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RESEARCH ARTICLE

Optimal design of structures with multiple natural frequency constraints using a hybridized BB-BC/Quasi-Newton algorithm

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Abstract

A hybridization of the Quasi-Newton with Big Bang-Big Crunch (QN-BBBC) optimization algorithm is proposed to find the optimal weight of the structures subjected to multiple natural frequency constraints. The algorithm is based on hybridizing a mathematical algorithm (quasi-Newton) for local search and a meta-heuristic algorithm (Big Bang-Big Crunch) for global search, and to help to leave the traps. Four examples are proposed for the optimization of trusses and two examples are studied for the optimization of frames with frequency constraints. The examples are widely reported and used in the related literature as benchmarks. The numerical results reveal the robustness and high performance of the suggested methods for the structural optimization with frequency constraints.

Keywords

frequency constraint structural optimization · hybridized quasi Newton and Big Bang-Big Crunch algorithms · truss and frame structures

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1 Introduction

It is well known that the natural frequencies are fundamental parameters affecting the dynamic behavior of the structures. Therefore, some limitations should be imposed on the natural frequency range to reduce the domain of vibration and also to prevent the resonance phenomenon in dynamic response of structures [1]. Weight optimization of structures with frequency constraints is considered to be a challenging problem. Mass reduction conflicts with the frequency constraints, especially when they are lower bounded. Also, frequency constraints are highly non-linear, non-convex and implicit with respect to the design variables [2].

Some progress has been made in optimum design of structures considering natural frequency constraints. This work started to be developed in 1980s with the paper of Bellagamba and Yang [3].

McGee and Phan [4] proposed an efficient optimality criteria (OC) method for cross-sectional optimization of two-dimensional frame structures. The iterative method involves alternately satisfying the constraints (scaling) and applying the Kuhn-Tucker (optimality) condition (resizing).

Grandhi and Venkayya [5] utilized an optimality criteria (OC) method based on uniform Lagrangian density for resizing and scaling procedure to locate the constraint boundary. Multiple equality and inequality frequency constraints were considered on truss structures.

Tong et al. [6] presented a basic theory for determining the solution existence of frequency optimization problems for truss structures. This method was based on the fact that the frequencies remain unchanged when the truss is modified uniformly.

Sedaghati [7] developed a new approach using combined mathematical programming based on the Sequential Quadratic Programming (SQP) technique, and the finite element technique based on the Integrated Force Method to optimize both frame and truss structures with frequency constraints.

Gholizadeh et al. [1] and Salajegheh et al. [8] presented a study where a Genetic Algorithm (GA) and a neural network (NN) were utilized to find the optimal weight of structures subjected to multiple natural frequency constraints.

Gomes [9] used the Particle Swarm Optimization (PSO) algorithm to investigate simultaneous shape and size optimization of truss structures with multiple frequency constraints.

Kaveh and Zolghadr [2] utilized a hybridization of the Charged System Search and the Big Bang-Big Crunch algorithms with trap recognition capability to shape and size optimization of truss structures with multiple frequency constraints.

Various methods with different levels of complexity and success have been proposed to solve optimal design problems. These methods can be divided into two general categories: 1. Mathematical methods such as quasi-Newton (QN) and dynamic programming (DP); 2. Meta-heuristic algorithms such as Genetic algorithms (GA), Particle swarm optimization (PSO), Ant Colony Optimization (ACO), Big Bang-Big Crunch (BB-BC), Charged system search (CSS), Magnetic charged system search (MCSS), and Ray optimization (RO). Some successful applications of meta-heuristic algorithms can be found in the work of Refs. [10–13].

Mathematical algorithms are hard to apply and time-consuming in these optimization problems. Furthermore, a good starting point is vital for these methods to be executed successfully and they may be trapped in local optima. On other hand, frequency constraints are highly non-linear, non-convex and implicit with respect to the design variables [9]. Simultaneous consideration of sizing and shape variables together with the vibration mode switching phenomenon which usually occurs while minimizing the weight, may cause some convergence difficulties. Hence, combining a meta-heuristic algorithm for global search for solving these difficulties seems to be inevitable.

The Big Bang and Big Crunch theory is introduced by Erol and Eksin [14] which has a low computational time and high convergence speed. According to this theory, in the Big Bang phase energy dissipation produces disorder and randomness is the main feature of this phase; whereas, in the Big Crunch phase, randomly distributed particles are drawn into an order. The Big Bang–Big Crunch (BB–BC) Optimization method similarly generates random points in the Big Bang phase and shrinks these points to a single representative point via a center of mass in the Big Crunch phase. After a number of sequential Big Bangs and Big Crunches where the distribution of randomness within the search space during the Big Bang becomes smaller and smaller about the average point computed during the Big Crunch, the algorithm converges to a solution. The BB–BC method has been shown to outperform the enhanced classical genetic algorithm for many benchmark test functions (Erol and Eksin [14]). This method was successfully applied to size optimization of space truss structures by Kaveh and Talatahari [15].

The Big Bang-Big Crunch optimization algorithm was hybridized with local directional moves and applied to target motion analysis problem by Genç et al. [16]. Another hybrid Big Bang–Big Crunch optimization (HBB–BC) was implemented to solve the truss optimization problems (Kaveh et al. [17]). The HBB–BC method consisted of two phases: a big bang phase where candidate solutions were randomly distributed over the search space, and a Big Crunch phase working as a convergence operator where the center of mass was generated. Then new solutions were created by using the center of mass to be used as the next Big Bang. These successive phases were carried repeatedly until a stopping criterion has been met. This algorithm not only considered the center of mass as the average point in the beginning of each Big Bang, but also similar to PSO-based approaches (Kenneday et al. [18]), utilized the best position of each particle and the best visited position of all particles.

The mathematical method utilized for local search in this study is the QN method. Similar to other mathematical methods, this method has one input and one output. On other hand, BB-BC has many inputs but only one output, which can be named as the center of 'mass'. In this paper, this similarity is utilized for the hybridization of these methods.

The present paper is organized as follows: In Section 2, we describe the QN and BB-BC. In Section 3, the new method is presented. Statement of the optimization design problem with frequency constraints is formulated in Section 4. Six examples are studied in Section 5. Conclusions are derived in Section 6.

