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An Investigation of the Recent Developments in Reliability-based Structural Topology Optimization

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Abstract

Optimizing a structure's topology involves finding the best possible distribution of material and connections within a given design space. It has a significant effect on its performance, which is why there has been a meteoric rise in the number of articles published on the topic over the last two decades. This work offers an investigation of reliability-based topology optimization of structures, in light of the recent development of several topology optimization techniques for both linear and nonlinear systems. Therefore, the emphasis of this study is on the latest advancements, enhancements, and applications of reliability-based topology optimization. This paper's primary objective is to provide an overview of the latest advancements in the reliability-based topology optimization of structures, with a particular emphasis on the recent improvement of integrating reliability-based design into the bi-directional evolutionary structural optimization (BESO) method, which accounts for the topological optimization of geometric and material nonlinearity as well as thermoelastic problems.

Keywords

topology optimization, reliability-based design, BESO, geometrically nonlinear, elasto-plastic

1 Introduction

The goal of structural optimization is to find the structure that is the lightest, stiffest, strongest, and most frequency-resistant while still meeting certain requirements like volume, mass, and deflection [1]. At the moment, structural optimization can be split into three different subfields: size optimization, topology optimization, and shape optimization [2–4]. It is important to emphasize that the optimization of topology is distinct from the optimization of size and shape. In shape optimization, design variables are the values that determine the shape of the structure of a given topology. Size optimization uses the geometrical dimensions of structural components as design variables, such as the thickness and diameter of a predefined geometrical shape. Topology optimization (TO) finds the ideal material distribution, hole placement, shape, size, and connection in a design domain. It is the most generic structural optimization method and has no constraints on design domain size or boundary, giving engineers greater design flexibility and search space to find the best solution, particularly during early design [5].

In the presence of sources of uncertainty, the optimum performance attained using typical deterministic approaches for structure optimization problems in the actual world might be drastically diminished. Variations in applied loads, material properties, and manufacturing tolerances may be the cause of uncertainty [6–10]. When the uncertainty comes simply from the variable directions in which a load is applied, Csébfalvi [11] offered a novel metaheuristic technique, where the randomness of the load directions were treated as uncertain but constrained parameters. Besides, Csébfalvi [12] presented benchmark results by considering the worst load direction-oriented technique. Kaveh and Zaerreza [13] introduced a novel approach for reliability-based design optimization (RBDO) that takes into account unpredictable characteristics like applied loads and material properties. Considering that the suggested technique uses decoupled metaheuristic algorithms. Lógó [14] developed a novel kind of probabilistic optimum topology design approach for continuum-type structures with random load application points.

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Furthermore, in the accessible literature, one can find a lot of research work considering the topology optimization of structures in the case of uncertain load conditions [15–20].

This paper aims to present an investigation of recent developments in the field of reliability-based topology optimization of structures, in which the BESO method was developed in the case of geometrically nonlinear elastoplastic designs as well as in thermoelastic structures.

2 State of the art

It is important to treat material properties, external loadings, and the errors of manufacturing as random variables with a specified probability distribution when working with the design of structures. Reliability-based topology optimization (RBTO), which takes into account random variables in the design process, has made significant strides in recent years. So, to take into consideration the uncertainties and potential failure mechanisms, the RBTO formulation includes further limitations on structural performance. However, RBTO approaches are intrinsically computationally costly due to the extra system analysis involved with RBDO and the huge number of design variables inherent in continuous topology optimization problems.

Reliability analysis was integrated into topology optimization problems by Kharmanda et al. [21]. Maute and Frangopol [22] suggested an approach of RBTO that combines recent developments in material-based topology optimization for compliant mechanisms of large displacements. Lógó and Ismail [23] provided an overview of some subjectively chosen examples of state-of-the-art accomplishments in topology optimization throughout the course of its history. Bruggi et al. [24] came up with an algorithm for optimizing the topology of composite structures by taking into account that the loading amplitude is uncertain. Also, Balogh et al. [25] investigated the effects of uncertain load amplitude on the optimization of structures. Based on first-order reliability analysis, Tauzowski et al. [26] suggested a technique for the efficient probabilistic topology optimization of elastoplastic structures. Topology optimization of stress-constrained structures was proposed by Blachowski et al. [27], who advocated for an elastoplastic materials.

