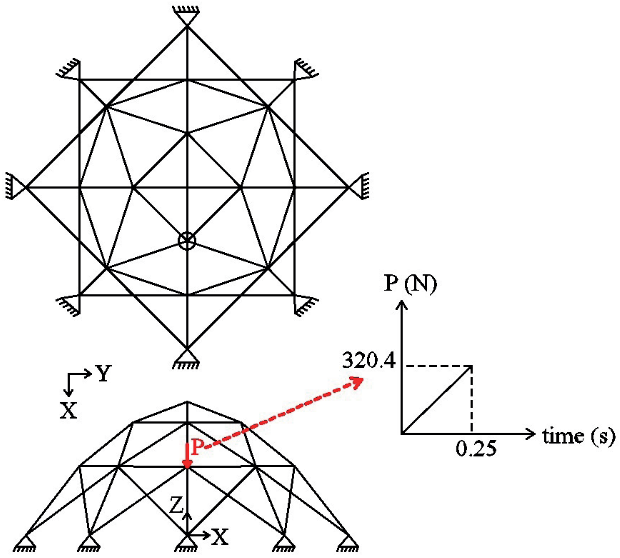


Fig. 10 Dome under dynamic loading

Table 3 DOFs corresponding to the optimal placement for sensors

Optimal placement for sensors							
6	4	3	9	12	15	16	20
21	38	39	25	1	19	18	27
7	13	37	26	10	33	31	28
23	35	30	32	8	2	34	14
24	11	36	5	29	17	22	

Table 4 Damaged scenarios (Element number- Damage severity %)

Scenario 1	Scenario 2	Scenario 3
Element 24- 30%	Element 19- 10%	Element 8- 15%
Element 43- 30%	Element 32- 15%	Element 16- 10%
-	Element 52- 15%	Element 20- 15%
-	-	Element 21- 10%

lesser than $\alpha (\alpha = 5\%)$. Therefore, the optimum number of sensors is 9 and the OSPs are 3, 4, 6, 9, 12, 15, 16, 20 and 21 DOFs. Fig. 13 demonstrates that damage detection using incomplete data obtained from 9 measurement points is accurate. In this figure, the error rate in damage detection using incomplete and complete data is almost the same. Therefore, the incomplete data recorded from the obtained OSP is suitable for the damage detection. The optimal location and number of sensors are illustrated in Fig. 14.

6 Conclusions

In this study, an algorithm called GVPSS is presented for the optimization of the number and location of sensors for structural damage detection. In GVPSS, the Geometrical Viewpoint (GV) of optimal sensor placement is combined with the Parameter Subset Selection (PSS) method to

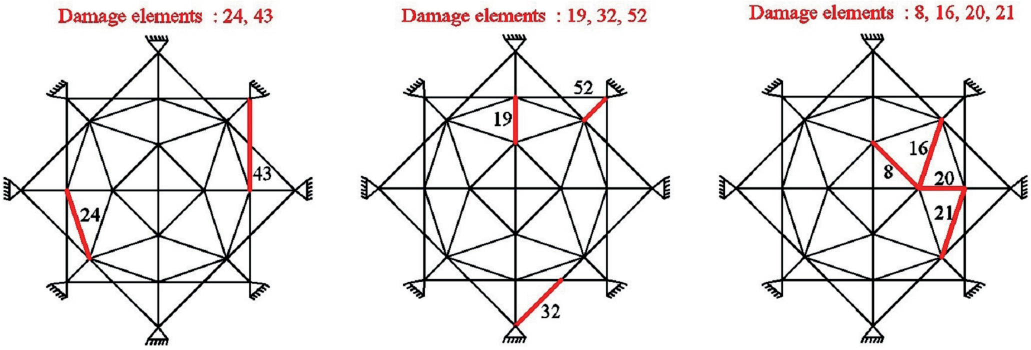


Fig. 11 Damage scenario for structure; (a) Scenario 1, (b) Scenario 2, (c) Scenario 3

