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Evaluating the Modified Uncertainty MCMC Approach to Obtain Safety-Quality Index in PC Construction with Current Models

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Abstract

Precast Concrete Construction (*PCC*) is a famous method in construction industry due to its different advantages. Nevertheless, this method has several discontinuous processes and they can enhance various uncertainty safety-quality issues. In this system, Building Information Modeling (*BIM*) technique plays a key role in managing of discontinuous processes to improve multidimensional safety and quality. Dimensionless parameters were used to evaluate the modified type of multidimensional *BIM* index (I_{MBIM}) as the target index using the Modified Fuzzy Analytic Network Process (*MFANP*) method. In fact, the importance weight of input indices on the target was determined by the *MFANP* approach. The results of statistical analysis in verification step indicate that *MFANP* is well compatible with other approaches, so this developed method was considered as a reference point. The modified uncertainty Markov Chain Monte Carlo (*MCMC*) analysis was used for the validated method to determine the optimal high occurrence rate of I_{MBIM}. The results demonstrate that a high percentage of occurrence frequency for I_{MBIM} is in the range between 0.9 and 0.925. In this range, the mean optimal value of I_{MBIM} as a management measure for multidimensional problems is 0.912. Stakeholders in *PCC* projects can use the proposed high occurrence range to assess the quality-safety of construction and make appropriate decisions.

Keywords

precast concrete construction, building information modelling, safety-quality index, modified fuzzy ANP, Markov chain Monte Carlo

1 Introduction

Over the last few decades, demands for utilizing precast concrete construction (*PCC*) has been increased in construction industry because it offers many advantages including cleaner site, better quality, and safer site [1]. However, this innovative method has several discontinuous processes including designing, producing, transporting, storing, and installing that cause various problems for stakeholders. These problems including reworks, schedule delay [2], imperfection in strength, safety problems [3], and poor quality arise from uncertainty issues [4]. Therefore, developing an appropriate coordination between different processes [5] and carrying out uncertainty analysis play as key roles to execute the *PCC* projects successfully [4]. Quality is very important in *PCC* and it is associated with various uncertainties. Quality in *PCC* generally depends on the stakeholder performance, the quality of materials, and the quality of procedure implementation [6]. It was revealed that the quality in *PCC* is prone to several unpredictable events such as machine break down, emergence of cracks in the elements [4], improper curing of structural elements, and the surface roughness [6]. In addition to quality, safety in *PCC* and Lightweight Steel Framing (LSF) system is a significant issue that should be controlled and managed accurately [3, 7]. Vamberský stated that safety is a broad topic and occupational safety is affected by structural safety significantly [8]. The structural safety in *PCC* depends on various parameters including element flawlessness during transportation, connections, bearing capacity, geometric accuracy, and durability performance [3]. Moreover, error is one of the main factors that enhance the uncertainty safety-quality issues [9]. Numerous studies implied that human error is one of the significant reasons of structural failures [8]. Construction stage which involves various processes such as transportation, inventory and installation has more challenging issues than prefabrication stage in plant [10]. This stage can be affected by many factors

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including human errors, equipment failures, inadequate elements delivery [5], batching error, and cumulative error that should be eliminated to achieve an ideal quality in *PCC* [6]. It was also revealed that installation in *PCC* has several complicated procedures and its schedule is significantly influenced by human errors [5]. In addition, uncertainty issues lead to reworks which has costly and time-consuming procedures [11]. Reworks can be reduced significantly by tolerance management during different stages including fabrication, assembly, and installation. In fact, tolerance analysis should be undertaken to reduce the uncertainties in related to geometric and dimensional variability [10]. Li et al. [12] stated that a virtual environment can improve the data management to trace the progression of the project in real-time manner. *BIM* as a virtual tool can identify deficiencies in the early stages [2] and it was emphasized that *BIM* technique can minimize the uncertainty issues particularly in context of dimensional and geometric variability [13]. *BIM* technique can promote the collaboration between the different stakeholders, develop site safety, and decrease the reworks [13, 14]. In the past studies some researchers considered the utilizing of *BIM* technique in *PCC* [15] like He et al. [16] developed a program using *BIM* to enhance the geometry design of the elements, Darko et al. [2] proposed an integrated framework based on *BIM* technique to promote the *PCC* management, and Kim et al. [17] presented a hybrid approach based on *BIM* technique to assess the dimensional and surface quality of precast concrete elements reliably. The results of past studies show that there are problems and limitations for evaluating the safety and quality of precast concrete. Problems can be solved by analyzing the uncertainty of all input variables. In this study, the Modified Fuzzy Analytic Network Process (*MFANP*) approach was developed to determine the importance weight of input variables. Then a relation based on the *MFANP* method was proposed to manage projects using the *BIM* technique. In this regard, the modified uncertainty Markov Chain Monte Carlo (*MCMC*) method was applied to better decide on the safety and quality level of these structures.

2 Methodology

2.1 Governing equation

Quality and safety of *PCC* are influenced by various parameters and several studies have been argued about the importance of these factors during different processes to enhance the performance of *PCC*. Quality should be evaluated to consider the appearance, strength, and durability of *PCC* [6]. Structural safety and quality are closely correlated [8] and it was found from past studies that they cannot be measured directly [18]. This study considered several dimensionless indices based on Ashtiani Araghi and Vosoughifar's study [19] as the primary equation to evaluate the suitability of safety and quality of precast concrete structures. In this regard, a novel approach of *BIM* technique was considered to investigate the safety and quality of *PCC* concurrently as *MBIM* [19]. This approach is validated by evaluating the *BIM* dimensionless index (I_{MBIM}). I_{MBIM} can be determined by various factors as given in Eq. (1).

$$I_{MBIM} = f\left(I_{sq}, I_{zd}, I_{ea}, I_{re}, I_{se}, I_{bc}, I_{cs}\right) \quad (1)$$

In Eq. (1), I_{sq}, I_{zd}, I_{ea}, I_{re}, I_{se}, I_{bc}, and I_{cs} are indices of the system quality, zero defect level, engineering accuracy, stable processes based on random errors, stable processes based on systematic errors, bearing capability, and construction safety, respectively. The system quality index (I_{sq}) can be determined by Eq. (2).

$$I_{sq} = n.\left(I_{pq}{}^{\alpha}.I_{mq}{}^{\beta}.I_{bq}{}^{\gamma}\right) \quad (2)$$