Also, by considering the uncertainties of the loading conditions and material properties, Jung and Cho [28] developed RBTO for volume minimization problems. Meng et al. [29] proposed a hybrid RBTO algorithm that takes into account a combining model of fuzzy and probabilistic uncertainties. Li et al. [30] introduced RBTO technique after studying hybrid uncertainties and defect damage in design. Yoo et al. [31] compared the consecutive standard response surface method (SRSM) with the conventional response surface method (RSM) using RBTO based on the BESO approach [31]. Since the computational time is a major factor when dealing with randomness in the case of topology optimization, Jalalpour and Tootkaboni [32] proposed an efficient computational RBTO algorithm which takes into account the randomness of the material Young's modulus.

Recently, RBTO was also integrated into various optimization problems such as in the case of vibrating continuum structures [28–30], buckling constraints [33, 34], and stress constraints [35, 36] problems. However, as was said before, this paper looks at how RBTO has changed recently in the cases of geometrically nonlinear elastic, elastoplastic, and thermoelastic problems. Therefore, the following sections examine these advancements in depth.

3 Theoretical background

3.1 Reliability analysis

Reliability analysis may characterize the failure issue as $\boldsymbol{X}_R \leq \boldsymbol{X}_S$ by assuming that $\boldsymbol{X}_R$ is the non-negative limit for $\boldsymbol{X}_S$. Independent random variables $\boldsymbol{X}_R$ and $\boldsymbol{X}_S$ have probability density functions $\boldsymbol{f}_R(\boldsymbol{X}_R)$ and $\boldsymbol{f}_S(\boldsymbol{X}_S)$ [37]. Then, failure probability ($\boldsymbol{P}_f$) is estimated as:

$$\boldsymbol{P}_f = \boldsymbol{P}\left[\boldsymbol{X}_R \leq \boldsymbol{X}_S\right] = \iint_{X_R \leq X_S} \boldsymbol{f}_R\left(\boldsymbol{X}_R\right) \boldsymbol{f}_S\left(\boldsymbol{X}_S\right) d\boldsymbol{X}_R d\boldsymbol{X}_S. \quad (1)$$

An optional limit state function formulation of the aforementioned issue is:

$$g\left(\boldsymbol{X}_R, \boldsymbol{X}_S\right) = \boldsymbol{X}_R - \boldsymbol{X}_S. \quad (2)$$

$g \leq 0$ defines the failure domain ($\boldsymbol{D}_f$). Thus, $\boldsymbol{P}_f$ is:

$$\boldsymbol{P}_f = \boldsymbol{F}_g\left(0\right), \quad (3)$$

$$\boldsymbol{P}_f = \int_{g\left(X_R, X_S\right) \leq 0} \boldsymbol{f}\left(X\right) dX = \int_{\boldsymbol{D}_f} \boldsymbol{f}\left(X\right) dX. \quad (4)$$

One of the efficient methods to estimate the probability of failure is Monte-Carlo technique. This technique involves creating realizations x of the random vector $\boldsymbol{X}$ out of probability density function $\boldsymbol{f}_x(x)$. $\boldsymbol{P}_f$ is measured through this technique by estimate the proportion of points in the failure domain to all points produced. The following function is called indicator function of $\boldsymbol{D}_f$ in which it can describe the concept:

$$\chi_{D_f}\left(x\right) = \left\{1 \quad if \quad x \in \boldsymbol{D}_f 0 \quad if \quad x \notin \boldsymbol{D}_f\right\}. \quad (5)$$

$\boldsymbol{P}_f$ then is computed by:

$$\boldsymbol{P}_f = \int_{-\infty}^{+\infty} \ldots \int_{-\infty}^{+\infty} \chi_{D_f}(x) \boldsymbol{f}_X(x) dx. \tag{6}$$

Thus, $\boldsymbol{\chi}_{D_f}(X)$ is a two-point random variable function:

$$P\left[\chi_{D_f}(X) = 1\right] = \boldsymbol{P}_f , \tag{7}$$

$$P\left[\chi_{D_f}(X) = 0\right] = 1 - \boldsymbol{P}_f , \tag{8}$$

where $\boldsymbol{P}_f = P[X \in D_f]$. The mean and the variance of $\boldsymbol{\chi}_{D_f}(X)$ are:

$$E\left[\chi_{D_f}(X)\right] = 1 \cdot \boldsymbol{P}_f + 0 \cdot \left(1 - \boldsymbol{P}_f\right) = \boldsymbol{P}_f , \tag{9}$$

$$\begin{aligned} Var\left[\chi_{D_f}(X)\right] &= E\left[\chi_{D_f}^2(X)\right] - (E\left[\chi_{D_f}(X)\right])^2 \\ &= \boldsymbol{P}_f - \boldsymbol{P}_f^2 = \boldsymbol{P}_f(1 - \boldsymbol{P}_f). \end{aligned} \tag{10}$$