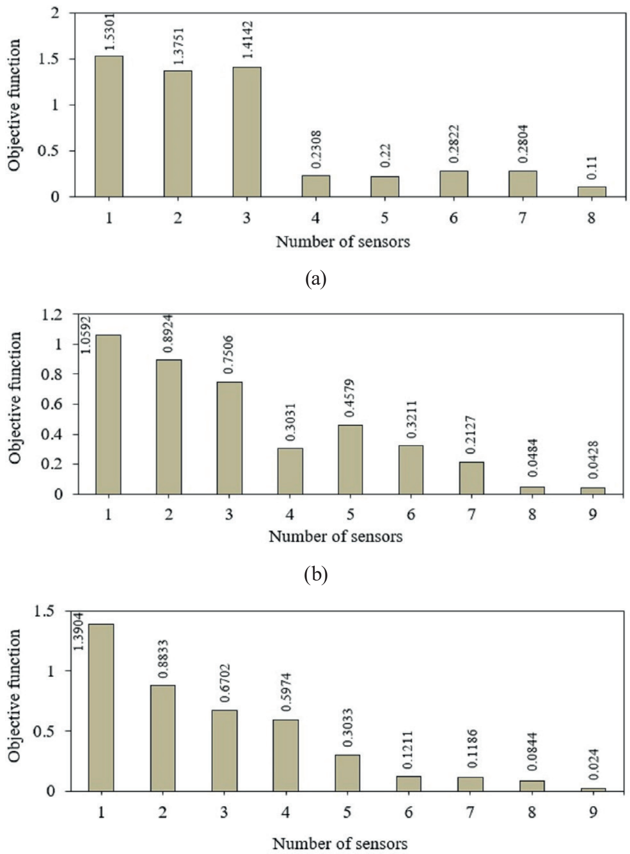


Fig. 12 Objective function value; (a) Scenario 1, (b) Scenario 2, (c) Scenario 3

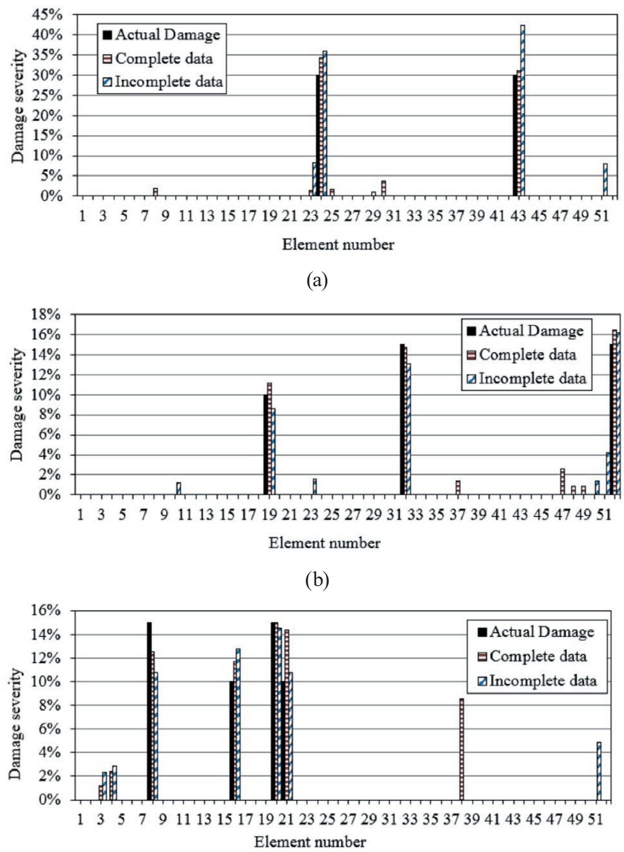


Fig. 13 Damage detection; (a) Scenario 1, (b) Scenario 2, (c) Scenario 3

achieve the optimum number of sensors. From GV, the optimal sensor placement is a projection of the elliptical noise on the side of the response change space. The degrees of freedom are sufficiently arranged according to decreasing order of the objective function values to place the sensors. In GVPSS, the optimum number of sensors is searched based on arranging DOFs obtained by GV. In GVPSS, the objective function is error in damage detection using parameter subset selection, which should be minimized. The GVPSS is tested on a 52-bar dome structure under static and dynamic loadings. Damages are detected in two states: 1) using the recorded responses of all DOFs, 2) using the recorded responses of DOFs corresponding to the optimal number and location of the sensors obtained by GVPSS. The results demonstrated that the number and location of the sensors obtained by GVPSS were optimum and damage detection using those locations was exact. Finally, the GVPSS algorithm can be utilized for the optimal number and location of sensors in structures.

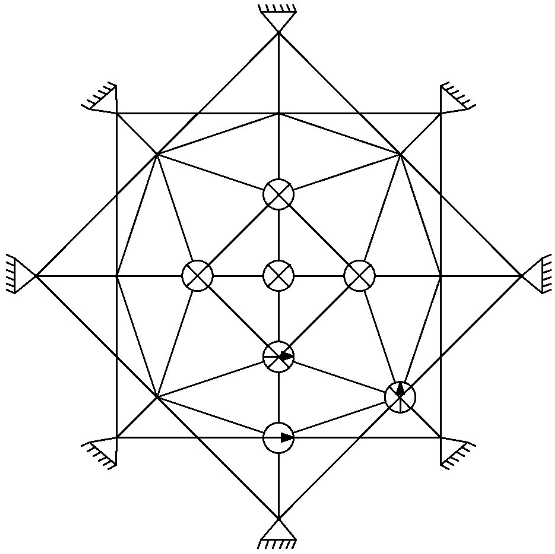


Fig. 14 Optimal location and number of sensors under dynamic loading (→, ↑ and × represents horizontal, vertical and downward directions, respectively)

References

[1] Ghiasi, P., Torkzadeh, P., Noori, M. "A machine-learning approach for structural damage detection using least square support vector machine based on a new computational kernel function", Structural Health Monitoring, 15, pp. 302–316, 2016. https://doi.org/10.1177/1475921716639587