In Eq. (2), I_{pq}, I_{mq}, and I_{bq} are project quality index, supply chain materials quality index, and building quality index, respectively. α, β, γ are weighting ratio that can be calculated by using fuzzy sets theory based on the error probabilities [19]. In this study they are considered 0.15, 0.25, 0.6, respectively. Various reports on *PCC* demonstrated that the suitable value of monitoring effectiveness (n) is 0.85. Tolerances analysis for evaluating acceptable appearance, bearing capacity, and feasibility play as a key role in *PCC* [6]. In this regard, zero defect level index (I_{zd}) and engineering accuracy index (I_{ea}) are proposed to consider the applicability of the precast element tolerances. I_{zd} and I_{ea} are calculated by statistical equations as given in Eqs. (3) and (4), respectively.

$$I_{zd} = \int_a^b f\left(x\right) d_x \quad (3)$$

In Eq. (3), a and b are the minimum and maximum tolerance range, respectively; $f(x)$ is probability density function.

$$I_{ea} = \frac{\Delta x}{2.t_{\alpha}.S_x} \quad (4)$$

In Eq. (4) Δx is specified limit based on ACI ITG-7-09 [20] and PCI MNL 116-99 [6]; S_x is a sample standard deviation; Inverse distribution of α level (t_{α}) can be determined based on elements' Degree of Freedom (*DOF*) and accuracy level (α). In this study, α was considered 0.01 to attain a desired accuracy in *PCC*. The stable processes based on systematic errors index (I_{se}) and the stable processes based on random errors index (I_{re}) were obtained with dividing the number of stable processes by the number of total processes based on systematic and random errors, respectively. In this study, the stable process is considered as a process that performs steadily over time. The bearing capacity index (I_{bc}) of precast structures can be investigated by division of actual bearing value to design bearing capacity value. In addition to bearing capacity, structural safety should be evaluated to ensure the reliability and robustness of the structures [8]. So, construction safety index (I_{cs}) was proposed in this study and it can be calculated with dividing the actual amount of construction safety by the amount of construction safety determined in the design stage. I_{zd} and I_{bq} are correlated [21] and their relation is given in Eq. (5).

$$I_{zd} = 1.368 I_{bq} - 0.314 \quad (5)$$

Finally, I_{MBIM} can be evaluated through the mentioned variables as input indices by Eq. (6).

$$I_{MBIM} = I_{sq}{}^{a} . I_{zd}{}^{b} . I_{ea}{}^{c} . I_{se}{}^{d} . I_{re}{}^{e} . I_{bc}{}^{f} . I_{cs}{}^{g}; \; I_{cs} \geq \left[I_{cs} \right], \; I_{bc} \geq \left[I_{bc} \right] \quad (6)$$

In Eq. (6), the pairwise comparison and fuzzy sets theory can be utilized to determine the weights of the variables (a, b, c, d, e, f, g).

2.2 Flowchart of solution

PCC has many discontinuous processes during its life cycle that depend on the design stage and when structures can provide services to the owner and stakeholders, so uncertainty becomes a major issue in this technology. This study proposed a holistic flowchart based on Markov Chain Monte Carlo method as illustrated in Fig. 1 to decrease challenges associated with unpleasant uncertainties in *PCC*. In this regard, the precast concrete structures which were used *BIM* technique in them were selected as case studies. The safety and quality of the projects were investigated by an appropriate questionnaire and the initial information was collected. Moreover, the importance weights of safety-quality variables were determined according to experts' opinion using Modified Fuzzy Analytic Network Process (*MFANP*). The value of inputs and target including I_{sq}, I_{zd}, I_{ea}, I_{se}, I_{re}, I_{bc}, I_{cs}, and I_{MBIM}, respectively were calculated according to the relations as given in Eqs. (2)–(6). Finally, the modified *MCMC* approach was applied to obtain the optimal value of each index with high occurrence frequency and consider whether the construction procedure is appropriate.

2.3 Obtaining the importance weights of variables

Several studies utilized multi-criteria decision making (*MCDM*) approaches to identify the relations between different variables. One of the most important *MCDM* approaches is Analytic Network Process (*ANP*) [22] which

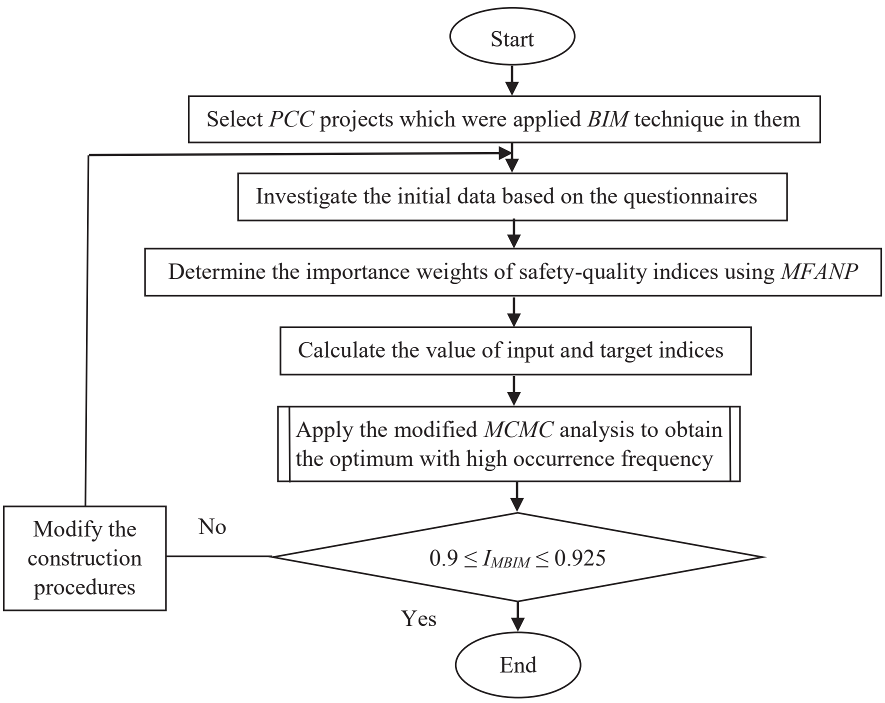

Fig. 1 Flowchart of research methodology

is obtained by developing Analytic Hierarchy Process (*AHP*) [23]. AHP can only solve the problem by considering hierarchical relation while *ANP* can also solve the problem by considering network relation [22]. Fuzzy *ANP* method was developed to eliminate the uncertainties that exist in human judgements [24]. In this study, the nine-scores with seven correlated fuzzy numbers were used to modify *ANP* method as *MFANP*. The *MFANP* approach was designed and developed to determine the importance weight of variables as a toolbox in MATLAB by authors of this paper. Total-influence fuzzy matrix was obtained based on expert's opinion and it was then normalized to gain fuzzy pair comparison matrix as given in Eq. (7). The expert's opinion was collected by using focus groups method which was first used as a research method in the 1940s [25].

