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Probabilistic Slope Stability Evaluation Using Hybrid Metaheuristic Approach

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Abstract

This paper develops an efficient evolutionary hybrid optimization technique based on the adaptive salp swarm algorithm (ASSA) and pattern search (PS) for the reliability evaluation of earth slopes considering spatial variability of soils under the framework of the limit equilibrium method. In the ASSA, to improve the salp swarm approach's exploration ability while also avoiding premature convergence, two new equations for the leaders' and followers' position updating procedure are introduced. The proposed hybrid algorithm (ASSPS) benefits from the effective global search ability of the adaptive salp swarm algorithm as well as the powerful local search capability of the pattern search method. The suggested ASSPS algorithm's efficiency is confirmed using mathematical test functions, and its findings are compared with the standard salp swarm algorithm as well as some efficient optimization techniques. Then, the ASSPS is applied for calculation of the lowest safety factor and reliability index of earth slopes. The safety factor is formulated using the Morgenstern and Price approach and the advanced first-order second-moment (AFOSM) method is implemented for the reliability calculation model. The ASSPS's efficacy for the evaluation of the minimum reliability index of slopes is investigated by considering two literature-based case studies. The numerical experiments demonstrate that the new algorithm could generate better optimal solutions and significantly outperform other methods in the literature.

Keywords

slope reliability, probability analysis, metaheuristic, hybrid approach

1 Introduction

During the lifetime of a structure, uncertainties in geometric and material characteristics, as well as external forces, are recognized. As a result, recent decades have seen a rising recognition that, for meaningful study of engineering structures, uncertainty must be included, and structural reliability theory provides a useful approach for doing so [1]. Reliability-based design optimization of frame structures [2, 3], reliability-based design optimization of composite structures [4], reliability-based geometrically nonlinear topology optimization of L-shape beam and U-shaped plate [5], pile foundation design under reliable conditions [6], reliability-based design optimization of offshore wind turbine [7], and reliability-based design optimization of bridges [8] are some examples of considering uncertainties in the optimum design of civil engineering problems. The demand is greater in geotechnical engineering because natural materials used to build dams, slopes, and retaining walls have more and higher uncertainties than other structures.

Slope failure is a common cause of fatalities and property damage. As a result, geotechnical engineers must determine the minimal slope safety factor and find the critical slip surface [9, 10]. The stability is often expressed in terms of factor of safety (FoS) in a standard deterministic slope stability testing and variables are represented by single values in this approach. However, because to the uncertainty in the input parameters (i.e., soil properties), assessing the slope safety with a single FoS value is difficult. Several studies have revealed that two nearly equivalent slopes with nearly similar FoS calculated from

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a deterministic approach might have noticeably differing failure probability due to uncertainty in geotechnical parameters and failure causes [11, 12]. This issue highlights the necessity for an even more objectively organized and quantitative method to deal with the computations' uncertainties. The probabilistic technique is a logical option for this sort of research since it allows for the direct insertion of uncertainties into the analysis. In this approach, instead of the traditional FoS, the safety of a slope is assessed by the probability of failure or the reliability index. As Duncan [13] presented, the reliability analysis is a good alternative to traditional stability assessments since the resulting reliability index provides more information than the deterministic FoS.

The Mean-Value First-Order Second-Moment (MFOSM) technique is a popular method for estimating the reliability index of earth slope [14]. In this approach, the performance function is extended for the mean values of the input variables, the first order terms are just retained, and the partial derivative of the performance function is required. Hasofer and Lind [15] provided an independent description of the reliability index to avoid the reliability index's dependency on the performance function. In this method, the reliability index was defined as the minimum distance from the origin of the standard normal space to the boundary limit state. The reliability assessment of an earth slope using the Hasofer-Lind reliability index (β_{HL}) can be formulated as an optimization problem and the solution of this problem is the lowest reliability index or highest probability of failure.

This optimization challenge may be solved using either traditional deterministic or newer metaheuristic optimization techniques. The factor of safety function is usually multimodal and complex due to variable soil qualities, ground conditions, and external forces. When the search space comprises numerous local minima and the computational complexity environment is high, traditional deterministic techniques fail to provide a feasible solution. Metaheuristic algorithms, on the other hand, have simple notions and structures, derivation-free methods, and are successful for both continuous and discrete functions. Accordingly, several research efforts have been attempted to implement various metaheuristic strategies for slope stability estimation based on these benefits.

Although metaheuristic approaches can produce acceptable results, no method is superior than another at solving all optimization issues. As a result, various research projects have indeed been carried out to improve the performance and efficiency of the initial metaheuristic algorithms and apply them to complex engineering challenges. Kaveh et al. [16] used a non-dominated sorting evolutionary algorithm to solve a multi-objective optimized design of structural steel structures while taking into account the initial cost and seismic damage costs. For determining the critical failure mode in slope stability evaluation, Li and Wu [17] suggested an enhanced slap swarm optimization. For automatic selection of orthophoto mosaic seamline network, Wang et al. [18] suggested a modified ant colony algorithm. Eslami et al. [19] introduced enhanced versions of particle swarm optimization for power system stabilization. For pile group foundation design, Chan et al. [20] employed a hybrid algorithm to develop an automatic optimal design technique for pile group foundations. Khajehzadeh et al. [21, 22] proposed modified versions of gravity search algorithm for reducing embedded emissions of CO2 and total cost of foundation and retaining wall.

Proposing novel optimization strategies to solve real-world issues, as evidenced by the literature analysis, is highly desirable. The Salp Swarm Algorithm (SSA) is a recently created bioinspired meta-heuristic optimization strategy simulates salp fish swarming in deep waters [23]. Compared to other metaheuristic algorithms, some advantages of SSA are as follows [24]: mixing with other algorithms is strangely satisfying; SSA has good convergence acceleration; is suitable for many kinds of optimization problems; has higher feasibility and efficiency in producing global optima; less likelihood of getting stuck in local optima; less reliance on initial solutions; reasonable execution time; and a few parameter tuning. However, similar to other metaheuristic approaches, the SSA can be expanded or hybridized with another algorithm to produce better solutions for future problems [17, 25]. This paper presents an adaptive salp swarm optimization algorithm by introducing new position updating equations for leaders and followers' salp (ASSA). This change significantly improves the algorithm's performance and convergence speed.

A balance of exploitation and exploration must be maintained throughout the search operation to achieve optimum performance utilizing any optimization technique. Because ASSA is a global search strategy, it searches a large area and may not provide the best result if used alone. Search engine techniques, such as Pattern Search (PS), take advantage of the local but can perform a comprehensive search. There is scope for hybridizing these methods due to their separate capabilities.

In light of the foregoing, a combination adaptive salp swarm and pattern search technique known as ASSPS was created and is being used in the current work to find the minimum reliability index of an earth slope. The objective function is modelled using the Hasofer-Lind reliability index (β_{HL}) and a new approach of the Morgenstern-Price method to find the most critical probabilistic failure surface of slope. The performance of the proposed ASSPS approach is assessed by comparing its findings in the two literature-based benchmark problems to those of other alternative techniques. The proposed method for computing the lowest FoS and reliability index delivers more effective performance than existing methods, as proven by the quantitative evaluation.