[2] Homaei, F., Shojaee, S., Ghodrati, A. G. "A direct damage detection method using multiple damage localization index based on mode shapes criterion", Structural Engineering and Mechanics, 49(2), pp. 183–202, 2014. https://doi.org/10.12989/sem.2014.49.2.183

[3] Torkzadeh, P., Fathnejat, H., Ghiasi, R. "Damage detection of plate-like structures using intelligent surrogate model", Smart Structures and Systems, 18(6), pp. 1233–1250, 2016. https://doi.org/10.12989/sss.2016.18.6.1233

[4] Kaveh, A., Dadras, A. "Structural damage identification using an enhanced thermal exchange optimization algorithm", Engineering Optimization, 50(3), pp. 430–451, 2017. https://doi.org/10.1080/0305215X.2017.1318872

[5] Dinh-Cong, D., Vo-Duy, T., Ho-Huu, V., Dang-Trung, H., Nguyen-Thoi, T. "An effective multi-stage optimization approach for damage detection in plate structures", Advances in Engineering Software, 112, pp. 76–87, 2017. https://doi.org/10.1016/j.advengsoft.2017.06.015

[6] Khatir, S., Benaiessa, B., Capozucca, R., Wahab, M. A. "Damage detection in CFRP composite beams based on vibration analysis using proper orthogonal decomposition method with radial basis functions and cuckoo search algorithm", Composite Structures, 187, pp. 344–353, 2018. https://doi.org/10.1016/j.compstruct.2017.12.058

[7] Kaveh, A., Zolghadr, A. "An improved CSS for damage detection of truss structures using changes in natural frequencies and mode shapes", Advances in Engineering Software, 80, pp. 93–100, 2015. https://doi.org/10.1016/j.advengsoft.2014.09.010