$$C^{(r)} = \begin{bmatrix} 1 & \tilde{C}_{12}^{(r)} & \cdots & \tilde{C}_{1n}^{(r)} \\ \tilde{C}_{21}^{(r)} & 1 & & \tilde{C}_{2n}^{(r)} \\ \vdots & \vdots & & \vdots \\ \tilde{C}_{n1}^{(r)} & \tilde{C}_{n2}^{(r)} & \cdots & 1 \end{bmatrix} = \begin{bmatrix} 1 & \tilde{C}_{12}^{(r)} & \cdots & \tilde{C}_{1n}^{(r)} \\ \frac{1}{\tilde{C}_{12}^{(r)}} & 1 & & \tilde{C}_{2n}^{(r)} \\ \vdots & \vdots & & \vdots \\ \frac{1}{\tilde{C}_{1n}^{(r)}} & \frac{1}{\tilde{C}_{2n}^{(r)}} & \cdots & 1 \end{bmatrix} \tag{7}$$

In Eq. (7), $\tilde{C}_{ij}^{(r)}$ is effect of element i over element j and its relative importance is (L_{ij}, m_{1ij}, m_{2ij}, m_{3ij}, m_{4ij}, m_{5ij}, U_{ij}). Each element should be normalized based on Eq. (8).

$$\tilde{D}_i = \left[\frac{\sum_{j=1}^{n} l_{ij}}{\sum_{j=1}^{n} l_{ij} + \sum_{i=1}^{n}\sum_{j=1}^{n} u_{ij}}, \frac{\sum_{j=1}^{n} m_{t_{ij}}}{\sum_{i=1}^{n}\sum_{j=1}^{n} m_{t_{ij}}}, \frac{\sum_{j=1}^{n} u_{ij}}{\sum_{j=1}^{n} u_{ij} + \sum_{i=1}^{n}\sum_{j=1}^{n} l_{ij}} \right]$$
$$(t = 1 \rightarrow 5) \tag{8}$$

The linguistic series of *MFANP* approach which consists of 7 fuzzy number including l, m_1, m_2, m_3, m_4, m_5 and u is demonstrated in Fig. 2.

In this study, the fuzzy linguistic scale of *MFANP* as μ_{MFANP} was determined according to the statements as given in Eq. (9) to assign value to each linguistic variable.

$$\mu_{MFANP}(x) = \begin{Bmatrix} 0 & x < l \\ (x-l)/(m_1-l) & l \le x \le m_1 \\ (x-m_1)/(m_2-m_1) & m_1 \le x \le m_2 \\ 1 & m_2 \le x \le m_4 \\ (x-m_5)/(m_4-m_5) & m_4 \le x \le m_5 \\ (x-u)/(m_5-u) & m_5 \le x \le u \\ 0 & x \ge u \end{Bmatrix} \tag{9}$$

The possibility degree of a convex fuzzy number for each criterion can be defined by Eq. (10).

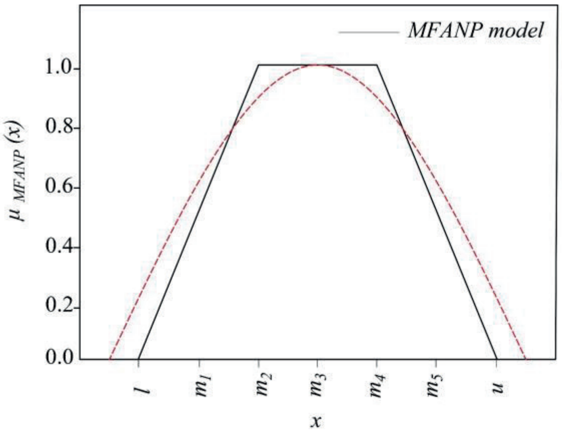

Fig. 2 The linguistic series of *MFANP* approach

$$W(\tilde{D}_i) = V(\tilde{D}_i \ge \tilde{D}_1, \tilde{D}_2, \ldots \tilde{D}_{i-1}, \tilde{D}_{i+1}, \ldots, \tilde{D}_n) = \min(\tilde{D}_i \ge \tilde{D}_k) \tag{10}$$

In Eq. (10), $V(\tilde{D}_i \ge \tilde{D}_k)$ is a degree of possibility with $\tilde{R}_i \ge \tilde{R}_k$ condition and the amount of k was considered as integer numbers between 1 to n. The normalized importance weight of each criterion is calculated by Eq. (11).

$$W_{MFANPi} = \frac{W(\tilde{D}_i)}{\sum_{i=1}^{n} W(\tilde{D}_i)} \tag{11}$$

2.4 Markov Chain Monte Carlo method

Monte Carlo is a repeating computational method and it is based on stochastics techniques. The problems which are complicated, nonlinear, and include uncertain parameters can be solved approximately by using Monte Carlo method [26]. Limited number of samples was the main imperfection of the experimental research, whereas Monte Carlo method performs by computer and can simulate the samples widely [27]. In fact, this great simulation tool can find out a close to reality solution for a complex problem [28]. In recent decades, Monte Carlo method has been developed broadly and has been applied to several fields including physics, medicine, material science, social science, and management science [29]. In this study, a modified uncertainty *MCMC* approach was applied as illustrated in Fig. 3 to determine the occurrence frequency of I_{MBIM} in different ranges. *MCMC* approach at first was recommended by Metropolis in 1954. Afterward, Hastings developed Metropolis's algorithm called Metropolis-Hastings (M-H) algorithm to converge the outcomes and it becomes the basis of *MCMC* approach [30]. The number of possible targets in *MCMC* approach is finite and the target in next sequence is only depends on the current sequence and it is independent from previous sequence [31].