2 Preliminaries

In order to make the paper self-explanatory, before proposing QN-BB-BC for optimal design of structures with frequency constraints, the characteristics of the QN and BB-BC are briefly explained in the following two subsections:

2.1 Quasi-Newton Method

Quasi-Newton method is a special case of variable metric methods for finding local maxima and minima of functions. Quasi-Newton methods build up curvature information. In Newton methods, let $\boldsymbol{H}$ denote the Hessian, $\boldsymbol{c}$ be a constant vector and b be a constant, then a quadratic model problem formulation can be constructed as

$$\min_{x} \left(\frac{1}{2}\mathbf{x}^T\mathbf{H}\mathbf{x} + \mathbf{c}^T\mathbf{x} + b\right) \tag{1}$$

If the partial derivatives of x tends to zero, i.e.$\nabla f(\mathbf{x}^*) = \mathbf{H}\mathbf{x}^* + \mathbf{c} = 0$, then the optimal solution for the quadratic problem occurs, and thus $\mathbf{x}^* = -\mathbf{H}\mathbf{c}$.

In quasi-Newton methods, the Hessian matrix of the second derivatives of the function to be minimized does not need to be computed. Instead the Hessian matrix is updated by analyzing successive gradient vectors. In this study, the quasi-Newton method of Matlab [19] is utilized which is called the Broyden-Fletcher-Goldfarb-Shannon (BFGS) method.

2.2 Big Bang-Big Crunch algorithm

The Big Bang-Big Crunch algorithm is proposed by Erol and Eksin [11] and it is based on one of the theories of evolution of the universe with the same name. The algorithm consists of two phases: a Big Bang phase and a Big Crunch phase. In the

Big Bang phase, solution candidates are distributed randomly within the search space. This randomness is regarded as energy dissipation in nature. Like any other meta-heuristic optimization technique, the initial population is produced by spreading the candidates all over the search space in a uniform manner. The Big Bang phase is followed by the Big Crunch phase.

The Big Crunch is a convergence operator that has many inputs but only one output, which is named as the center of 'mass', since the calculations of this output are similar to those of the center of mass. Here, the term mass refers to the inverse of the fitness function value. The point representing the center of mass is calculated according to the following formula:

$$x_j^c = \frac{\sum_{i=1}^{N} \frac{1}{f^i} x_j^i}{\sum_{i=1}^{N} \frac{1}{f^i}} \qquad i = 1, 2, \ldots, N \tag{2}$$

Where x_j^c is the jth component of the center of mass, x_j^i is the jth component of ith candidate, f^i is the fitness of the ith candidate, and N is the number of candidates in the population. When the Big Crunch phase is completed, the Big Bang phase is carried out to produce new candidates for the next iteration. These new candidates are produced using a normal distribution around the center of mass of the previous iteration. The standard deviation of this normal distribution decreases as the optimization process proceeds:

$$x_j^{i,new} = x_j^c + r \times c_1 \times \frac{(x_j^{\max} - x_j^{\min})}{1 + k/c_2} \tag{3}$$

where $x_j^{i,new}$ is the new value of the jth component from ith candidate, r is a random number with a standard normal distribution, c_1 and c_2 are two constants and k is the iteration index. $x_j^{\max}$ and $x_j^{\min}$ are the maximum and minimum values of the jth component of the variable x, respectively.

3 A hybrid Quasi-Newton and Big Bang-Big Crunch

The hybrid QN and BB-BC algorithm is presented in this section. The main algorithm is based on the local search property of the QN and the global search property of the BB-BC is added to it. As mention before, one of disadvantages of the QN algorithm is that it may be trapped in local optima and a good starting point is vital for this method. Therefore BB-BC is utilized to help the QN method to leave the local optima and be independent of a good starting point. Therefore, similar to the other mathematical methods, QN has one input and output. On other hand, BB-BC has many inputs but only one output, which is known as the center of 'mass'. This similarity is one of the reasons for choosing the BB-BC for hybridization of these methods.

In the present optimization algorithm the method is divided into some stage (NS stages), and each stage is continued until QN method gets trapped in a local optima. Then, the Big Bang phase is carried out to leave the local optima, and the Big Crunch phase helps to change all the values of the candidate solutions to a single value as the input of the quasi-Newton method in the next stage. Also, with increasing the number of stages, the search space decreases and the local search increases.

The proposed method is summarized as:

Step 1: Initialization. A point is initialized with a random position in the search space.

Step 2: Big Bang phase. In this step, for having a good starting point (in the first iteration) and to escape from the trap (in the subsequent iterations), Big Bang phase is carried out by the following equation:

$$x_j^{i,new} = x_j^c + r \times c_1 \times \frac{(x_j^{\max} - x_j^{\min})}{1 + SN/c_2} \tag{4}$$

Where *SN* is the stage number that is replaced with k in Eq. (3). In fact, by performing this change the search space is decreased and the stage number is increased. The center of mass in Eq. (4) is the starting point (in the first iteration) or the output of the QN method (in the subsequent iterations).

Step 3: Big Crunch phase. The center of mass of the generated points in the previous step is determined by Eq. (3).

Step 4: Local search: In this step, as the local search in the search space, quasi-Newton method is used. Input point in this method is the calculated center of mass in the previous step. Different terminating criterion can be applied to this method, such as the number of iterations, number of function count, tolerance variable and tolerance function. In this study, the number of iterations is utilized as the terminating criterion in each stage. Number of iterations is increased in each stage, e.g. 10**SN*.

Step 5: Terminating criterion control. Steps 2-5 are repeated after some stages. In step 2, the center of mass is the new point utilized in the QN method.

4 Formulation of the structural optimal design for frequency constraints

In structural optimization problems with frequency constraints, the objective is to minimize the weight of the structure while satisfying multiple constraints on natural frequencies. Cross-sectional areas of the members along with the coordinates of some nodes are considered as the design variables and assumed to change continuously. The connectivity information of the structure is predefined and kept unchanged during the optimization process. A lower and upper bound may also be prescribed for each variable. The optimization problem is formally stated as follows:

$$\begin{aligned} &\text{Find } \mathbf{X} = [x_1, x_2, x_3, \ldots, x_n] \\ &\text{to minimizes } \mathrm{Mer}(\mathbf{X}) = \mathrm{f}(\mathbf{X}) \times \mathrm{f}_{\mathrm{penalty}}(\mathbf{X}) \\ &\text{subjected to} \\ &g_i(\mathbf{X}) \le 0, \; i = 1, 2, \ldots, m \\ &x_{imin} \le x_i \le x_{imax} \end{aligned} \tag{5}$$

where $\boldsymbol{X}$ is the vector of design variables with n unknowns, g_i is ith constraint from m inequality constraints, and $Mer(\boldsymbol{X})$ is the merit function; $f(\boldsymbol{X})$ is the cost function; $f_{penalty}\ (\boldsymbol{X})$ is the penalty function which results from the violations of the constraints corresponding to the response of the structure. Also, x_{imin} and x_{imax} are the lower and upper bounds of design variable vector, respectively.