The remainder of this paper is organized as follows. In Section 2, the formulation of the safety factor of an earth slope is presented. A probabilistic slope stability evaluation procedure has been developed in Section 3. The proposed hybrid optimization algorithm is presented in Section 4. Section 5 conducts a comparative analysis of ASSPS on a large test environment. In Section 6, a numerical investigation of probabilistic slope stability analysis using the ASSPS algorithm is conducted. Finally, the conclusions of the study and suggestions for further research are presented in Section 7.

2 Safety factor formulation

Geotechnical engineering includes seismic performance of earth slopes, especially in seismic zones [26]. To assess the stability of an earth slope, many traditional approaches are used, such as finite elements, strength reduction, and the limit equilibrium [27], is the most widely used analytical method for geotechnical problems, and it evaluates the factor of safety (FOS) using Mohr's coulomb criteria. Several procedures of analysis based on the limit equilibrium technique are available, which Duncan [28] has thoroughly reviewed and summarized. The simple or basic methods, like the ordinary method of slices and the Bishop method, are relevant to a particular shape of slip surface, whereas the rigorous methodologies, such as Spencer an Morgenstern and Price method, are applicable to any shape of failure surface. Determining the exact behavior of the soil slope becomes more complex when earthquake loads are applied. As a result, an effective pseudo-static approach can be used to assess the reliability of the earth's steep hills under earthquake stresses. The Morgenstern and Price method of slices, as well as the pseudo-static approach, were adjusted for seismological slope stability analysis in this study.

Morgenstern and Price [29] established a holistic and rigorous method for general form failure surfaces that fulfils both the force as well as the moment equilibrium. In order to accept the seismic load inside the pseudo-static assessment, an inertial force (F_h) is applied at the center of each slice in the horizontal direction, which can be computed by:

$$F_h = \left(W \times a_h / g\right) = W \times k_h \,, \tag{1}$$

in which k_h stands for horizontal acceleration coefficient, a_h stands for lateral ground motions and g stands for gravity acceleration. The Morgenstern and Price (MaP) model are used to analyze the safety factor under seismic load in this study. The M–P method divides slippery mass into a number of vertical segments, just like other limit equilibrium techniques. Take into account the forces acting on a standard slice of a slope, as shown in Fig. 1, with the overall shape of the slope.

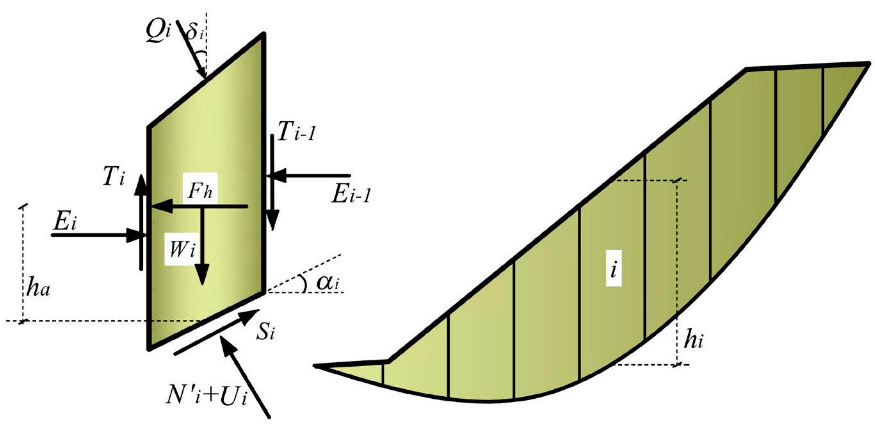

Fig. 1 Forces acting on a typical slice, W: wight of the slice, S: shear strength, N: effective normal force, U: resultant water force, E: normal inter-slice force, T: shear inter-slice force, h: average height of the slicem b: width of the slice, h_a: height of the slice center, F_h: horizontal seismic force, α:inclination of slice base, Q: external surcharge

The scaling factor λ and *FOS* are two unknown factors in the MaP method, which are deduced from moment and vertical force equilibriums [29]. Because of the complexity of the obtained equations, evaluating the *FOS* and λ is often complicated. In order to solve the aforementioned challenges, Zhu et al. [30] developed a concise version of the MaP approach. In this concise method, the inclination of a resultant inter-slice force varies symmetrically over the slide mass, and the relationship between the shear (T) and normal (E) inter-slice forces is offered as:

$$T = f(x).\lambda.E \; , \tag{2}$$

where $f(x)$ denotes the assumed inter-slice force function and λ denotes the scaling factor.

The following is a detailed description of the *FOS* evaluation procedure:

Step 1. Create a trial slip surface and divide it into n vertical segments

Step 2. Determine R_i and T_i using the equations below:

$$R_i = \left[W_i \cos\alpha_i - F_h \sin\alpha_i + Q_i \cos\left(\delta_i - \alpha_i\right) - U_i\right] \times \tan\varnothing_i' + c_i' b_i \sec\alpha_i , \tag{3}$$

$$T_i = W_i \sin\alpha_i + F_h \cos\alpha_i - Q_i sin\left(\delta_i - \alpha_i\right) . \tag{4}$$

Step 3. Select the function for inter-slice forces. A unit inter-slice function ($f(x) = 1$) is assumed here.

Step 4. Choose *FOS* and λ initial values according to the following criteria:

$$FOS > -\frac{\sin\alpha_i - \lambda f_i \cos\alpha_i}{\cos\alpha_i + \lambda f_i \sin\alpha_i} \tan\varnothing' . \tag{5}$$

The *FOS* and λ should be set to 1 and 0, respectively, as their initial values [30].

Step 5. Evaluate Φ_i and Ψ_{i-1} based on Eqs. (6) and (7).

$$\Phi_i = \left(\sin\alpha_i - \lambda f_i \cos\alpha_i\right)\tan\varnothing_i' + \left(\cos\alpha_i + \lambda f_i \sin\alpha_i\right) \times FOS \tag{6}$$

$$\Psi_{i-1} = \left[\begin{array}{l} \left(\sin\alpha_i - \lambda f_{i-1} \cos\alpha_i\right)\tan\varnothing_i' \\ +\left(\cos\alpha_i + \lambda f_{i-1} \sin\alpha_i\right) \times FOS \end{array}\right] / \, \Phi_{i-1} \tag{7}$$

Step 6. Calculate *FOS* based on Eq. (8).

$$FOS = \frac{\sum_{i=1}^{n-1}\left(R_i \prod_{j=1}^{n-1} \Psi_j\right) + R_n}{\sum_{i=1}^{n-1}\left(T_i \prod_{j=1}^{n-1} \Psi_j\right) + T_n} \tag{8}$$

Step 7. Compute Φ_i and Ψ_{i-1} and calculate *FOS* again by repeating steps 5 and 6.