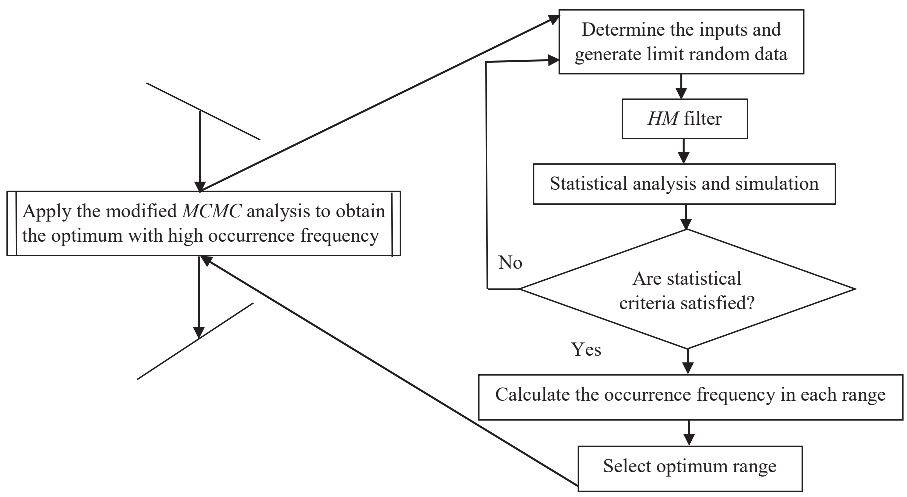

Fig. 3 The processes of applying modified MCMC analysis

In *MCMC* method, it is necessary to define an initial relation between inputs and target [32]. The *MFANP*-relation which depends on I_{sq}, I_{zd}, I_{ea}, I_{se}, I_{re}, I_{bc} and I_{cs} as input indices was used for a preliminary function. A set of input data should be sampled for a large number called N times [32] and this number should be selected according to the trial-error when there is no significant change in functional distribution. The range of each input index was defined based on the *PCC* projects data. Hidden Markov (*HM*) filtering was then applied to estimate the target optimally and predict uncertain targets [31]. The transition probability which only depends on the current distribution was performed as illustrated in Eq. (12) [30].

$$P\left(x^{(N)}|x^{(N-1)},\ldots,x^{(2)},x^{(1)}\right)=P\left(x^{(N)}|x^{(N-1)}\right) \tag{12}$$

In Eq. (12), $x^{(N-1)}$ and $x^{(N)}$ are the sample values in the current and next sequences, respectively; $P(x^{(N)}|x^{(N-1)})$ is the occurrence probability of $x^{(N)}$ and it is corresponded to the circumstances of $x^{(N-1)}$. The occurrence probability of each sequence can be evaluated based on its mean and variance as given in Eqs. (13) and (14), respectively [32, 33].

$$\bar{y}=\frac{1}{N}\sum_{i=1}^{N}y_i\,, \tag{13}$$

$$\sigma_y^2=\frac{1}{N}\sum_{i=1}^{N}\left(y_i-\bar{y}\right)^2\,, \tag{14}$$

where N, y_i, $\bar{y}$, and σ_y are the number of data points, the value of each sample, the mean of samples in each sequence, and the standard deviation of samples in each sequence, respectively. Appropriate sampling in Markov Chain can be attained by creating a stationary distribution. In fact, when the detailed balance distribution is satisfied as illustrated in Eq. (15) the chain is reversible and the mentioned condition can be achieved [30].

$$\pi\left(x^{(N)}\right)T\left(x^{(N-1)}|x^{(N)}\right)=\pi\left(x^{(N-1)}\right)T\left(x^{(N)}|x^{(N-1)}\right) \tag{15}$$

In Eq. (16), $\pi(x^{(N)})$ and $T(x^{(N)}|x^{(N-1)})$ are the probability of occurrence according to the current sequence and the probability matrix of transferring from sequence (N–1) to sequence (N). The mentioned relation in Eq. (15) allows the Markov chain to return to any sequences [30]. The proposed samples should be considered to control whether they are accepted according to the probability [30, 34] which is given in Eq. (16). Indeed, The Markov chain must converge to the balance distribution criteria in each element using M-H pattern [30].

$$\lambda=\begin{cases} if\ \pi\left(x^{(N)}\right)T\left(x^{(N-1)}\middle|x^{(N)}>0\right) & \min\left[1,\dfrac{\pi\left(x^{(N-1)}\right)T\left(x^{(N)}\middle|x^{(N-1)}\right)}{\pi\left(x^{(N)}\right)T\left(x^{(N-1)}\middle|x^{(N)}\right)}\right] \\ if\ \pi\left(x^{(N)}\right)T\left(x^{(N-1)}\middle|x^{(N)}\leq 0\right) & 1 \end{cases} \tag{16}$$

In Eq. (16) if the sample value is accepted the system will be updated ($x^{(N+1)} = x^{(N+1)}$), otherwise the next sample value takes the current sample value ($x^{(N+1)} = x^{(N)}$). When the simulation was finished, the occurrence frequency for each configuration in different ranges was determined. The acceptance probability value of λ is between [0, 1], and it must convert the results of the Markov chain to the balance condition. The optimal value with high frequency was selected for each input and target in different configurations. It should be noted that all the mentioned modified *MCMC* processes were performed by designing new toolbox in MATLAB.

2.5 Case studies

Many precast concrete structures have been built in different parts of the world. Several precast structures were chosen as case studies to evaluate the feasibility of proposed approach. The *BIM* technique was used for all construction processes during the entire project life cycle, which depends on the design phase and when the structures can serve the owner and stakeholders in the selected case studies. It is worth noticing that the demolition processes are not considered in the life cycle assessment of the projects. Fig. 4 shows the location of the case studies and 3D view of the structures [35–39]. Project No. 1 is called Parseh and is a five-story residential complex in Isfahan, Iran. This *PCC* building is in Baharestan, a new city located 20 km southeast of Isfahan and north of the Lashtar mountains along the Isfahan-Shiraz Road. Project No. 2 called Rafsanjan is a five-story municipal parking located in Rafsanjan, Iran. Project No. 3, CONRAC, is a five-story car rental building in Honolulu, Hawaii, located at Daniel K. Inouye international airport. Project No. 4, called Hale Mahana, is a 14-story dormitory located on South King Street, a short distance from the University of Hawaii at Manoa and the University of Chaminade in Honolulu, Hawaii. Project No. 5, called Fifteen Fifty, is a 39-story apartment located at the corner of Mission Street and south Van Ness Avenue in San Francisco, California. Projects No. 6 to 13 were selected from Baiburin's research [21] on concrete