Exterior penalty function method is employed to transform constrained optimization problem into unconstrained one as follows:

$$f_{penalty}(\mathbf{X}) = 1 + \gamma_p \sum_{i=1}^{m} \max(0, g_j(x))^2 \qquad (6)$$

where γ_p is the penalty multiplier.

5 Numerical examples

In this section, common structural optimization examples, as benchmark problems, are optimized with the proposed method. The final results are compared to the solutions of other methods to demonstrate the efficiency of the present approach. Four trusses and two frames are optimized in this section.

For the proposed algorithm, five stages are used in the optimization process. As mentioned before, the number of iterations for the quasi-Newton method in each stage is equal to 10*SN. Therefore, the number of iterations in each stage is equal to 10, 20, 30, 40 and 50. Thus, the total number of iterations is equal to 150 in the all examples. In the BB-BC method, the population of the solution candidates is taken as the number of variables; the value of constants c_1 and c_2 are set to 1 and 2, respectively.

It should be mentioned that when the natural frequencies are optimized to the edge, then the structure might be sensitive to small implementation errors. In order to avoid this problem, the target frequencies can be considered slightly higher than the necessary magnitudes.

5.1 A 10-Bar planar truss

The 10-bar truss problem has become a common problem in the field of structural design with frequency constraints to test and verify the efficiency of many different optimization methods. Fig. 1 shows the geometry and support conditions for this planar truss. Table 1 shows the material properties, variable bounds, and the frequency constraints for this example. There are 10 design variables in this example and a set of pseudo variables larger than 0.645 cm^2.

This example was first solved by Grandhi and Venkayya [5] using the optimality algorithm. Lingyun et al. [20] have used a niche hybrid genetic algorithm to optimize this truss. Gomes has analyzed this problem using the particle swarm algorithm [9]. Kaveh and Zolghadr [2] have investigated the problem utilizing the standard and hybridized CSS-BBBC algorithm with trap recognition capability.

Table 2 compares the results obtained in this research with the outcome of the other researches. It can be seen that the results of the proposed algorithm are better than those of the previously reported methods and the standard QN. Table 3 shows the natural frequencies of the optimized structure obtained by

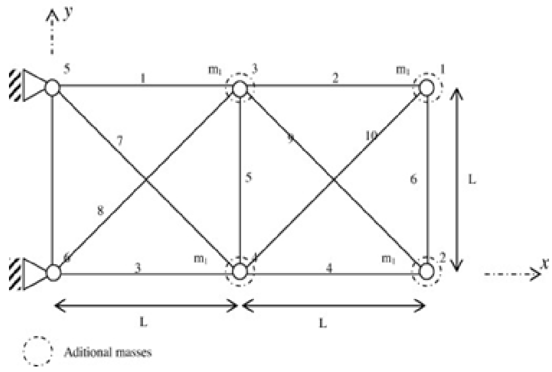


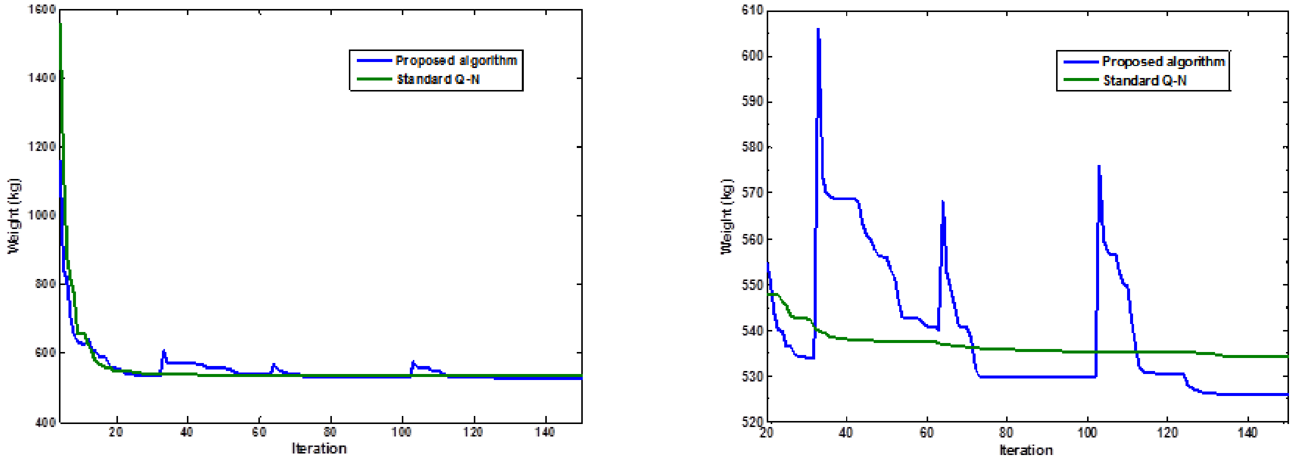
Fig. 1. A 10-bar planar truss.

several authors in the literature and those of the present work. It is clear that none of the frequency constraints are violated. Fig. 2 provides a comparison of the convergence rates of the QN and QN-BBBC in the all iterations and the 20-150 iterations, respectively. As it can be seen, the standard QN is trapped in a local optimum after the 40^{th} iteration and the function value is changed slowly. In the QN-BBBC method, the BBBC occurs in the 10^{th}, 30^{th}, 60^{th} and 100^{th} iterations and the QN method escapes from the local optima. Also, the function value in each stage is lower than the previous stage.