The following is a mean value estimator used in Monte-Carlo sampling to calculate failure probability:

$$\hat{E}\left[\chi_{D_f}(X)\right] = \frac{1}{Z} \sum_{z=1}^{Z} \chi_{D_f}\left(X^{(z)}\right) = \hat{\boldsymbol{P}}_f . \tag{11}$$

where $X^{(z)}$ are independent randoms ($z = 1,...,Z$) with *PDF* given by $\boldsymbol{f}_X(x)$.

3.2 Basic concept of BESO

Most of the time, the goal of topology optimization is to find the stiffest structure for a given amount of material. The pioneering study of BESO was carried out by Yang et al. [38]. With BESO methods, an optimized solution is found by taking away and adding elements. That is to say, the design variable is the element itself, not the physical or material parameters that go with it.

Detailed descriptions of the BESO method can be found in the accessible papers and literature. In this section, a summary of the BESO method's foundational concepts is provided. Also, it is important to note that the recent enhancement of BESO in the case of reliability-based design for material nonlinearity and large deformations is discussed later. So, the basic formulation of the optimization problem subjected to the volume constraint is constructed as:

$$Minimize: C = \frac{1}{2} f^T u , \tag{12}$$

$$Subject\ to: V^* - \sum_{i=1}^{N} V_i x_i = 0 , \tag{13}$$

$$\boldsymbol{x}_i = 0 \ \ or \ \ 1 , \tag{14}$$

where f, u, and C are the load vector, the displacement vector and the mean compliance, respectively. Element's volume is denoted by V_i, total structural volume is denoted by V^*. N is the total number of system elements. x_i is a design variable indicating whether an element is absent (0) or present (1).

3.3 Integration of reliability-based design into BESO

To consider reliability design into BESO method we need to define a reliability constraint, one of the suggested techniques is to adopt reliability index as a limit or constraint to control the optimization process:

$$\beta_{target} - \beta_{calc} \leq 0 , \tag{15}$$

where the following equations are used to calculate $\boldsymbol{\beta}_{target}$ and $\boldsymbol{\beta}_{calc}$:

$$\beta_{target} = -\Phi^{-1}\left(\boldsymbol{P}_{f,target}\right), \tag{16}$$

$$\beta_{calc} = -\Phi^{-1}\left(\boldsymbol{P}_{f,calc}\right). \tag{17}$$

Therefore, the optimization problem in the case of reliability-based design is defined as:

$$Minimize: C = u^T K u , \tag{18}$$

$$Subject\ to: V^* - \sum_{i=1}^{N} V_i x_i = 0 , \tag{19}$$

$$\beta_{target} - \beta_{calc} \leq 0 , \tag{20}$$

$$\boldsymbol{x}_i \in \{0,1\} . \tag{21}$$

Here Eqs. (18), (19) and (20) have same roles as ordinary deterministic equations. While Eq. (20) shows the reliability constraint in which in this case is applied on the volume fraction.

4 Recent developments of evolutionary RBTO in the case of mechanical applied loads

As was said in earlier parts of this paper, this work focuses on the current progress of reliability-based evolutionary topology optimization algorithms in the cases of elastoplastic materials, and large displacements problems. Therefore, two different research works are going to be discussed here in this section which are the work of Rad et al. [39], and Habashneh and Rad [40]. It is worth mentioning that since the aim of this paper is to provide an investigation, we only focus on the basic concepts and the results of the proposed algorithms.

4.1 Reliability-based topology optimization of geometrically nonlinear structures

The proposed approach could be seen as a significant step toward a more rational and comprehensive paradigm for the topological design of geometrically nonlinear problems. The work adopted the idea of reliability analysis by assuming the volume fraction which is one of the BESO's parameters as a random variable with two parameters of mean value and standard deviation.