[8] Khatir, S., Wahab, M. A., Boutchicha, D., Khatir, T. "Structural health monitoring using modal strain energy damage indicator coupled with teaching-learning-based optimization algorithm and isogoemetric analysis", Journal of Sound and Vibration, 448, pp. 230–246, 2019. https://doi.org/10.1016/j.jsv.2019.02.017

[9] Krishnanunni, C. G., Sethu Raj, R., Nandan, D., Midhun, C. K., Sajith, A. S., Ameen, M. "Sensitivity-based damage detection algorithm for structures using vibration data", Journal of Civil Structural Health Monitoring, 9, pp.137–151, 2019. https://doi.org/10.1007/s13349-018-0317-0

[10] Ghannadi, P., Kourehli, S. S., Noori, M., Altabey, W. A. "Efficiency of grey wolf optimization algorithm for damage detection of skeletal structures via expanded mode shapes", Advances in Structural Engineering, 23(13), pp. 2850–2865, 2020. https://doi.org/10.1177/1369433220921000

[11] Mishra, M., Barman, S. K., Maity, D., Maiti, D. K. "Ant lion optimization algorithm for structural damage detection using vibration data", Journal of Civil Structural Health Monitoring, 9, pp. 117–136, 2019. https://doi.org/10.1007/s13349-018-0318-z

[12] Baneen, U., Kausar, Z. "A baseline-free modal strain energy method for damage localization", International Journal of Civil Engineering, 16, pp. 607–618, 2017. https://doi.org/10.1007/s40999-017-0149-z

[13] Alexandrino, P. S. L., Gomes, G. F., Cunha, Jr., S. S. "A robust optimization for damage detection using multiobjective genetic algorithm, neural network and fuzzy decision making", Inverse Problems in Science and Engineering, 28, pp. 21–46, 2020. https://doi.org/10.1080/17415977.2019.1583225

[14] Blachowski, B. "Modal sensitivity based sensor placement for damage identification under sparsity constraint", Periodica Polytechnica Civil Engineering, 63(2), pp. 432–445, 2019. https://doi.org/10.3311/PPci.13888

[15] Liu, W., Gao, W., Sun, Y., Xu, M. "Optimal sensor placement for spatial lattice structure based on genetic algorithms", Journal of Sound and Vibration, 317, pp. 175–189, 2008. https://doi.org/10.1016/j.jsv.2008.03.026

[16] Papadimitriou, C. "Optimal sensor placement methodology for parametric identification of structural systems", Journal of Sound and Vibration, 278, pp. 923–947, 2004. https://doi.org/10.1016/j.jsv.2003.10.063

[17] Yi, T.-H., Li, H.-N., Gu, M. "Optimal sensor placement for health monitoring of high-rise structure based on genetic algorithm", Mathematical Problems in Engineering, 2011, Article ID 395101, 2011. https://doi.org/10.1155/2011/395101

[18] Shahbaznia, M., Dehkordi, M. R., Mirzaee, A. "An improved time-domain damage detection method for railway bridges subjected to unknown moving loads", Periodica Polytechnica Civil Engineering, 64(3), pp. 928–938, 2020. https://doi.org/10.3311/PPci.15813

[19] Talebpour, M. H., Goudarzi, Y., Sharifnezhad, M. "Clustering elements of truss structures for damage identification by CBO", Periodica Polytechnica Civil Engineering, 65(1), pp. 120–133, 2021. https://doi.org/10.3311/PPci.16636

[20] Yi, T.-H., Li, H.-N. "Methodology developments in sensor placement for health monitoring of civil infrastructures", International Journal of Distributed Sensor Networks, 8(8), Article ID 612726, 2012. https://doi.org/10.1155/2012/612726

[21] Dinh-Cong, D., Dang-Trung, H., Nguyen-Thoi, T. "An efficient approach for optimal sensor placement and damage identification in laminated composite structures", Advances in Engineering Software, 119, pp. 48–59, 2018. https://doi.org/10.1016/j.advengsoft.2018.02.005