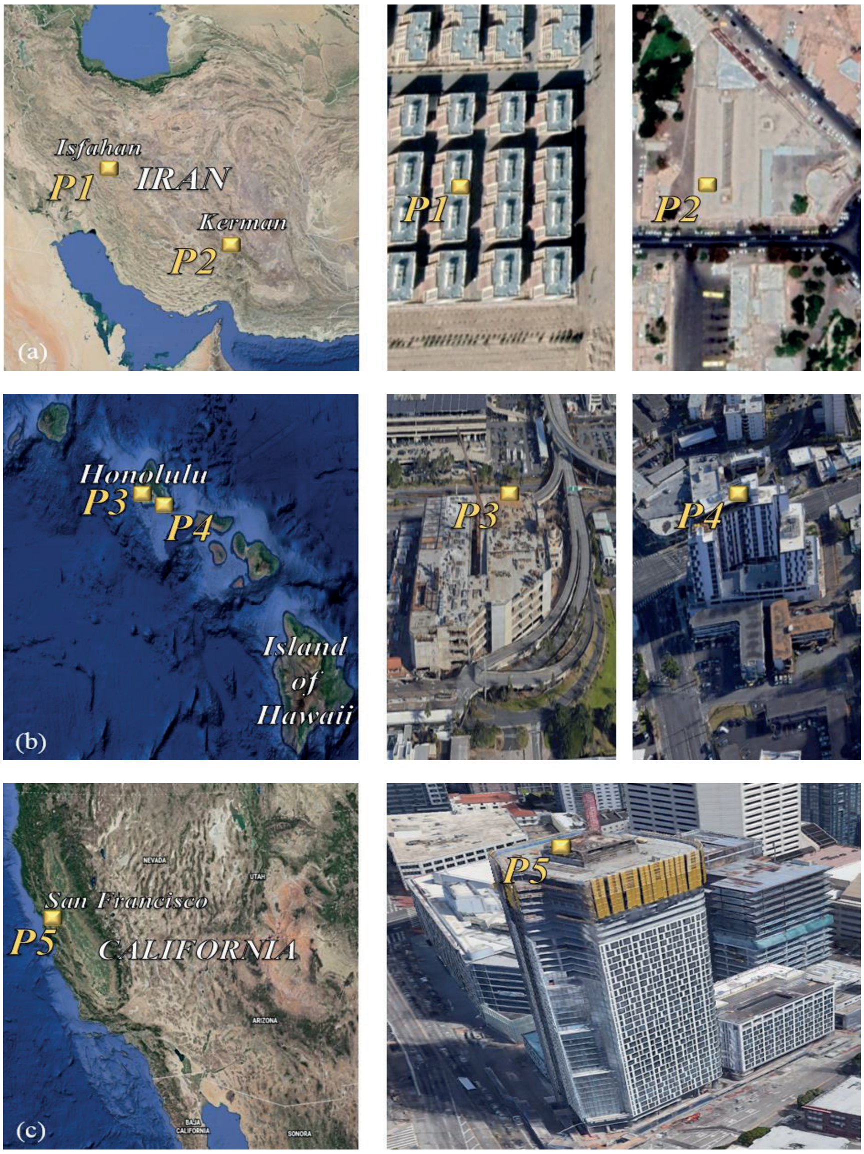

Fig. 4 Location of the case studies, (a) Location of projects 1 & 2 in Iran [35, 36], (b) Location of projects No. 3 & 4 in Hawaii [37, 38], (c) Location of project No. 5 in California [39]

structures. It should be noted that Several questions were asked from project managers to evaluate the indices including I_{sq}, I_{se}, I_{re}, I_{bc} and I_{cs}. The variation level of the elements from specified limit were checked both in manufacturing and installation stage to assess I_{zd} and I_{ea}. The result of evaluating I_{zd} and I_{ea} for all the precast concrete structures are given in Table 1. In Table 1, N and $\bar{x}$ are the number of the observed elements and the mean deviation value of the elements.

2.6 Overview on statistical method

Boxplot is one of the most important analytical tools and it can indicate the variation of data sets by proposing an illustrative diagram. A boxplot consists of a box that specifies the interquartile range and two whiskers. In fact, the box demonstrates the middle 50% of all data and it consists of 3 quartiles including Q1, Q2, Q3. The first, second, and third quartiles indicate the values that 25%, 50%, and 75% of data situated below them, respectively [40]. In other

Table 1 The results of tolerance control of the precast concrete elements

Controlled parameters of precast elements	*Unit*	Δx	N	$\bar{x}(\mu)$	$S_x(\sigma)$	I_{zd}	I_{ea}
Wall panel strength	*Psi*	-500, +0	610	258	105.33	0.89	0.98
Slab strength	*Psi*	-500, +0	140	376.2	126.2	0.84	0.73
Column strength	*Psi*	-500, +0	681	383.32	133.32	0.81	0.8
Beam strength	*Psi*	-500, +0	292	250.36	65.36	0.96	1.91
Deviation from specified location:							
Wall panel reinforcement position	*in.*	±1/4	610	0.04	0.107	0.91	0.89
Slab reinforcement position	*in.*	±1/4	560	0.03	0.117	0.87	0.82
Column reinforcement position	*in.*	±1/4	787	0.066	0.114	0.88	0.83
Beam reinforcement position	*in.*	±1/4	640	0.063	0.113	0.89	0.85
Wall panel depth	*in.*	±1/4	610	0.027	0.107	0.91	0.89
Slab depth	*in.*	±1/4	560	0.048	0.115	0.88	0.83
Column depth	*in.*	±1/4	681	0.069	0.117	0.87	0.80
Beam depth	*in.*	±1/4	356	0.063	0.123	0.85	0.77
Wall panel length	*in.*	±1/2	610	0.07	0.23	0.88	0.82
Slab length	*in.*	±1/2	560	0.1	0.233	0.87	0.82
Column length	*in.*	±1/2	681	0.148	0.248	0.84	0.75
Beam length	*in.*	±3/4	382	0.174	0.349	0.87	0.82
Wall panel width	*in.*	±1/4	610	0.042	0.122	0.85	0.78
Slab width	*in.*	±1/4	560	0.025	0.108	0.9	0.88
Column width	*in.*	±1/4	681	0.069	0.109	0.9	0.87
Beam width	*in.*	±1/4	347	0.048	0.113	0.89	0.85
Wall panel blockout's location	*in.*	±1	919	0.167	0.5	0.84	0.79
Slab blockout's location	*in.*	±2	300	0.2	0.867	0.89	0.91
Beam blockout's location	*in.*	±1	1354	-0.05	0.5	0.84	0.78
Blockout's size	*in.*	±1/2	2573	0.15	0.27	0.80	0.71
Plane surface between wall panel's embedments	*in.*	–1/4, + 0	610	-0.153	0.058	0.87	0.84
Local exposed surface roughness (per 10 ft.)	*in.*	1/4	510	0.12	0.05	0.93	0.97
Slab deviation (horizontal dimension)	*in.*	1	501	0.283	0.217	0.9	0.84
wall panel deviation (horizontal dimension)	*in.*	1/2	610	0.15	0.1	0.93	0.94
Column deviation (horizontal dimension)	*in.*	1/2	681	0.15	0.1	0.93	0.93
Beam deviation (horizontal dimension)	*in.*	1/2	355	0.238	0.263	0.82	0.74
Column deviation (vertical dimension)	*in.*	–1/2, +1/4	681	-0.28	0.155	0.86	1.04
Slab deviation (vertical dimension)	*in.*	±3/4	501	0.1	0.333	0.89	0.82
Wall panel deviation from joint width	*in.*	±3/8	610	0.087	0.167	0.88	0.89
Beam deviation from joint width	*in.*	±1/2	220	0.113	0.213	0.91	0.87

words, the Q1, Q2, and Q3 are 25th, 50th, and 75th quartiles, respectively [40–42]. The difference between the first and third quartiles represents the interquartile range (*IQR*). This means that 50% of the data lie between Q1 and Q3 [40, 42]. Another analysis was carried out by using Taylor method to compare the calculated and predicted data statistically. The statistical analysis including correlation coefficient (*R*), root-mean-square-differences (*RMSD*), and standard deviation are shown in a single diagram to demonstrate the degree of correspondence between different methods. The standard deviation proportional of a pattern, the *RMSD*, and the correlation coefficient are obtained by calculating, radial distance of the methods from origin, their distance apart proportional, and the azimuthal angle of the predicted method, respectively as given in Eqs. (17), (18) and (19) [43, 44].