5.2 A 52-bar dome-like truss

Fig. 3 shows the initial topology of a 52-bar dome-like space truss. This example is optimized for shape and configuration with constraints on the first two natural frequencies. The space truss has 52 bars, and non-structural masses of $m = 50\ kg$ are added to the free nodes. The material density is $7800 kg/m^3$ and the modulus of elasticity is $2.1 \times 10^{11} N/m^2$. The structural members of this truss are arranged into eight groups, where all members in a group share the same material and cross-sectional properties. Table 4 shows each element group by member numbers. The range of the cross-sectional areas varies from 0.0001 to $0.001 m^2$. The shape optimization is performed taking into account that the symmetry is preserved in the process of design. Each movable node is allowed to vary $\pm 2m$. For the frequency constraints $\omega_1 \leq 15.916 HZ$ and $\omega_2 \geq 28.649 HZ$ are considered. This example is considered to be a truss optimization problem with two natural frequency constraints and 13 design variables (five shape variables plus eight size variables).

Table 5 compares the results of this paper with the outcomes of other researches. It can be seen that the results obtained by the proposed algorithm are better than those of the previously reported ones and the standard QN. Table 6 shows the natural frequencies of optimized structure obtained by different authors in the literature and the results obtained by the present work. Here, the 1^{th} natural frequency is closer to the constraint being considered (equal to 15.916) than other results. Fig. 4 provides a comparison of the convergence rates of the QN and QN-BBBC.



(a) all iterations

(b) 20-150 iterations

Fig. 2. Comparison of the convergence rates between the two algorithms for the 10-bar planar truss.

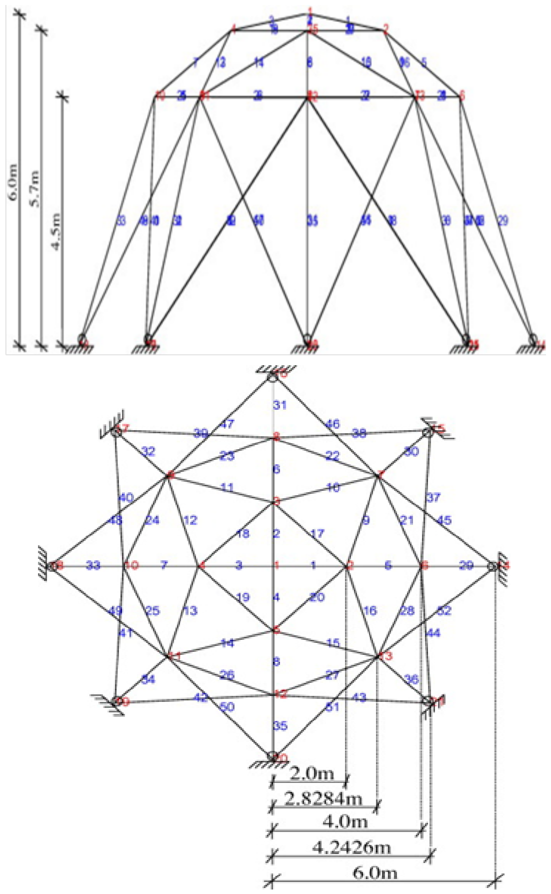


Fig. 3. A 52-bar dome-like space truss (initial shape).

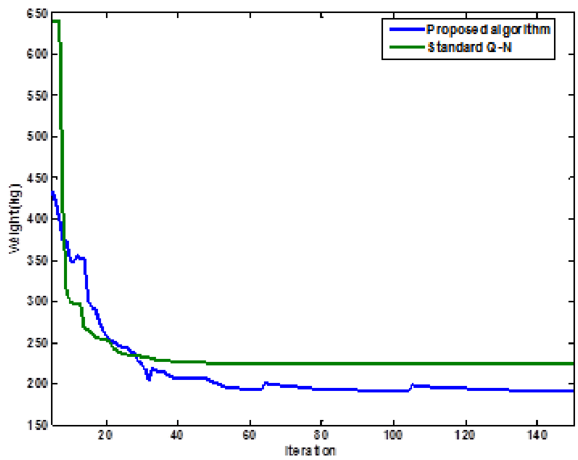


Fig. 4. Comparison of the convergence rates between the two algorithms for the 52-bar planar truss

5.3 A 120-bar dome truss

A 120-bar dome truss, shown in Fig. 5, was first analyzed by Soh and Yang [21] to obtain the optimal sizing and configuration variables with stress constraint. In this example, similar to Kaveh and Zolghadr [2], only sizing variables are considered to minimize the structural weight with frequency constraints. Non-structural masses are attached to all free nodes as follows: 3000 *kg* at node one, 500 *kg* at nodes 2–13 and 100 *kg* at the remaining nodes. The material density is taken as $7971.810kg/m^3$ and the modulus of elasticity is $2.1\times10^{11}N/m^2$. The structural members of this truss are arranged into seven groups. Fig. 5 shows each element group by member numbers. The range of cross-sectional areas varies from 0.0001 to $0.01293m^2$. For the frequency constraints, $\omega_1 \geq 9HZ$ and $\omega_2 \leq 11HZ$ are considered.

Table 7 compares the results obtained in this research with the outcome of other researches. It can be seen that the results of the proposed algorithm are better than those of the previously reported algorithms and the standard QN. Table 8 shows the natural frequencies of optimized structure obtained by different authors in literature and the results obtained by the present work. Fig. 6 provides a comparison of the convergence rates of the QN and QN-BBBC.