[22] Li, J., Zhang, X., Xing, J., Wang, P., Yang, Q., He, C. "Optimal sensor placement for long-span cable-stayed bridge using a novel particle swarm optimization algorithm", Journal of Civil Structural Health Monitoring, 5, pp. 677–685, 2015. https://doi.org/10.1007/s13349-015-0145-4

[23] Hou, L.-Q., Zhao, X.-F., Han, R.-C. "Optimal sensor placement for stay cable damage identification of cable-stayed bridge under uncertainty", International Journal of Distributed Sensor Networks, 9(12), Article ID 361594, 2013. https://doi.org/10.1155/2013/361594

[24] Beygzadeh, S., Salajegheh, E., Torkzadeh, P., Salajegheh, J., Naseralavi, S. S. "An improved genetic algorithm for optimal sensor placement in space structures damage detection", International Journal of Space Structures, 29(3), pp. 121–136, 2014. https://doi.org/10.1260/0266-3511.29.3.121

[25] Wu, Z. Y., Zhou, K., Shenton, H. W., Chajes, M. J. "Development of sensor placement optimization tool and application to large-span cable-stayed bridge", Journal of Civil Structural Health Monitoring, 9, pp. 77–90, 2019. https://doi.org/10.1007/s13349-018-0320-5

[26] Chisari, C., Macorini, L., Amadio, C., Izzuddin, B. A. "Optimal sensor placement for structural parameter identification", Structural and Multidisciplinary Optimization, 55, pp. 647–662, 2017. https://doi.org/10.1007/s00158-016-1531-1

[27] Sadat Shokouhi, S. K., Vosoughifar, H. R. "Optimal sensor placement in the lightweight steel framing structures using the novel TTFD approach subjected to near-fault earthquakes", Journal of Civil Structural Health Monitoring, 3, pp. 257–267, 2013. https://doi.org/10.1007/s13349-013-0053-4

[28] Ostachowicz, W., Soman, R., Malinowski, P. "Optimization of sensor placement for structural health monitoring: a review", Structural Health Monitoring, 18(3), pp. 963–988, 2019. https://doi.org/10.1177/1475921719825601

[29] Gomes, G. F., Pereira, J. V. P. "Sensor placement optimization and damage identification in a fuselage structure using inverse modal problem and firefly algorithm", Evolutionary Intelligence, 13, pp. 571–591, 2020. https://doi.org/10.1007/s12065-020-00372-1

[30] Friswell, M. I., Mottershead, J. E., Ahmadian, H. "Combining subset selection and parameter constraints in model updating", Journal of Vibration and Acoustics, 120(4), pp. 854–859, 1998. https://doi.org/10.1115/1.2893911

[31] Naseralavi, S. S., Salajegheh, E., Salajegheh, J., Fadaee, M. J. "Detection of damage in cyclic structures using an eigenpair sensitivity matrix", Computers & Structures, 110–111, pp. 43–59, 2012. https://doi.org/10.1016/j.compstruc.2012.06.003

[32] Li, X. Y., Law, S. S. "Adaptive Tikhonov regularization for damage detection based on nonlinear model updating", Mechanical Systems and Signal Processing, 24, pp. 1646–1664, 2010. https://doi.org/10.1016/j.ymssp.2010.02.006

[33] Weber, B., Paultre, P., Proulx, J. "Consistent regularization of nonlinear model updating for damage identification", Mechanical Systems and Signal Processing, 23, pp. 1965–1985, 2009. https://doi.org/10.1016/j.ymssp.2008.04.011

[34] Wu, Y., Hu, D., Wu, M., Hu, X. "Unscented Kalman filtering for additive noise case: augmented versus nonaugmented", IEEE Signal Processing Letters, 12(5), pp. 357–360, 2005. https://doi.org/10.1109/LSP.2005.845592