Step 8. Define E_i and λ based on the Eqs. (9) and (10).

$$E_i \Phi_i = \Psi_{i-1} E_{i-1} \Phi_{i-1} + FOS \times T_i - R_i \tag{9}$$

$$\lambda = \frac{\sum\left[b_i\left(E_i + E_{i-1}\right)\tan\alpha_i + F_h h_i + 2Q \sin\delta_i h_i\right]}{\sum\left[b_i\left(f_i E_i + f_{i-1} E_{i-1}\right)\right]} \tag{10}$$

Step 9. Reevaluate *FOS* by the computed λ and the iterative process is completed when the distinction between the computed *FOS* and λ becomes lower than 0.005 and 0.01, respectively.

3 Probabilistic slope stability approach

Generally, the deterministic method's safety factor is not a consistent measure of safety since numerous uncertainties are not taken into account. As a result, the probabilistic approach has been proposed as an alternate tool for determining earth slope safety in which different soil parameter uncertainties may be sensibly incorporated. In probabilistic analysis, to assess the safety of a slope, the reliability index (β) or the probability of failure (P_f) are used. The failure-safety status of a slope may be described in a probabilistic analysis by the performance function $G(\boldsymbol{X})$, and $\boldsymbol{X} = [X_1, X_2, X_3, \ldots, X_n]$ represents the vector of random variables of an earth slope.

The performance function $G(\boldsymbol{X})$ or limit state function divides the vector space X into two distinct areas: the safety zone, denoted by $G(\boldsymbol{X}) > 0$, and the failure region, denoted by $G(\boldsymbol{X}) < 0$, with the limit state surface denoted by $G(\boldsymbol{X}) = 0$. In general, the slope's factor of safety (*FoS*) determines the performance function, which is defined as:

$$G(\boldsymbol{X}) = FoS(\boldsymbol{X}) - 1 . \tag{11}$$

Using the system's limit state function, the probability of failure of the slope (P_f) may be calculated by the following integral equation:

$$P_f = P\left[G\left(\boldsymbol{X} \le 0\right)\right] = \int_{G(\boldsymbol{X}) \le 0} f_X\left(\boldsymbol{X}\right) dX \; , \tag{12}$$

where $f_X(\boldsymbol{X})$ denotes the vector of random variables' joint probability density function, and the integral is performed over the failure domain.

The reliability index (β) was created to assess a system's comparative reliability when the actual probability distribution function is unknown. Hasofer and Lind [15] introduced an invariant way to calculating the reliability index known as the advanced first order second moment approach (AFOSM).

In this method, using the mean value (μ_i) and standard deviation (σ_i) the random variables (x_i) are converted into a normalized and uncorrelated set of reduced variables z based on the following equation:

$$z_i = \frac{x_i - \mu_i}{\sigma_i}, \tag{13}$$

where z_i denotes a normalized variable with a mean of zero and a standard deviation of one. Based on the transformation of Eq. (13), the mean value point in the original space (***X***-space) is mapped into the origin of the normal space (***Z***-space) as presented in Fig. 2 [31]. The failure surface $G(\boldsymbol{X}) = 0$ in X-space is mapped into the corresponding failure surface $G(\boldsymbol{Z}) = 0$ in ***Z***-space. The Hasofer and Lind (HL) reliability index (β_{HL}) is defined as the shortest distance in the normalized coordinate system from the origin of the normalized fundamental variables to the limit state function $G(\boldsymbol{X})$.

The matrix formulation of the Hasofer-Lind reliability index (β_{HL}) is presented as follows [32]:

$$\beta_{HL} = \min_{X \in F} \sqrt{\left[\frac{x_i - \mu_i}{\sigma_i}\right]^T \left[\boldsymbol{R}\right]^{-1} \left[\frac{x_i - \mu_i}{\sigma_i}\right]}, \tag{14}$$

where F represents the failure domain and $\boldsymbol{R}$ represents the correlation matrix. $\boldsymbol{R}$ is a square matrix that contains the correlations among a set of n random variables and defined by $\boldsymbol{R} = [\rho_{ij}]$ $(i,j = 1,2,\ldots,n)$. Although the correlation coefficient among two random variables has a range $-1 < \rho_{ij} < 1$, the correlation matrix cannot be assigned any value within this range. It should be noted that the correlation matrix must be positive definite. It must be emphasized that the correlation matrix has to be positive definite [33].

The search for the lowest reliability index ($\beta_{\min}$) may be expressed as an optimization problem based on the following equation:

$$\begin{aligned} &\text{Minimize } \beta_{HL} \\ &\text{Subject to } G(\boldsymbol{Z}) = 0, \end{aligned} \tag{15}$$

where $G(\boldsymbol{Z})$ is the normalized coordinate system limit state function. the reliability index and most probable failure surface are the results of the aforesaid optimization task.

4 Adaptive salp swarm – pattern search

4.1 Salp swarm algorithm

A salp is a type of marine organism in the Salpidae family. It has a cylindrical structure with apertures at the ends, similar to jellyfish, which move and eat by pumping water through internal feeding filters in their gelatinous bodies. The salp swarm algorithm (SSA), a population-based optimization technique, was developed by Mirjalili et al. [23]. The salp chain can be used to calculate the SSA's behavior while hunting for optimal feeding sources (i.e., the target of this swarm is a food position in the search space called FP). To mathematically model salp chains, the sample into two groups: leaders and followers. The salp at the head of the chain is known as the leader, while the others are known as followers. The swarm is led by the leader of these salps, and the followers follow in his footsteps. The chain begins with a leader, who is followed by the followers to guide their movements.

Similar to other swarm-based algorithms, Salps' location is specified in a n–dimensional search space, where n is the number of variables in a given problem. As a result, the positions of all salps are recorded in a two-dimensional matrix known as X, as shown in Eq. (16).