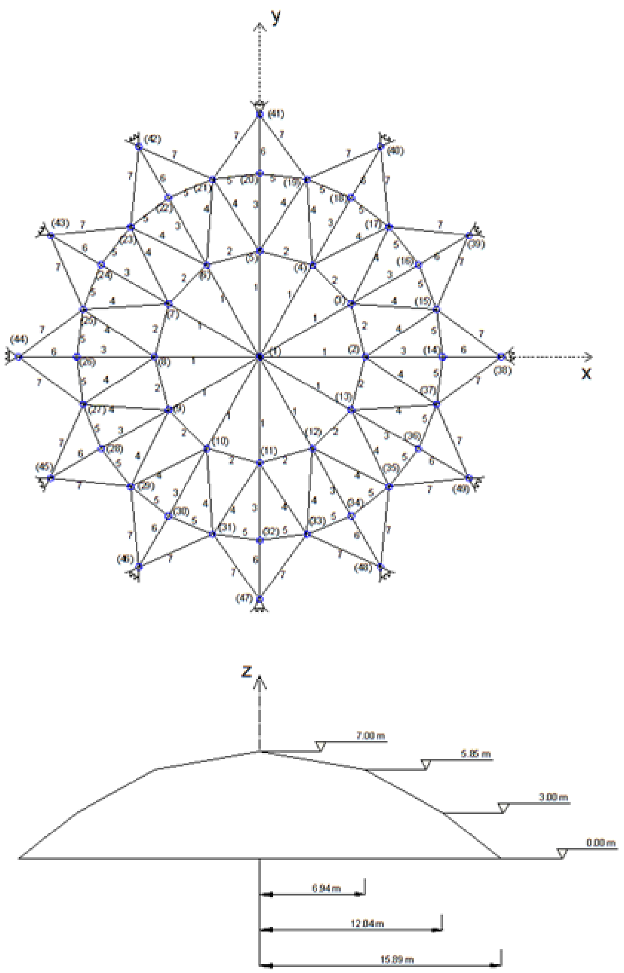


Fig. 5. A 120-bar dome truss.

5.4 A 200-bar planar truss

The 200-bar plane truss, shown in Fig. 7, was analyzed with static condition by Kaveh et al. [21]. This truss has been investigated using the standard CSS and CSS-BBBC algorithms as a frequency constraint weight optimization problem by Kaveh and Zolghadr [2]. The material density and modulus of elasticity of members are $7860kg/m^3$ and $2.1\times10^{11}N/m^2$, respectively. Non-structural masses of $100kg$ are attached to the upper nodes. A lower bound of 0.1 cm^2 is assumed for the cross-sectional areas. For the frequency constraints $\omega_1 \geq 5HZ$, $\omega_2 \geq 10HZ$ and $\omega_3 \geq 15HZ$ are considered. The elements are divided into 29 groups.

Table 9 compares the results of this research with those of the other researches. It can be seen that the results obtained by the

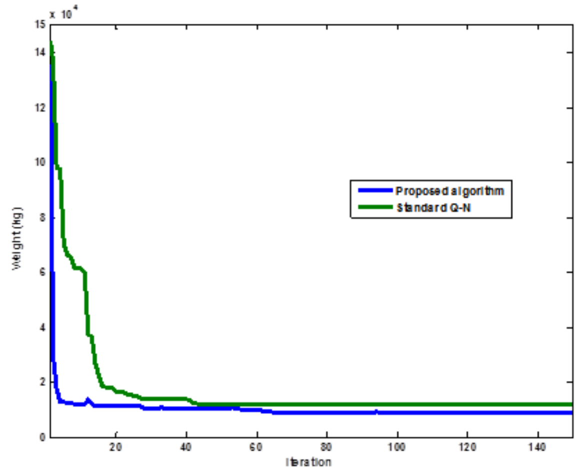


Fig. 6. Comparison of the convergence rates between the two algorithms for the 120-bar planar truss

proposed algorithm are better than those previously reported and the standard QN. Table 10 shows the nature frequencies of the optimized structure obtained by several authors in the literature and the results obtained by the present work. Fig. 8 provides a comparison of the convergence rates of the QN and QN-BBBC.

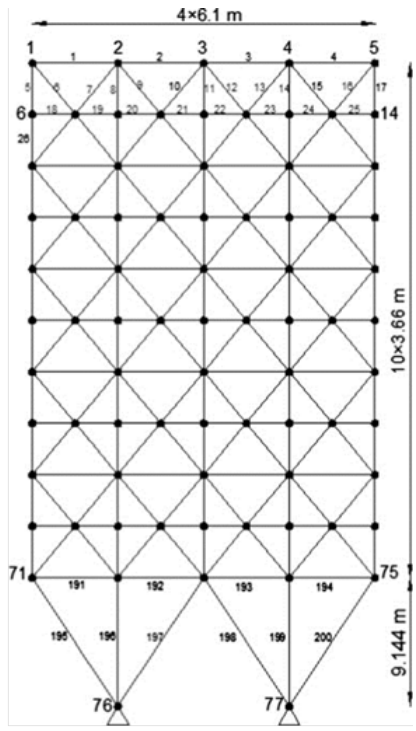


Fig. 7. A 200-bar planar truss.

5.5 Member frame (two-story and one-bay) with non-structural distributed mass

This six-member frame, studied in [4,22] is depicted in Fig. 9. A uniformly distributed non-structural weight of 178.740 kg/m is imposed on the horizontal members of the frame. This prob-

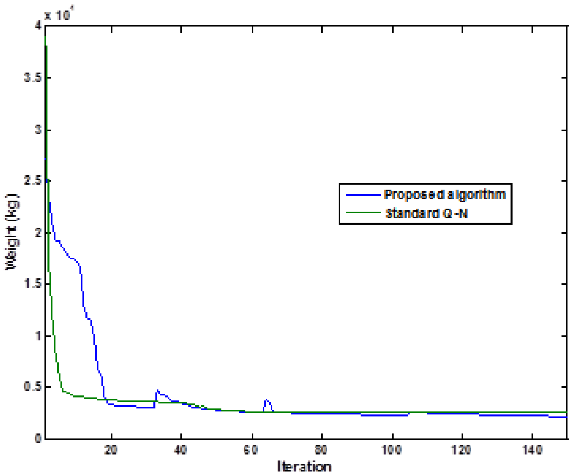


Fig. 8. Comparison of the convergence rates between the two algorithms for the 200-bar planar truss

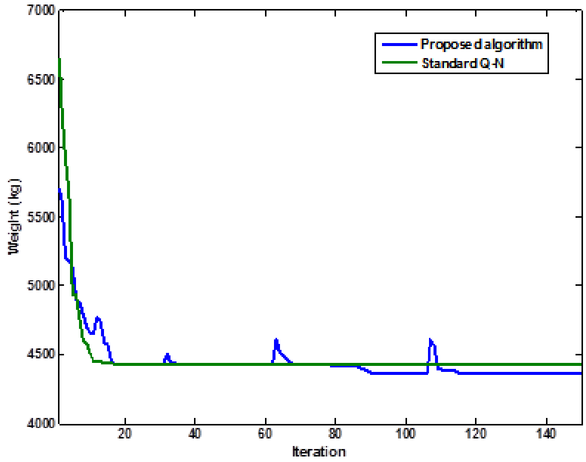


Fig. 10. Comparison of the convergence rates between the two algorithms for the two-story and one-bay frame

lem is solved with a weight density of $7830kg/m^3$and an elastic modulus $2.07 \times 10^{11} N/m^2$. The range of cross-sectional areas varies from 51.088 to $569.55cm^2$. The cross-sectional areas of the members, A, are considered as design variables and the principal moments of inertia, I_z, are expressed in terms of A as follows (McGee and Phan [4]):

$$\begin{aligned} I_Z &= 4.6248A^2 & 0in^2 < A < 44.2in^2 \\ I_Z &= 256A - 2300 & 44.2in^2 < A < 88.28in^2 \end{aligned} \tag{7}$$