$$\sigma_c = \sqrt{\frac{\sum_{i=1}^{N}\left(C_i - \bar{C}\right)}{N-1}};\ \sigma_P = \sqrt{\frac{\sum_{i=1}^{N}\left(P_i - \bar{P}\right)}{N-1}}, \tag{17}$$

$$RMSD = \left[\frac{1}{N}\sum_{i=1}^{N}\left(C_i - P_i\right)^2\right]^{1/2}, \tag{18}$$

$$R = \frac{\frac{1}{N}\sum_{i=1}^{N}\left(C_i - \bar{C}\right)\left(P_i - \bar{P}\right)}{\sigma_C . \sigma_P}, \tag{19}$$

where σ_c, C_i, $\bar{C}$, N, σ_p, P_i, and $\bar{P}$ are the standard deviation of calculated data, the value of each calculated data, the mean of all calculated data, the number of data points, the standard deviation of predicted data, the value of each predicted data, and the mean of all predicted data, respectively.

3 Result

3.1 *MFANP* results

The *MFANP* model was used to determine the weight of each input variable on the target. The importance weights of the indices (i.e., I_{sq}, I_{zd}, I_{ea}, I_{se}, I_{re}, I_{bc} and I_{cs}) were determined of 0.2, 0.153, 0.129, 0.096, 0.099, 0.1614, and 0.1611, respectively. The importance weights were assigned to *a*, *b*, *c*, *d*, *e*, *f*, and *g*, respectively. The super decision matrix obtained from the *MFANP* method is given in Table 2.

Preliminary information was reviewed based on project manager's opinion through the interview process. The collected data were used to determine the input indices including I_{sq}, I_{zd}, I_{ea}, I_{se}, I_{re}, I_{bc} and I_{cs}. The target index (i.e., I_{MBIM}) was then calculated based on the input indices and their importance weight as shown in Eq. (6). The input and target value of the case studies are summarized in Table 3.

3.2 Statistical comparison

The statistical result between different methods for obtaining I_{MBIM} is shown in Fig. 5. This figure illustrates boxplot analysis of *MFANP*, genetic expression programming (*GEP*) [19], fuzzy decision making trial and evaluation laboratory (*FDEMATEL*) [19], and Baiburin [21] methods to identify their accuracy. The midpoint of the distribution is called median and marked with a red line inside the box. The first and third quartiles of the distribution are shown with the bottom and top edges of the box, respectively. The whiskers, which extends from the bottom of the box, are below and above the 5th and 95th percentile, respectively. The outliers indicated by the upper and lower circles of the whiskers are minimum and maximum of data sets, respectively.

It is clear from Fig. 5 that there is no significant difference between *MFANP*, *GEP*, and *FDEMATEL* methods for I_{MBIM} prediction. However, there is no good agreement between Baiburin's study and other methods. The lower, upper and middle bands indicate that the *MFANP* method is compatible with the calculated method, so this approach was considered as a reference point in another statistical analysis. The boxplot indicates that the minimum value of

Table 2 Super matrix between inputs and target

	I_{MBIM}	I_{sq}	I_{zd}	I_{ea}	I_{se}	I_{re}	I_{bc}	I_{cs}
I_{MBIM}	0	0.034483	0.017241	0.017241	0.017241	0.017241	0.034483	0.034483
I_{sq}	0.155172	0	0.137931	0.155172	0.12069	0.12069	0.155172	0.155172
I_{zd}	0.12069	0.103448	0	0.12069	0.103448	0.12069	0.12069	0.103448
I_{ea}	0.103448	0.086207	0.068966	0	0.051724	0.103448	0.12069	0.137931
I_{se}	0.086207	0.103448	0.086207	0.086207	0	0.051724	0.068966	0.051724
I_{re}	0.086207	0.086207	0.068966	0.086207	0.017241	0	0.086207	0.103448
I_{bc}	0.068966	0.155172	0.12069	0.137931	0.051724	0.068966	0	0.137931
I_{cs}	0.12069	0.137931	0.12069	0.12069	0.068966	0.086207	0.137931	0

Table 3 The value of the dimensionless parameter for case studies

No. of the projects	Case study	The values of indices for the case studies.							
		I_{sq}	I_{zd}	I_{ea}	I_{se}	I_{re}	I_{bc}	I_{cs}	I_{MBIM}
P1	Parseh	0.74	0.86	0.84	0.95	0.85	1.1	1.1	0.908
P2	Rafsanjan	0.75	0.88	0.93	0.95	0.88	1.1	1.1	0.927
P3	CONRAC	0.74	0.92	0.98	0.73	0.85	1.1	1.1	0.911
P4	Hale Mahana	0.73	0.85	0.76	0.9	0.98	1.14	0.98	0.889
P5	Fifteen Fifty	0.75	0.88	0.88	0.98	0.98	1.1	1.04	0.925
P6	Sel P6	0.66	0.66	1	0.5	0.07	1	0.98	0.619
P7	Sel P7	0.79	0.82	1	0.6	0.6	1.06	1	0.846
P8	Sel P8	0.79	0.76	0.59	0.86	0.79	1.03	0.99	0.825
P9	Sel P9	0.79	0.81	0.96	0.64	0.73	1.02	0.99	0.855
P10	Sel P10	0.65	0.66	0.6	0.9	0.9	1.02	1.11	0.805
P11	Sel P11	0.69	0.7	0.96	0.75	0.75	1	0.962	0.822
P12	Sel P12	0.79	0.77	1	0.25	0.38	1.03	0.97	0.729
P13	Sel P13	0.79	0.88	1	0.64	0.6	1.01	1.04	0.859