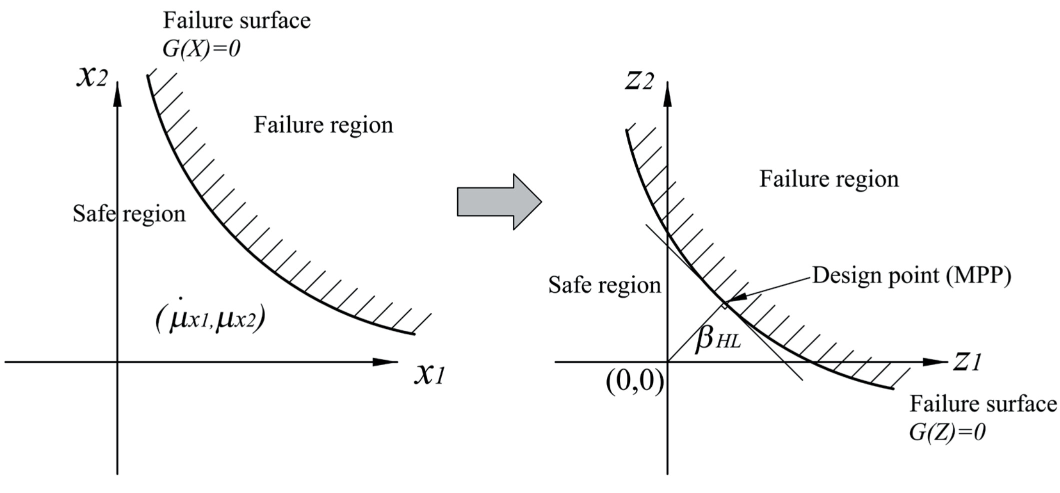

Fig. 2 The geometrical definition of β_{HL}

$$X_i = \begin{bmatrix} x_1^1 & x_2^1 & \dots & x_d^1 \\ x_1^2 & x_2^2 & \dots & x_d^2 \\ \vdots & \vdots & \dots & \vdots \\ x_1^n & x_2^n & \dots & x_d^n \end{bmatrix} \quad (16)$$

The fitness of each salp is then determined in order to determine which salp has the best fitness. It's also assumed that the swarm's target is a food position called *FP* in the search space.

The following equation can be used by the leader salp to change positions:

$$x_i^1 = \begin{cases} FP_i + r_1\left(\left(ub_i - lb_i\right)r_2 + lb_i\right) & r_3 \geq 0 \\ FP_i - r_1\left(\left(ub_i - lb_i\right)r_2 + lb_i\right) & r_3 < 0 \end{cases}, \quad (17)$$

where x_i^1 denotes the first salp's position in the *i*th dimension and FP_i denotes the food position in the *i*th dimension. The lower and upper bounds of the *i*th dimension are represented by lb_i and ub_i, respectively, and the coefficient r_1 is calculated by Eq. (18). The random numbers r_2 and r_3 are between 0 and 1.

$$r_1 = 2e^{-\left(\frac{4t}{t_{\max}}\right)^2} \quad (18)$$

The maximum number of iterations is $t_{\max}$ is the current iteration is t. It's worth noting that the r_1 coefficient is critical in SSA because it balances exploration and exploitation throughout the search. The following equations are used to change the position of the followers.

$$x_i^j = \frac{1}{2}\left(x_i^j + x_i^{j-1}\right), \quad (19)$$

where $j \geq 2$. In case some salps move outside of the search space, Eq. (20) shows how to bring salps back into the search space if they leave it.

$$x_i^j = \begin{cases} lb_i & if\ x_i^j \leq lb_i \\ ub_i & if\ x_i^j \geq ub_i \\ x_i^j & otherwise \end{cases} \quad (20)$$

The pseudocode of SSA is shown in Algorithm 1.

Algorithm 1 Salp swarm algorithm

```
Initialize the salp population x_i (i = 1, 2, ..., n) considering lb_i and ub_i
while t ≤ t_max
   Calculate the fitness of each search agent (salp)
   Put the best search agent as FP (Food position)
   Update r_1 by Eq. (18)
      for each salp (x_i)
         if i==1
            Update the position of the leading salp by Eq. (17)
         else
            Update the position of the follower salp by Eq. (19)
         end
      end
   Amend the salps based on the upper and lower bounds of variables
Calculate the fitness of each search agent FP
Update the food position
t = t + 1
end
return the food position FP and its best fitness
```

4.2 Adaptive salp swarm algorithm

In spite of the SSA's aptitude to generate effective outcomes in contrast to other famous algorithms, it is prone towards becoming stuck in a local optimum, making it unsuitable for very complex problems with multiple local optima.

As observed in Eq. (17), the leading salp modifies its position in SSA in accordance with the availability of food. At each generation, the SSA approach adjusts the leader salp's location around a single point, and additional salps follow the leader. Because it doesn't know the food position (FP), the procedure will fail if it is unable to recover. In other words, when an approach converges, it stops being able to find new objects and goes dormant. This technique causes the SSA algorithm to become unreachable at locally optimal points. Given these facts, an adaptive SSA (ASSA) is suggested to fix the aforementioned issue while also enhancing the algorithm's flexibility and search ability.

In the proposed ASSA, the performance and exploring capabilities are enhanced by considering half of the population as leader and the other salps as followers. The following equation is then used to update the position of the leader salps:

$$x_i^j = \begin{cases} x_i^j + r_1\left(FP_i - x_i^j\right) & r_3 \geq 0.5 \\ x_i^j - r_1\left(FP_i - x_i^j\right) & r_3 < 0.5 \end{cases}. \quad (21)$$

The leaders adjust their positions in response to the state of the food source as well as their previous position, as shown in Eq. (21).

This procedure encourages exploration while also allowing the SSA algorithm to conduct a more powerful global search across the entire search space. To improve the proposed ASSA's search efficiency, the followers will update their positions according to the following equation:

$$x_i^j = rand^2\left(x_i^j + x_i^{j-1}\right). \quad (22)$$

In addition, in the suggested ASSA, at each iterative process, the worst salp with the highest objective function value will be replaced with a completely random salp. The flowchart of the proposed ASSA algorithm is shown in Fig. 3.

4.3 Pattern search (PS)

PS is a gradient-free approach for fine-tuning local search that can be easily implemented. The PS method produces a group of locations that may or may not be near to the ideal point [34]. A mesh (a combination of elements) is formed around an existing element in the first round. In the next round, if a new element in the mesh has a smaller fitness, it becomes the current element.

The PS commences the investigation with a user-defined initial location P_0. The mesh level is taken as 1 in the first round, and the pattern elements are generated as $[0\ 1] + P_0$, $[1\ 0] + P_0$, $[-1\ 0] + P_0$ and $[0\ -1] + P_0$, and novel mesh elements are added as depicted in Fig. 4. The fitness function is then computed for each created sample element until a value less than P_0 is discovered. The poll is successful if there is such an element ($f(P_1) < f(P_0)$), and the PS method assumes this element as basis point. The method doubles the existing mesh size by 2 (called the expanding factor) after a successful poll and moves on to the second round with the following new elements: $2 \times [0\ 1] + P_1$, $2 \times [1\ 0] + P_1$, $2 \times [-1\ 0] + P_1$ and $2 \times [0\ -1] + P_1$. Then, P_2 is established if a value less than P_1 is discovered, the mesh size is expanded by two, and iterations proceed.

The current element is not modified and the mesh size is decreased by a contraction factor if the poll is unsuccessful at any round. These steps were continued until the lowest value was reached or the termination criteria were fulfilled.