Two cases for constraints are considered as follows:

Case 1: Single frequency $\omega_1 = 78.5\frac{rad}{s}$

Case 2: Multiple frequencies $\omega_1 = 78.5\frac{rad}{s}$ $\quad \omega_2 \geq 180\frac{rad}{s}$

Table 11 lists the optimal values of the six size variables obtained by the present algorithm, and compares them with other results. It can be seen that the results of the proposed algorithm are slightly lighter than the other results. Fig. 10 compares the convergence rate of the two algorithms.

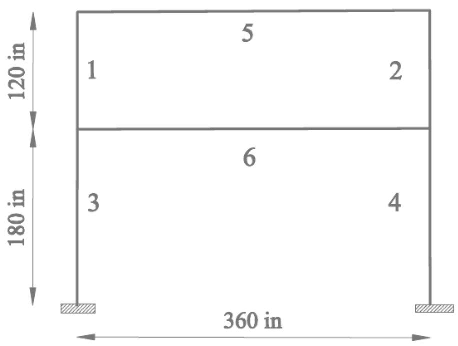


Fig. 9. A two-story and one-bay frame with non-structural distributed mass

5.6 A seven-story and one-bay frame

The 21-member frame, shown in Fig. 11, is examined as the final design problem to demonstrate the efficiency of the algorithm. A uniformly distributed non-structural weight of 10 lb/in is superimposed on the horizontal members of the frame. This problem is solved with a weight density of $0.283Ib - \sec^2/in^3$and an elastic modulus of $30 \times 10^6 Psi$. The lower value of the cross-sectional areas is 7.9187 in^2. The lowest allowable natural frequency is taken as 10.2 rad/s.

Table 12 lists the optimal values of the size variables obtained by the present algorithm and compares them with other results. It can be seen that the results of the proposed algorithm are slightly lighter than those of the others. Fig. 12 compares the convergence rate of the two algorithms.

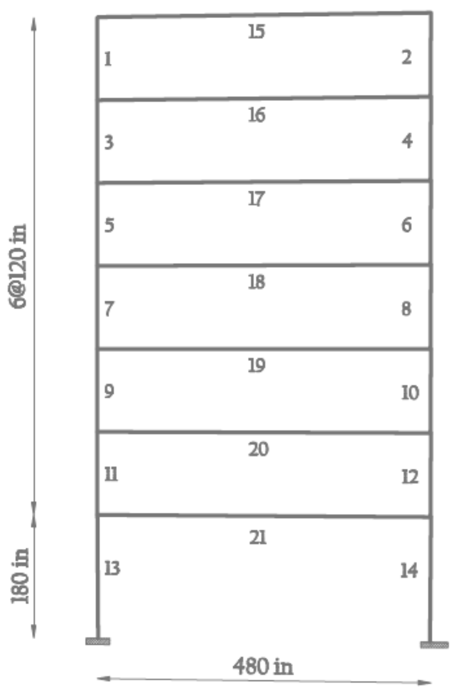


Fig. 11. A seven-story and one-bay frame with non-structural distributed mass

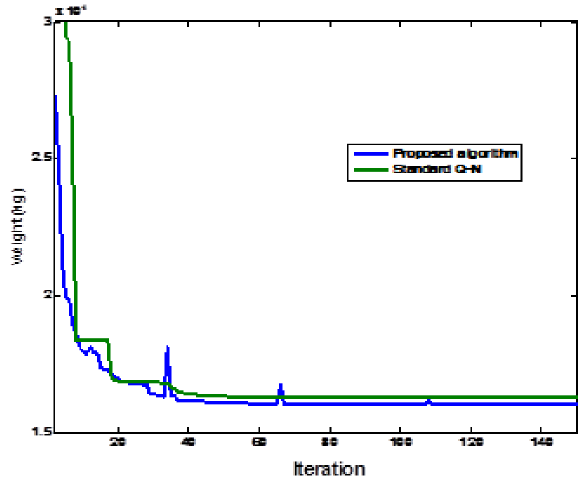


Fig. 12. Comparison of the convergence rates between the two algorithms for the two-story and one-bay frame

Tab. 1. Material properties, variable bounds and frequency constraints for the 10-bar truss structure.

Property/unit	Value
E (Modulus of elasticity)/N/m^2	6.98×10^{10}
ρ (Material density)/kg/m^3	2770.0
Added mass/kg	454.0
Design variable lower bound/m^2	0.645
L (Main bar's dimension)/m	9.144
Constraints on first three frequencies/Hz	$\omega_1 \geq 7, \omega_2 \geq 15, \omega_3 \geq 20$

Tab. 2. Optimal design cross-sections (cm^2) using different methods for the 10-bar planar truss

Element number	Grandhi et al. [5]	Lingyun et al. [20]	Gomes [9]	Kaveh and Zolghadr [2]		Present Work	
				Standard CSS	CSS-BB-BC	Standard QN	Proposed algorithm
1	36.584	42.23	37.712	37.831	35.274	38.1483	34.9513
2	24.658	18.555	9.959	9.586	15.463	11.3936	14.3598
3	36.584	38.851	40.265	29.498	32.11	40.796	34.9224
4	24.658	11.222	16.788	16.096	14.065	18.5863	14.3612
5	4.167	4.783	11.576	4.536	0.645	5.1369	0.645
6	2.070	4.451	3.955	4.868	4.88	4.2316	4.5997
7	27.032	21.049	25.308	27.12	24.046	20.9597	24.0205
8	27.032	20.949	21.613	25.066	24.34	19.8113	24.0034
9	10.346	10.257	11.576	18.604	13.343	13.5018	12.4454
10	10.346	14.342	11.186	10.08	13.543	11.2133	12.5438
Weight (*kg*)	**594.0**	**542.75**	**537.98**	**549.09**	**529.09**	**534.1966**	**524.5499**

Tab. 3. Natural frequencies (Hz) of the optimized 10-bar planar truss.