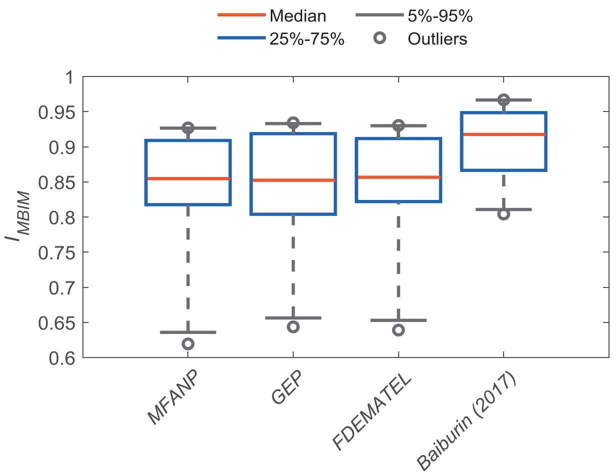

Fig. 5 The result of boxplot analysis using different methods to calculate I_{MBIM}

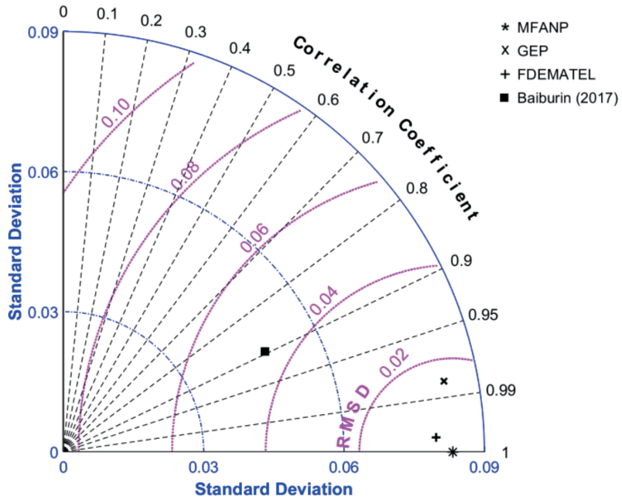

Fig. 6 Taylor diagram of I_{MBIM} with *MFANP* value as reference point

I_{MBIM} for *MFANP* method is different from other methods because of 2 reasons: The first reason is that the weight of I_{re} (i.e., *e*) is 0.099 for *MFANP*, while it is 0.086 for *FDEMATEL* [19] and the second reason is the low value of I_{re} in the project No. 6. So, according to the mentioned reasons the minimum values are different. Further comparative analysis was performed with Taylor criteria to evaluate the results. Fig. 6 shows the Taylor diagram for comparison of *MFANP*, *GEP* [19], *FDEMATEL* [19] and Baiburin [21] methods. In this diagram, the predicted I_{MBIM} value of the *MFANP* method was considered as a reference value for comparison with other methods.

The results of Taylor diagram show that the standard deviation of I_{MBIM} from *MFANP*, *GEP*, *FDEMATEL*, Baiburin's study are 0.083, 0.082, 0.08 and 0.048, respectively. The overall results of *MFANP*, *GEP* and *FDEMATEL* show that there are a low RMSD of 0, 0.015, and 0.004, and high RMSD of 0.046 for Baiburin's study. The correlation coefficients for *MFANP*, *GEP*, *FDEMATEL* are 1, 0.983, 0.999, respectively, while for the Baiburin's study it is 0.895. Thus, *MFANP* can be used as a reliable approach to calculate I_{MBIM}, but it is better to use uncertainty analysis to achieve the optimal value with a high occurrence rate.

3.3 The results of uncertainty analysis

Four configurations of random sets were defined in *MCMC* analysis. Fig. 7(a) and (b) show configuration number 1 with 144 random sets in two-dimensional and three-dimensional views, respectively. Configuration No. 2 with 484 random sets is shown in Fig. 7(c) and (d). Fig. 7(e) and (f) show con-