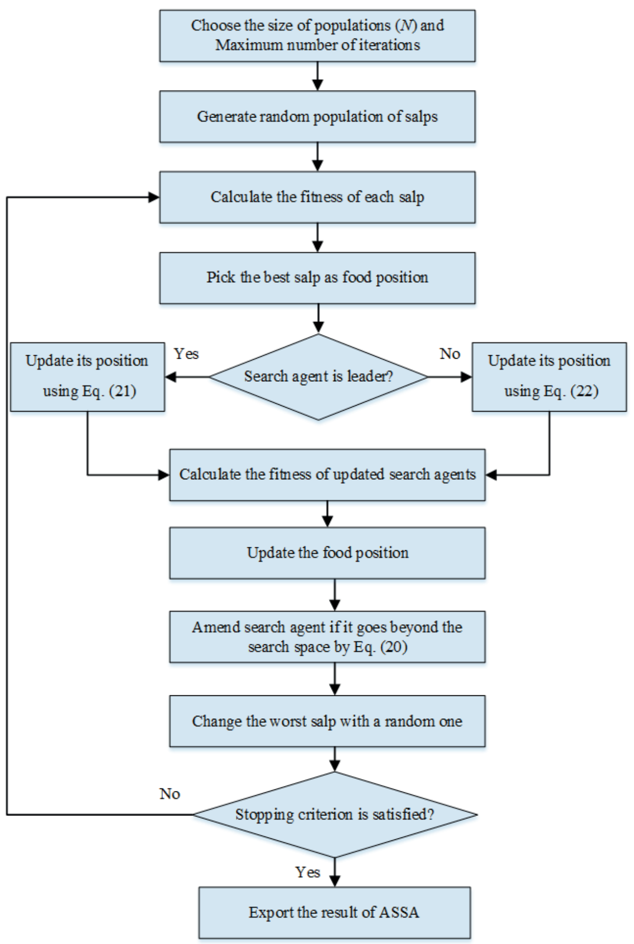

Fig. 3 Flowchart of ASSA

4.4 Adaptive salp swarm - pattern search

A hybrid approach is one that solves the same issue by combining two or more methods. Hybridization aims to integrate the benefits of each method to improve the result's accuracy [35].

In the current research, the adaptive salp swarm - pattern search (ASSPS) approach, which combines adaptive salp swarm algorithm and pattern search methods is developed. The adaptive salp swarm algorithm (ASSA) presented in Section 4.2 is a global optimization technique that investigates the solution space effectively and, is likely to provide an optimum or near-optimal solution. As a result, it may be used in conjunction with local optimization approaches such as pattern search. Pattern search is useful for exploiting a small area, but it is rarely useful for exploring a larger area. The suggested hybrid approach may take the ASS's powerful global searching capabilities as well as the PS algorithm's strong local searching capabilities. The global optimum performance of adaptive salp swarm algorithm (ASSA) is excellent, and it's easy to get out of local minima. The ASSA can increase the precision of the results by raising the number of iterations.

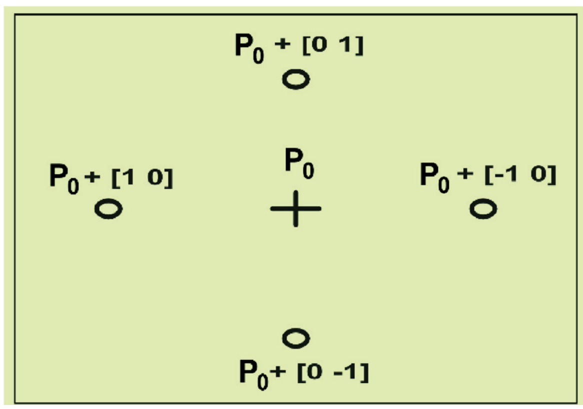

Fig. 4 Pattern Search mesh elements

When the number of generations is great enough, however, ASSA is unable to enhance the results' accuracy. As a result, ASSA's local search capability remains poor. Pattern search is a local optimization methodology, and the beginning point has a significant impact on the algorithm's output, with different initial points resulting in significant differences in the outcomes. However, if a great starting point is chosen, pattern search will be a simple and effective strategy. In this study, we successfully combine the benefits of ASSA as a global optimization and pattern search as a local optimization to identify the best answer. Because the PS is dependent to the first solution, the suggested hybrid method starts with the ASSA. The ASSA is used to keep searching for a certain number of iterations. The PS is then enabled to do a local search utilizing ASSA's best solution as an initial point. Fig. 5 shows the process flow of the suggested hybrid algorithm.

5 Model verification

A set of numerical reference test functions has been used in this section to compare and confirm the achievement and effectiveness of the proposed adaptive salp swarm - pattern search (ASSPS). In the empirical evidence literature, these functions are commonly used to determine the performance of optimizers [36].

The mathematical model and characteristics of these test functions are shown in Tables 1 and 2. This standard set is divided into two categories: unimodal functions with a single global best for testing algorithm convergence pace and enslavement ability, and multi - modal functions with multiple local minimums and a global ideal for testing an algorithm's local optima avoidance and exploratory capacity. MATLAB R2020b was used to create the suggested algorithms. All of these functions, should be minimized. Furthermore, all functions have a dimension of 30.

The proposed ASSPS is compared to the original SSA as well as some well-known optimization methods such as Particle Swarm Optimization (PSO), Firefly Algorithm (FA), Multi-Verse Optimizer (MVO), Tunicate Swarm Algorithm (TSA), and Sand Cat Optimization (SCO). For all methodologies, the size of solutions (N) and the maximum number of iterations number (t_{max}) are set to 30 and 1000, respectively, in order to make a fair comparison between them.