Frequency number	Grandhi et al. [5]	Lingyun et al. [20]	Gomes [9]	Kaveh and Zolghadr [2]		Present Work	
				Standard CSS	CSS-BB-BC	Standard QN	Proposed algorithm
1	7.059	7.008	7.000	7.000	7.000	7.0000	7.0000
2	15.895	18.148	17.786	17.442	16.238	18.1978	16.1633
3	20.425	20.000	20.000	20.031	20.000	20.0687	20.0000
4	21.528	20.508	20.063	20.208	20.361	20.4220	20.0000
5	28.978	27.797	27.776	28.261	28.121	28.5573	28.6501
6	30.189	31.281	30.939	31.139	28.610	31.2076	28.9504
7	54.286	48.304	47.297	47.704	48.390	48.2783	48.3637
8	56.546	53.306	52.420	52.420	52.291	53.3562	50.8435

Tab. 4. Element grouping

Group number	Elements
1	1-4
2	5-8
3	9-16
4	17-20
5	21-28
6	29-36
7	37-44
8	45-52

Tab. 5. Cross-sectional areas and nodal coordinates obtained by different researchers for the 52-bar space truss.

Variable	Initial	Lingyun et al. [20]	Gomes [9]	Kaveh and Zolghadr [2]		Present Work	
				Standard CSS	CSS-BB-BC	Standard QN	Proposed algorithm
$Z_A(m)$	6.000	5.8851	5.5344	4.000	5.331	7.0503	5.4785
$X_B(m)$	2.000	1.7623	2.0885	1.955	2.134	2.2246	2.4517
$Z_B(m)$	5.700	4.4091	3.9283	3.742	3.719	4.4334	3.7027
$X_F(m)$	4.000	3.4406	4.0255	3.841	3.935	2.0000	4.1190
$Z_F(m)$	4.500	3.1874	2.4575	2.500	2.500	2.5000	2.5000
$A_1(cm^2)$	2.0	1.0000	0.3696	1.0000	1.0000	1.0000	1.0000
$A_2(cm^2)$	2.0	2.1417	4.1912	1.0000	1.3056	1.2803	1.3620
$A_3(cm^2)$	2.0	1.4858	1.5123	2.3858	1.4230	2.6617	1.2585
$A_4(cm^2)$	2.0	1.4018	1.5620	1.0000	1.3851	1.3243	1.3809
$A_5(cm^2)$	2.0	1.9110	1.9154	1.4659	1.4226	1.0000	1.3551
$A_6(cm^2)$	2.0	1.0109	1.1315	1.0000	1.0000	1.8673	1.0000
$A_7(cm^2)$	2.0	1.4693	1.8233	2.9158	1.5562	1.0000	1.3485
$A_8(cm^2)$	2.0	2.1411	1.0904	1.0000	1.4485	1.4977	1.5730
Weight (*kg*)	**338.69**	**236.046**	**228.381**	**235.931**	**197.309**	**233.6367**	**193.3183**

Tab. 6. Natural frequencies (*HZ*) of the optimized 52-bar planar truss.

Frequency number	Initial	Lingyun et al. [20]	Gomes [9]	Kaveh and Zolghadr [2]		Present Work	
				Standard CSS	CSS-BB-BC	Standard QN	Proposed algorithm
1	22.69	12.81	12.751	14.984	12.987	15.9159	15.9154
2	25.17	28.65	28.649	28.649	28.648	28.6490	28.8070
3	25.17	28.65	28.649	28.672	28.679	28.6480	28.8070
4	31.52	29.54	28.803	28.7228	28.713	28.6480	28.8070
5	33.80	30.24	29.230	29.3432	30.262	28.6483	30.0307

Tab. 7. Optimal cross-sectional areas (cm^2) for the 120-bar dome truss.

Element number	Kaveh and Zolghadr [2]		Present Work	
	Standard CSS	CSS-BB-BC	Standard QN	Proposed algorithm
1	21.710	17.478	19.9686	20.0325
2	40.862	49.076	45.0818	38.2935
3	9.048	12.365	10.8889	11.7403
4	19.673	21.979	18.7818	21.9118
5	8.336	11.190	8.3964	1.02E+01
6	16.120	12.590	9.9785	10.9328
7	18.976	13.585	20.3965	14.6337
Weight (*kg*)	**9204.51**	**9046.34**	**9109.5**	**8789.50**

Tab. 8. Natural frequencies (*HZ*) of the optimized 120-bar dome truss.

Frequency number	Kaveh and Zolghadr [2]		Present Work	
	Standard CSS	CSS-BB-BC	Standard QN	Proposed algorithm
1	9.002	9.000	9.001	9.000
2	11.002	11.007	11.023	11.000
3	11.006	11.018	11.034	11.002
4	11.015	11.026	11.034	11.0210
5	11.045	11.048	11.087	11.0863

Tab. 9. Optimal cross-sectional areas for the 200-bar planar truss (cm^2).