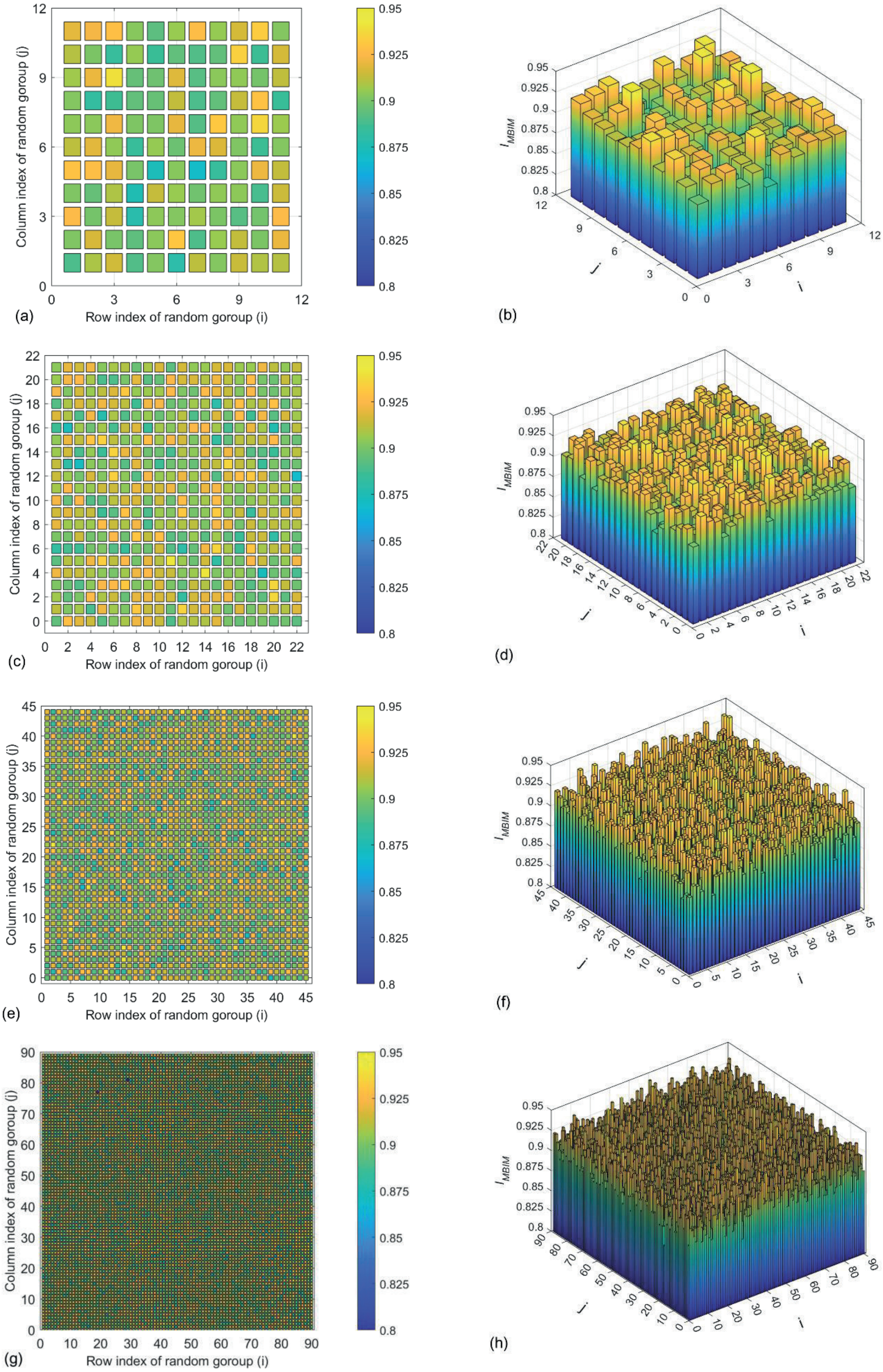

Fig. 7 2D and 3D view of uncertainty analysis; (a, b) Configuration No. 1 with 144 random sets, (c, d) Configuration No. 2 with 484 random sets, (e, f) Configuration No. 3 with 2025 random sets, (g, h) Configuration No. 4 with 8100 random sets

figuration number 3 with 2025 random sets. Configuration No. 4 with 8100 random sets is shown in Fig. 7(g) and (h). It should be noted that each set of configurations has random values including I_{sq}, I_{zd}, I_{ea}, I_{se}, I_{re}, I_{bc} and I_{cs} as input values for calculating I_{MBIM} based on the *MFANP* equation.

Table 4 summarizes the value and percentage of occurrence in the intended ranges using *MCMC* stochastic series analysis.

The results of Table 4 show the occurrence frequency of *MCMC* analysis in specific limitation. These results indicate that the maximum frequency of occurrence in the range A and E are1.65 and 0.02, respectively, so these ranges can be ignored. The frequency of occurrence in configuration 1 for ranges B, C and D are 29.75%, 57.85% and 10.74%, respectively. These results are 26.45%, 64.67% and 8.06% for configuration 2 and 28.89%, 60.44 and 9.98 for configuration 3, respectively. In addition, the percentages for configuration 4 are set to 28.90%, 60.62% and 10.04%, respectively. The maximum frequency of occurrence in range C was obtained for all configurations. In fact, the priority of frequency of occurrence in the considered range based on *MCMC* analysis is C, B, D, A and E. The optimal values of input and target indices including I_{sq}, I_{zd}, I_{ea}, I_{se}, I_{re}, I_{bc}, I_{cs} and I_{MBIM} are given in Table 5.

From Table 5 it can be concluded that the mean value ranges for I_{sq}, I_{zd}, I_{ea}, I_{se}, I_{re}, I_{bc}, and I_{cs} are 0.740, 0.886, 0.879, 0.865, 0.918, 1.121, and 1.044, respectively. Another result from this table indicates that the mean of I_{MBIM} value is 0.912.

4 Discussion

PCC technique was taken by stakeholders and project managers as an efficient method to increase the quality of construction. However, there is several problems due to discontinuous processes of this system including factory activities, transportation, inventory, assembly, and installation. Therefore, it is necessary to evaluate a comprehensive uncertainty analysis between discontinuous processes. The review of past studies indicate that most researchers focused on benefits of this system instead of evaluating a comprehensive criterion. A few numbers of researchers presented one dimensional uncertainty analysis for *PCC* structures. Meanwhile, Kim et al. [45], Arashpour et al. [9], and Zhai et al. [4] researched on uncertainties of schedule, on-site and off-site processes, and logistics problem, respectively. In this study, a governing comprehensive model was selected to perform uncertainty analysis. Ashtiani Araghi and Vosoughifar [19] found that the

Table 4 Number (percentage) of occurrence frequency for different configurations

Configuration	Range				
	A (0.85–0.875)	B (0.875–0.9)	C (0.9–0.925)	D (0.925–0.95)	E (0.95–0.975)
1	2 (1.65%)	36 (29.75%)	70 (57.85%)	13 (10.74%)	0 (0.00%)
2	4 (0.83%)	128 (26.45%)	313 (64.67%)	39 (8.06%)	0 (0.00%)
3	14 (0.69%)	585 (28.89%)	1224 (60.44%)	202 (9.98%)	0 (0.00%)
4	34 (0.42%)	2341 (28.90%)	4910 (60.62%)	813 (10.04%)	2 (0.02%)

Table 5 Optimal value of indicators from *MCMC* analysis

Configuration	Range	I_{sq}	I_{zd}	I_{ea}	I_{se}	I_{re}	I_{bc}	I_{cs}	I_{MBIM}
1	B	0.74	0.884	0.818	0.802	0.903	1.12	1.015	0.891
	C	0.74	0.883	0.881	0.851	0.916	1.122	1.042	0.910
	D	0.741	0.889	0.929	0.915	0.936	1.129	1.073	0.931
2	B	0.739	0.878	0.823	0.816	0.907	1.117	1.026	0.893
	C	0.74	0.886	0.887	0.864	0.915	1.122	1.042	0.912
	D	0.74	0.892	0.928	0.929	0.934	1.124	1.069	0.931
3	B	0.74	0.88	0.821	0.811	0.902	1.118	1.025	0.893
	C	0.74	0.886	0.886	0.862	0.918	1.120	1.042	0.912
	D	0.74	0.897	0.935	0.927	0.932	1.122	1.062	0.931
4	B	0.739	0.88	0.823	0.812	0.904	1.118	1.025	0.893
	C	0.74	0.886	0.883	0.866	0.918	1.120	1.043	0.912
	D	0.741	0.893	0.938	0.925	0.934	1.122	1.064	0.931

PCC quality can be evaluated by a comprehensive model called I_{MBIM}. They proposed an integrated approach based on *FDEMATEL* and *GEP* which consist of several important indices. This comprehensive model was selected as initial relation and *MFANP* approach was used to modify this model. The verification process of new model was carried out with boxplot and Taylor analysis. The results of verification indicate that the modified model has better accuracy in comparison with *GEP*, *FDEMATEL*, and Baiburin's study. Consequently, the achieved equation was applied to find the optimum value with high occurrence frequency of I_{MBIM} based on modified uncertainty *MCMC* approach.

5 Conclusions

The uncertainty analysis for range consideration demonstrates that the high occurrence frequency of I_{MBIM} is in the range of 0.9 to 0.925. The overall results show that the mean percentage of the occurrence frequency for all configurations in governing range is 60.90%. The results obtained from this study can be used in the construction industry as a management issue to consider the multidimensional factor in decision making. Therefore, managers can use the optimum range with high frequency to evaluate the quality-safety of construction and make a suitable decision based on the proposed approach. In other words, if the value of indicators in a project is less than the optimal values of *MCMC* analysis, the construction type should be modified according to the project policy.

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