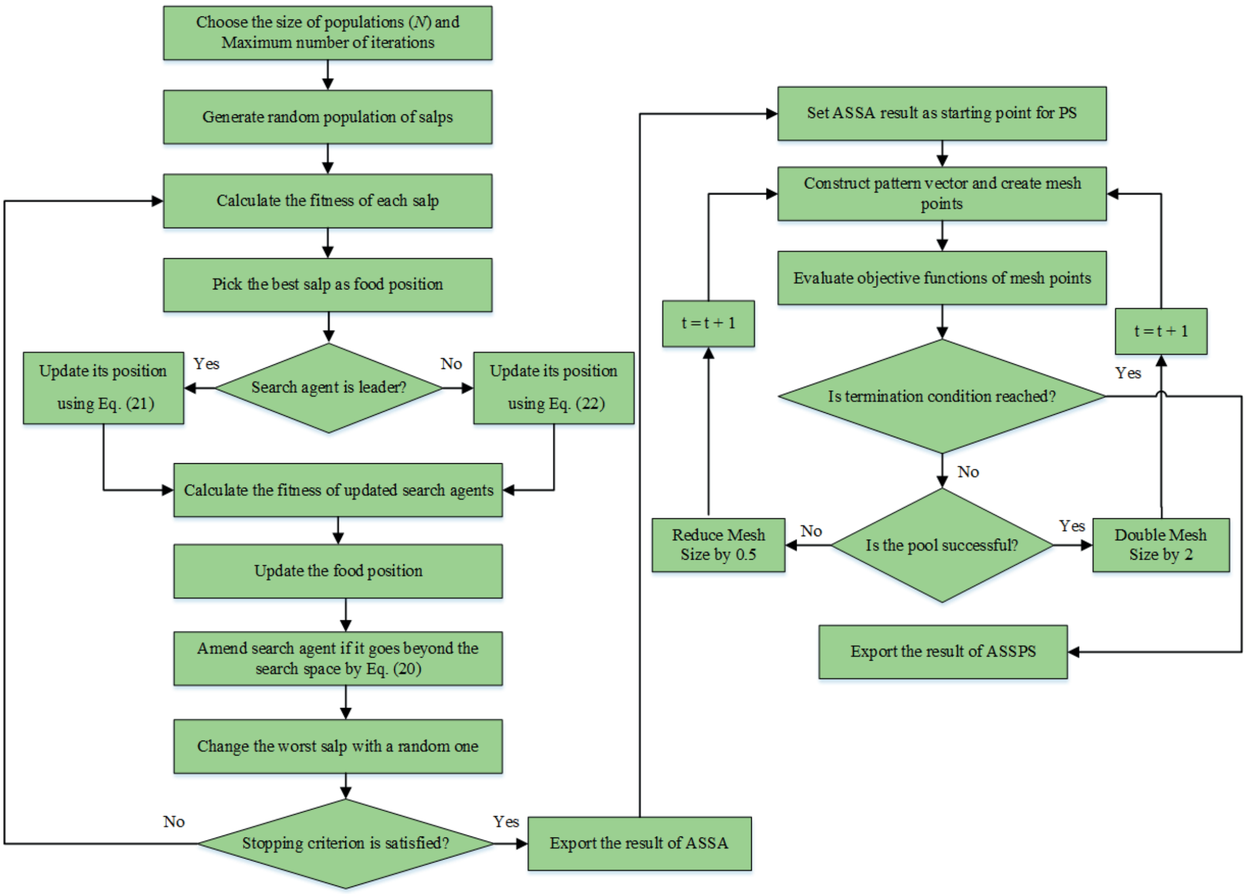

Fig. 5 Hybrid adaptive salp swarm pattern search algorithm

Table 1 Unimodal benchmark functions

Function	Range	$f_{\min}$	n (Dim)
$F_1(X)=\sum_{i=1}^{n} x_i^2$	$[-100, 100]^n$	0	30
$F_2(X)=\sum_{i=1}^{n} \lvert x_i\rvert+\prod_{i=1}^{n} \lvert x_i\rvert$	$[-10, 10]^n$	0	30
$F_3(X)=\sum_{i=1}^{n}\left(\sum_{j=1}^{i} x_j\right)^2$	$[-100, 100]^n$	0	30
$F_4(X)=\max_i\left\{\lvert x_i\rvert, 1\le i\le n\right\}$	$[-100, 100]^n$	0	30
$F_5(X)=\sum_{i=1}^{n-1}\left[100\left(x_{i+1}-x_i^2\right)^2+\left(x_i-1\right)^2\right]$	$[-30, 30]^n$	0	30
$F_6(X)=\sum_{i=1}^{n}\left(\left[x_i+0.5\right]\right)^2$	$[-100, 100]^n$	0	30
$F_7(X)=\sum_{i=1}^{n} i x_i^4+random[0,1)$	$[-1.28, 1.28]^n$	0	30

Table 2 Multimodal benchmark functions

Function	Range	$f_{\min}$	n (Dim)
$F_8(X)=\sum_{i=1}^{n}-x_i\sin\left(\sqrt{\lvert x_i\rvert}\right)$	$[-500, 500]^n$	$428.9829\times n$	30
$F_9(X)=\sum_{i=1}^{n}\left[x_i^2-10\cos\left(2\pi x_i\right)+10\right]$	$[-5.12, 5.12]^n$	0	30
$F_{10}(X)=-20\exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n}x_i^2}\right)-exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos\left(2\pi x_i\right)\right)+20+e$	$[-32, 32]^n$	0	30
$F_{11}(X)=\frac{1}{4000}\sum_{i=1}^{n}x_i^2-\prod_{i=1}^{n}\cos\left(\frac{x_i}{\sqrt{i}}\right)+1$	$[-600, 600]^n$	0	30
$F_{12}(X)=\frac{\pi}{n}\left\{10\sin\left(\pi y_1\right)+\sum_{i=1}^{n-1}\left(y_i-1\right)^2\left[1+10\sin^2\left(\pi y_{i+1}\right)\right]+\left(y_n-1\right)^2\right\}+\sum_{i=1}^{n}u\left(x_i,10,100,4\right)$ $y_i=1+\frac{x_{i+4}}{4}$ $u\left(x_i,a,k,m\right)=\begin{cases}k\left(x_i-a\right)^m & x_i>a\\ 0 & a<x_i<a\\ k\left(-x_i-a\right)^m & x_i<-a\end{cases}$	$[-50, 50]^n$	0	30
$F_{13}(X)=0.1\left\{\sin^2\left(3\pi x_1\right)+\sum_{i=1}^{n}\left(x_i-1\right)^2\left[1+\sin^2\left(3\pi x_i+1\right)\right]+\left(x_n-1\right)^2\left[1+\sin^2\left(2\pi x_n\right)\right]\right\}$ $+\sum_{i=1}^{n}u\left(x_i,5,100,4\right)$	$[-50, 50]^n$	0	30

Because the results of a single run of metaheuristic methods are stochastic, they may be incorrect. As a result, statistical analysis should be performed in order to provide a fair comparison and evaluate the algorithms' efficacy. To address this issue, 30 times runs for the mentioned methods are performed, with the results presented in Tables 3 and 4.

Tables 3 and 4 show that, for all functions, ASSPS might provide better solutions in terms of mean value of the objective functions than conventional SSA as well as other optimization techniques.

The results also show that the mean and standard deviation of the ASSPS algorithm are significantly lower than those of the other strategies, indicating that the algorithm is stable. ASSPS outperforms both the standard method and alternative optimization approaches, according to the findings.