Element number	Members in the group	Kaveh and Zolghadr [2]		Present Work	
		Standard CSS	CSS-BB-BC	Standard QN	Proposed algorithm
1	1, 2, 3, 4	1.2439	0.2934	1.0559	0.5111
2	5, 8, 11, 14, 17	1.1438	0.5561	1.3172	0.4944
3	19, 20, 21, 22, 23, 24	0.3769	0.2952	0.346	0.1043
4	18, 25, 56, 63, 94, 101, 132, 139, 170, 177	0.1494	0.1970	0.2763	0.1099
5	26, 29, 32, 35, 38	0.4835	0.8340	1.4495	0.4649
6	6, 7, 9, 10, 12, 13, 15, 16, 27, 28, 30, 31, 33, 34, 36, 37	0.8103	0.6455	1.9043	0.8925
7	39, 40, 41, 42	0.4364	0.1770	0.1	0.1363
8	43, 46, 49, 52, 55	1.4554	1.4796	1.2411	1.3464
9	57, 58, 59, 60, 61, 62	1.0103	0.4497	0.1	0.1207
10	64, 67, 70, 73, 76	2.1382	1.4556	1.2208	1.5715
11	44, 45, 47, 48, 50, 51, 53, 54, 65, 66, 68, 69, 71, 72, 74, 75	0.8583	1.2238	1.1829	1.1589
12	77, 78, 79, 80	1.2718	0.2739	1.1551	0.1943
13	81, 84, 87, 90, 93	3.0807	1.9174	3.546	2.9144
14	95,96, 97, 98, 99, 100	0.2677	0.1170	0.888	0.1482
15	102, 105, 108, 111, 114	4.2403	3.5535	5.737	3.1653
16	82, 83, 85, 86, 88, 89, 91, 92, 103, 104, 106, 107, 109, 110, 112, 113	2.0098	1.3360	2.0844	1.5704
17	115, 116, 117, 118	1.5956	0.6289	2.8868	0.3248
18	119, 122, 125, 128, 131	6.2338	4.8335	7.283	4.9532
19	133, 134, 135, 136, 137, 138	2.5793	0.6062	0.3799	0.1907
20	140, 143, 146, 149, 152	3.0520	5.4393	8.2783	5.3614
21	120, 121, 123, 124, 126, 127, 129, 130, 141, 142, 144, 145, 147, 148, 150, 151	1.8121	1.8435	2.0724	2.0871
22	153, 154, 155, 156	1.2986	0.8955	0.8176	0.6755
23	157, 160, 163, 166, 169	5.8810	8.1759	12.4391	7.3919
24	171, 172, 173, 174, 175, 176	0.2324	0.3209	0.3288	0.3577
25	178, 181, 184, 187, 190	7.7536	10.98	6.7231	7.8055
26	158, 159, 161, 162, 164, 165, 167, 168, 179, 180, 182, 183, 185, 186, 188, 189	2.6871	2.9489	2.7784	2.7029
27	191, 192, 193, 194	12.5094	10.5243	9.6504	10.1453
28	195, 197, 198, 200	29.5704	20.4271	22.5904	20.918
29	196, 199	8.2910	19.0983	9.631	10.952
Weight (kg)		**2559.86**	**2298.61**	**2562.17**	**2140.89**

Tab. 10. Natural frequencies (*HZ*) of the optimized 200-bar planar truss.

Frequency number	Kaveh and Zolghadr [2]		Present Work	
	Standard CSS	CSS-BB-BC	Standard QN	Proposed algorithm
1	5.00	5.010	5.0088	5.0000
2	15.961	12.911	15.9617	13.7774
3	16.407	15.416	17.3448	15.3509
4	20.748	17.033	21.5145	17.3697
5	21.903	21.426	24.7663	21.8335
6	26.995	21.613	29.6299	22.8342

Tab. 11. Final design for the cross-sectional areas (cm^2) for different sets of frequency constraints (rad/s) for the 6-member frame

Element number	Sedaghati [7]		Present Work			
			Standard QN		Proposed algorithm	
	$\omega_1 = 78.5$	$\omega_1 = 78.5$ $\omega_2 \geq 180$	$\omega_1 = 78.5$	$\omega_1 = 78.5$ $\omega_2 \geq 180$	$\omega_1 = 78.5$	$\omega_1 = 78.5$ $\omega_2 \geq 180$
1	51.088	206.289	135.1755	135.766	214.0794	206.2409
2	215.551	62.746	135.1246	122.9037	51.088	58.4141
3	365.982	138.767	283.8704	251.9183	51.088	142.4868
4	51.088	297.876	284.0923	284.3097	366.9939	291.4753
5	51.088	51.088	51.088	51.088	51.088	51.088
6	253.799	256.361	200.5152	218.7014	252.6131	257.773
Weight (kg)	**4272.32**	**4365.56**	**4438.1**	**4427.0**	**4263.9**	**4355.6**

Tab. 12. Final design for the cross-sectional areas (in^2) for the seven-story and one-bay frame

Element number	Khan and Willmart [23]	McGee and Phan [4]	Present Work	
			Standard QN	Proposed algorithm
1	7.9187	7.9187	7.9187	7.9187
2	7.9187	7.9187	7.9187	7.9187
3	7.9187	7.9187	7.9187	7.9187
4	7.9187	7.9187	7.9187	7.9187
5	8.0937	7.9187	7.9187	7.9187
6	8.0937	7.9187	7.9187	7.9187
7	9.3640	8.7612	8.1101	9.5085
8	9.3640	8.7612	7.9187	7.9187
9	10.3130	9.6766	7.9187	7.9187
10	10.3130	9.6766	10.8463	11.2192
11	11.0870	10.2988	9.5177	14.0188
12	11.0870	10.2988	7.9187	7.9187
13	17.5070	25.5192	41.7525	40.9802
14	17.5070	25.5192	7.9187	7.9187
15	7.9187	7.9187	7.9187	7.9187
16	7.9787	7.9187	7.9187	7.9187
17	10.7220	10.4736	12.0382	9.4789
18	12.8020	13.0765	9.2532	12.7474
19	14.9460	14.5608	16.0758	14.3229
20	15.8780	15.7920	16.8055	14.271
21	15.0900	7.9187	7.9187	7.9187
Weight (kg)	**16901**	**16537**	**16328**	**16050**

6 Concluding remarks

In this study, a new efficient optimization algorithm, named as QN-BBBC, is proposed. This algorithm is a hybrid method which is a combination of mathematical method (Quasi-Newton) for local search and a recently developed meta-heuristic algorithm (Big Bang-Big Crunch) for global search. In the mathematical algorithms, a good starting point is vital for these methods to be executed successfully and they may be trapped in local optima. On other hand, frequency constraints are highly non-linear, non-convex and implicit with respect to the design variables. Hence, combining a meta-heuristic algorithm for global search and help to escape the trap seems to be inevitable. In this method, the Big Bang phase is carried out to leave the local optima and Big Crunch phase helps to obtain a value for all candidate solutions as an input to the quasi-Newton method.

The proposed algorithm is applied to the optimization of truss and frame structures with frequency constraints. Comparison of the results with those of the other meta-heuristic methods demonstrates that the proposed approach has a good capability of determining the approximate optimum solutions.

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