Table 3 Results comparison of unimodal test functions

F	Index	ASSPS	SSA	SCO	PSO	FA	MVO	TSA
F_1	Mean	**0.00 E+00**	3.29E-07	2.42 E–97	4.98E–09	7.11E–03	2.81E–01	8.31E–56
	Std.	**0.00 E+00**	5.92E–07	7.22 E–97	1.40E–08	3.21E–03	1.11E–01	1.02E–58
F_2	Mean	**0.00 E+00**	1.9111	1.16 E–52	7.29E–04	4.34E–01	3.96E–01	8.36E–35
	Std.	**0.00 E+00**	1.6142	2.55 E–52	1.84E–03	1.84E–01	1.41E–01	9.86E–35
F_3	Mean	**0.00 E+00**	1.50E+03	7.84 E–81	1.40E+01	1.66E+03	4.31E+01	1.51E–14
	Std.	**0.00 E+00**	707.05	3.49 E–80	7.13E+00	6.72E+02	8.97E+00	6.55E–14
F_4	Mean	**0.00 E+00**	2.44E–05	4.57 E–46	6.00E–01	1.11E–01	8.80E–01	1.95E–05
	Std.	**0.00 E+00**	1.89E–05	9.98 E–46	1.72E–01	4.75E–02	2.50E–01	4.49E–04
F_5	Mean	**8.22E–08**	136.56	2.80 E+01	4.93E+01	7.97E+01	1.18E+02	28.4E+00
	Std.	**5.78E–09**	154.00	8.73 E–01	3.89E+01	7.39E+01	1.43E+02	0.842
F_6	Mean	**0.00 E+00**	5.72E–07	2.15 E+00	6.92E–02	6.94E–03	2.02E–02	3.67E+00
	Std.	**0.00 E+00**	2.44E–07	4.47 E–01	2.87E–02	3.61E–03	7.43E–03	0.3353
F_7	Mean	**2.39E–05**	8.82E–05	1.51 E–04	8.94E–02	6.62E–02	5.24E–02	0.0018
	Std.	**3.65E–05**	6.94E–05	1.33 E–04	0.0206	4.23E–02	1.37E–02	4.62E–04

Table 4 Results comparison of multimodal test functions

F	Index	CSCPS	SSA	SCO	PSO	FA	MVO	TSA
F_8	Mean	**–1.25E+04**	–7.46E+03	–1.01 E+04	–6.01E+03	–5.85E+03	–6.92E+03	–7.89E+03
	Std.	**0.00 E+00**	634.67	1.70 E+03	1.30E+03	1.61E+03	9.19E+02	599.26
F_9	Mean	**0.00 E+00**	55.45E+00	**0.00** E+00	4.72E+01	1.51E+01	1.01E+02	151.45
	Std.	**0.00 E+00**	18.27E+00	**0.00**E+00	1.03E+01	1.25E+01	1.89E+01	35.87
F_{10}	Mean	**8.88E–16**	2.84E+00	**8.77E–16**	3.86E–02	4.58E–02	1.15E+00	2.409
	Std.	**0.00 E+00**	6.58 E–01	**0.00** E+00	2.11E–01	1.20E–02	7.87E–01	1.392
F_{11}	Mean	**0.00 E+00**	2.29 E–01	**0.00** E+00	5.50E–03	4.23E–03	5.74E–01	0.0077
	Std.	**0.00 E+00**	1.29 E–01	**0.00** E+00	7.39E–03	1.29E–03	1.12E–01	0.0057
F_{12}	Mean	**2.57E–32**	6.82E+00	1.25E–01	1.05E–02	3.13E–04	1.27E+00	6.373
	Std.	**3.88E–48**	2.72E+00	5.41E–02	2.06E–02	1.76E–04	1.02E+00	3.458
F_{13}	Mean	**2.35E–32**	21.31E+00	1.99E+00	4.03E–01	2.08E–03	6.60E–02	2,897
	Std.	**3.95E–48**	16.99E+00	2.51E–01	5.39E–01	9.62E–04	4.33E–02	0.643

6 Model application

The feasibility and reliability of the suggested approach (ASSPS) as an effective optimization method were validated in the preceding section by evaluating a variety of benchmark issues. In this part, the adaptability and efficacy of the suggested approach for exploring the least FoS and reliability index will be explored by considering two instances of slope stability challenges from the previous research. The first one is a slope in uniform soil, whereas the second is a multilayered slope. The FoS is determined using the Morgenstern and Price, while the reliability index is derived using AFOSM. The ASSPS was implemented for slope stability assessment using a program written in MATLAB R2017a, which can find the lowest FoS and reliability index and their related critical slip surfaces.

6.1 Slope in a uniform soil

The first instance is a slope in a homogenous soil with an elevation of 10 m and a slope gradient of 18.4°, which Chowdhury and Xu [37] presented and investigated for the first time. Fig. 6 depicts the cross section and topology of the slope.

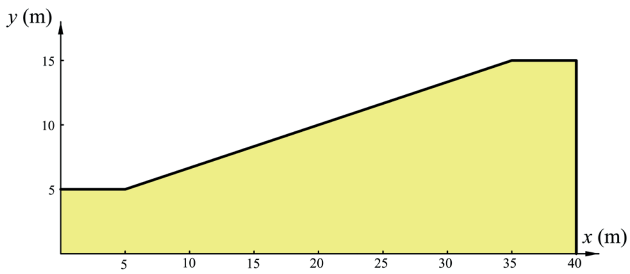

Fig. 6 Slope in a uniform soil

In this experiment, four parameters include effective cohesion (c'), effective friction angle (φ'), unit weight of soil (γ) and pore water pressure ratio (r_u) are random variables. Table 5 displays the random variables' mean and standard deviation (i.e., first and second statistical moments). All of the random variables in this experiment are supposed to be uncorrelated and normally distributed.

Chowdhury and Xu [37] solved the problem and evaluated the minimum factor of safety based on simplified Bishop's method [38] and the Hasofer and Lind reliability index. In this case, the minimum factor of safety and its associated critical deterministic slip surface is computed based on the mean values of the soil properties. Khajehzadeh et al. [31] proposed modified version of gravitational search algorithm (MGSA) for searching the minimum FoS and lowest reliability index. This problem is solved using the proposed algorithm and the evaluated minimum safety factors and minimum reliability indexes are shown in Table 6. In Table 6, $FS_{\min}$ and $\beta_{\min}$ are the minimum factor of safety and the minimum HL reliability index associated with the critical deterministic and probabilistic slip surfaces, respectively. From the results of this table, it can be observed that the minimum factor of safety obtained by ASSPS is 1.4432, which is almost 4.7 percent lower than the value achieved by SSA (1.5142) and approximately 10 percent lower than that reported by Chowdhury and Xu [37]. In addition, the minimum reliability index calculated by the presented ASSPS method is 9.7 percent and 15.6 percent lower than those obtained by SSA and Chowdhury and Xu [37], respectively.

Fig. 7 depicts and compares the final probabilistic and deterministic slip surfaces identified by both methods (SSA and ASSPS). As shown in this figure, the final probabilistic and deterministic slip surfaces are rather near to each other, as would be anticipated in a homogenous slope [39]. While, the failure surfaces estimated by SSA differ somewhat from those produced by ASSPS.

Table 5 Statistical properties of soil parameters for homogeneous slope

Random variable	Mean	Standard deviation	Coefficient of variation
c' (kN/m^2)	12	2.0	16.67%
φ' (°)	15	2.5	16.3%
γ (kN/m^3)	20	1.2	6%
r_u	0.2	0.02	10%

Table 6 Results comparison for the homogeneous slope

Reference	Minimum FoS $FS_{\min}$	Minimum Reliability Index $\beta_{\min}$
Chowdhury and Xu [37]	1.6044	2.946
MGSA [31]	1.4526	2.5133
SSA (Present study)	1.5142	2.7646
ASSPS (Present study)	1.4432	2.4965

6.2 Slope in multilayered soil

This experiment is taken from the study of Hassan and Wolff [40], which is a multilayer clay slope confined by a hard layer below the ground surface, as illustrated in Fig. 8. The soil strength characteristics associated to slope stability, such as effective cohesion c' and effective friction angle φ' are considered as random variables. In this case, the random variables are supposed to have a lognormal distribution and to be uncorrelated. Table 7 summarizes the statistical properties (i.e., mean and standard deviation) of the considered variables. The slope's unit weight (γ) is considered to be 18 kN/m^3.

This case has already been investigated by Hassan and Wolff [40], Bhattacharya et al. [39] and Khajehzadeh et al. [31]. To calculate the lowest FoS and reliability index, Hassan and Wolff [40] suggested a novel search technique. They used the Spencer technique to calculate the *FoS* and the MFOSM approach to calculate the reliability index, assuming a lognormal distribution for the *FoS*. Bhattacharya et al. [39] used the direct search approach

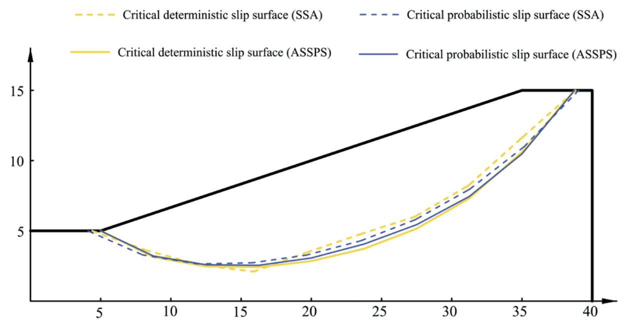

Fig. 7 Critical slip surfaces explored by SSA and ASSPS for the first experiment

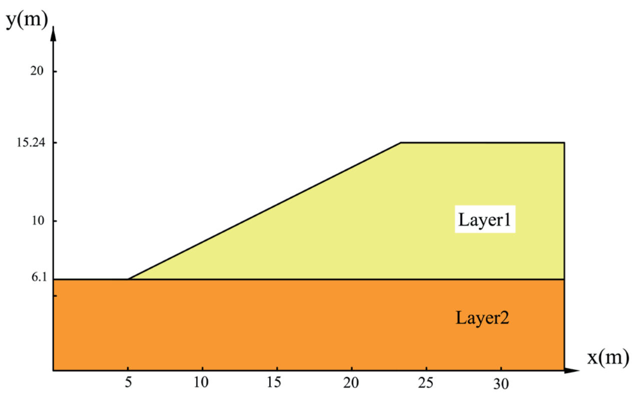

Fig. 8 Cross section of a multi-layered slope

Table 7 Statistical properties of soil parameters for homogeneous and stratified slope

Layer of soil	Random variable	Mean	Standard deviation	Coefficient of variation	Distribution
Layer 1	c_1' (kN/m^2)	38.31	7.662	20%	Lognormal
	φ_1' (°)	0	-	-	-
Layer 2	c_2' (kN/m^2)	23.94	4.788	20%	Lognormal
	φ_2' (°)	12	1.2	10%	Lognormal

Table 8 Results comparison for the multi-layered slope

Method	FS_{min}	β_{min}
Hassan and Wolff [40] – MVFOSM, Spencer method, non-circular slip surface	1.663	2.869
Bhattacharya et al. [39] – MVFOSM, Spencer method, non-circular slip surface by direct search	1.665	2.861
Khajehzadeh et al. [31]– AFOSM, Morgenstern and Price method, non-circular slip surface by MGSA	1.6453	2.767
Present study– AFOSM, Morgenstern and Price method, non-circular slip surface by SSA	1.6616	2.8134
Present study– AFOSM, Morgenstern and Price method, non-circular slip surface by ASSPS	1.6376	2.7532

in conjunction with the Spencer method and MFOSM to solve this issue. Khajehzadeh et al. [31] suggested a modified gravitational search algorithm (MGSA) combined with Morgenstern and Price and AFOSM for the solution. Table 8 describes the present study's findings, as well as a comparison to those published by earlier studies. The findings demonstrate that the suggested ASSPS's minimal *FoS* is 1.6376, which is almost 2 percent lower than the other approaches studied. Furthermore, the lowest reliability index obtained by ASSPS is 2.7532, the least among the other alternatives.

Fig. 9 displays the crucial deterministic and probabilistic failure surfaces produced from SSA and ASSPS, respectively. When the top layer of a multilayered slope is weaker than the bottom layer, the critical surfaces are placed significantly apart, as seen in this figure. The slip surfaces found by the given approach are in fair agreement with those published by Hassan and Wolff [40], Bhattacharya et al. [39] and Khajehzadeh et al. [31].

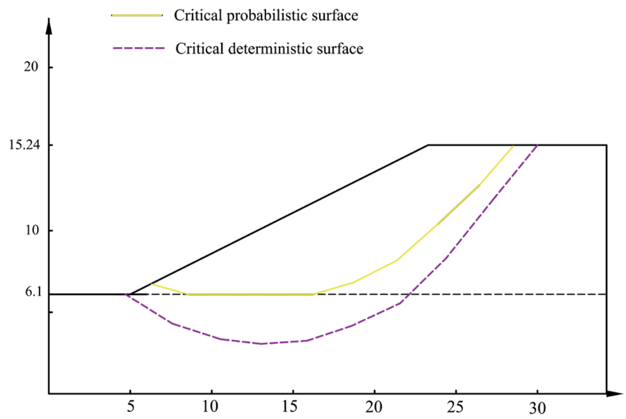

Fig. 9 Critical failure surfaces of a multi-layered slope

7 Conclusions

For the reliability assessment of soil slopes, this research develops a hybrid optimization technique based on the adaptive salp swarm as well as pattern search (ASSPS). The suggested methodology uses the powerful exploratory ability of the adaptive salp swarm algorithm as well as efficient local search capacity of the pattern search technique. The new method's performance is evaluated using a variety of unimodal and multimodal benchmark functions. According to the obtained results, the ASSPS outperforms basic SSA and other methodologies in aspects of finding the global solution for most test functions. In order to investigate the effectiveness of the proposed method for practical engineering problems, the ASSPS is used to find the earth slope's minimum reliability index and its associated critical probabilistic failure surface. Two series of experiments were used to investigate the effectiveness of the new ASSPS algorithm for reliability analysis of soil slopes. The obtained results indicate that the minimum factors of safety obtained by ASSPS are almost 4.7 and 2 percent lower than the values achieved by SSA for single-layer and multi-layer earth slopes, respectively, and approximately 10 percent lower than the results previously published. Furthermore, for single-layer and multi-layer slopes, the minimum reliability indexes calculated by the presented ASSPS method are 9.7 and 2% lower, respectively, than those obtained by SSA. The findings show that the newly suggested methodology can improve prior methods' results, indicating that ASSPS is quite efficient for trying to solve such a difficult engineering problem.

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