In the case of large displacements consideration, the nonlinear Green-Lagrange finite element (FE) concept was recalled as Eq. (22), and the resultant FE equilibrium Eq. (23) was solved in iterative way by using Newton-Raphson technique. The following equation was used to express the nonlinear Lagrange FE:

$$\eta_{ij} = \frac{1}{2}\left(\boldsymbol{u}_{i,j} + \boldsymbol{u}_{j,i} + \boldsymbol{u}_{k,i}\boldsymbol{u}_{k,j}\right), \tag{22}$$

where $\boldsymbol{u}$ is the point-wise displacement.

$$R(U) = P - \int_V B^T \boldsymbol{s} dV, \tag{23}$$

where P stands for the applied force, B represents the finite element matrix, $\boldsymbol{s}$ is the Piola-Kirchhoff stress vector, and V stands for the undeformed volume.

4.2 Geometrically nonlinear optimization results

A slender rectangular plate problem of geometrically nonlinear elastic design for probabilistic analysis was considered in the work of Rad et al. [39] and Habashneh and Rad [40]. Fig. 1 shows the geometry and the loading conditions of the considered problem.

In this problem, the volume fraction V_f was assumed a random variable with two parameters (mean value and standard deviation). Taking into account that the Monte-Carlo sampling method was used and for reliability purposes. It is worth noting that in the work of Rad et al. [39], two values of $\boldsymbol{\beta}_{target}$ were considered, which were 4.30 and 4.34. While in the work of Habashneh and Rad [40], more than two $\boldsymbol{\beta}_{target}$ values were considered, because one of the primary aims of the work was to shed light on the effects of changing $\boldsymbol{\beta}_{target}$. Therefore, only two values of $\boldsymbol{\beta}_{target}$ will be considered for each work, and a comparison is made according to these results.

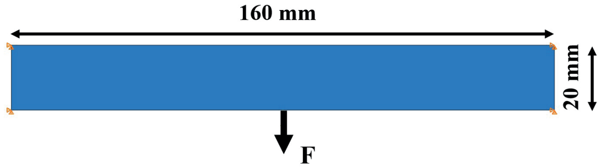


Fig. 1 Simply supported plate problem

The displacement measurements were considered based on the complementary work (W^C) for both problems. So, both works' W^C and their optimized solutions are added up in Table 1. As can be seen, the final optimal solutions for the two different works were different when the designs were deterministic vs. probabilistic. Also, when reliability-based design was integrated into a deterministic problem, the obtained W^C in the case of a probabilistic approach was smaller compared to the deterministic design. In general, the results of probabilistic design suggest that there is an inverse relationship between W^C and $\boldsymbol{\beta}_{target}$.

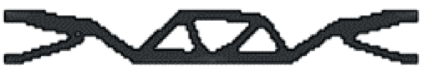
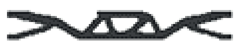
4.3 Reliability-based topology optimization of elastoplastic structures

The second application of RBTO is the optimization problem in the case of elastoplastic models in which the proposed technique adopted the ultimate plastic limit analysis. The static principle of limit analysis was considered to help in the explaining the new technique. This illustrates limit analysis: Consider an elasto-plastic body exposed to a force $\boldsymbol{F}_i$ that increases continuously. Formulating proportional or one-parameter loading:

$$\boldsymbol{F}_i = m\boldsymbol{F}_0, \tag{24}$$

where $\boldsymbol{F}_0$ is the initial external applied forces and m is the load multiplier, a monotonically rising scalar quantity. As m grows, the plastic regions of the body progressively widen, and at a highly defined intensity $\boldsymbol{m}_p$, a condition of

Table 1 Obtained W^C and optimized shapes

Deterministic results	Optimal solution				
	W^C (kJ)	1.43			
Study		Movahedi et al. [39]		Habashneh and Movahedi [40]	
Probabilistic results	$\boldsymbol{\beta}_{target}$	4.30	3.34	3.87	3.15
	Optimal solution				
	(kJ)	1.22	1.41	1.35	1.42

unrestricted plastic flow will finally be established, allowing an increase in plastic stresses and displacements under consistent external pressures throughout the loading process. According to the accepted definition of the plastic limit, the work done by the applied forces is nonnegative, hence $\boldsymbol{m}_s - \boldsymbol{m}_p \leq 0$. Consequently, the elasto-plastic probabilistic optimization problem is formulated as:

$$Minimize : C = u^T Ku , \tag{25}$$

$$Subject\ to : V^* - \sum_{i=1}^{N} V_i x_i = 0 , \tag{26}$$

$$\beta_{target} - \beta_{calc} \leq 0 , \tag{27}$$

$$\boldsymbol{x}_i \in \{0,1\} , \tag{28}$$

$$\boldsymbol{m}_s - \boldsymbol{m}_p \leq 0 . \tag{29}$$

4.4 Elastoplastic optimization results

A 2D L-shaped beam problem of geometrically nonlinear elasto-plastic design for probabilistic analysis, which was considered in the work of Rad et al. [39], is considered as the first example in this section. Fig. 2 shows the geometry and the loading conditions of the considered problem. In this problem, the volume fraction V_f was assumed random variable with mean value and standard deviation. Besides, to show the effect of plastic ultimate load multiplier, four load cases were considered ($\boldsymbol{F}_1 = 0.5\ \boldsymbol{F}_0$, $\boldsymbol{F}_2 = 2\ \boldsymbol{F}_0$, $\boldsymbol{F}_3 = 2.8\ \boldsymbol{F}_0$, $\boldsymbol{F}_4 = 3\ \boldsymbol{F}_0$). The plastic limit load multiplier (m_p) was equal to 4.25 and the initial load ($\boldsymbol{F}_0$) was 4 kN. Taking into account that the Monte-Carlo sampling method was used and for reliability purposes.

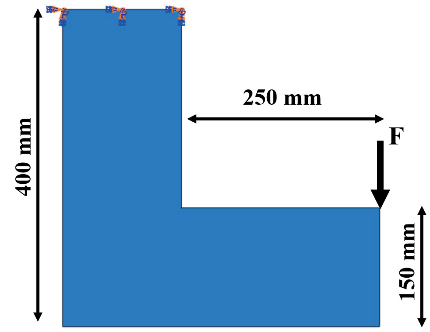
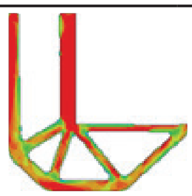
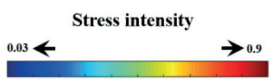
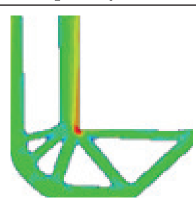
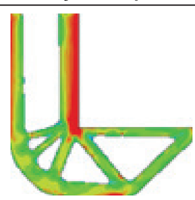
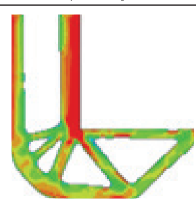
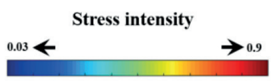


Fig. 2 2D L-shaped beam

A comparison was made to explain how the developed approach enhances the resulting optimal topologies, which are represented in Table 2. In the case of deterministic design, the resulting optimal solution related to the fourth loading case was considered. While in the case of probabilistic design, a specific value of $\boldsymbol{\beta}_{target}$ was considered and the corresponding topologies according to different load cases were discussed. The optimized solutions according to the different load cases show that the absence of plastic zones was particularly evident at the lightest load condition. In contrast, we saw plastic zones in situations two and three. Large plastic zones were achieved in the fourth scenario. Also, the study showed that the number of yielded elements is fewer in the probabilistic situation than in the deterministic case.

The second example in this section is the 2D U-shaped problem of elasto-plastic design for probabilistic analysis was considered in the work of Habashneh and Rad [40]. It is worth mentioning that in this example, the primary aim was to show how the changing of $\boldsymbol{\beta}_{target}$ constraint would affect the results. Fig. 3 shows the geometry and the

Table 2 Yielding conditions

Deterministic results - ($\boldsymbol{F}_4 = 3\boldsymbol{F}_0$)				
				
Probabilistic results - ($\boldsymbol{\beta}_{target} = 3.79$)				
Ratio of the applied load	$\boldsymbol{F}_1 = 0.5\boldsymbol{F}_0$	$\boldsymbol{F}_2 = 2\boldsymbol{F}_0$	$\boldsymbol{F}_3 = 2.8\boldsymbol{F}_0$	$\boldsymbol{F}_4 = 3\boldsymbol{F}_0$
Optimal solution				
				

loading conditions of the considered problem. In this problem, V_f was assumed random variable with mean value and standard deviation. Besides, to show the effect of plastic ultimate load multiplier, four load cases were considered ($\boldsymbol{F}_1 = 0.9\ \boldsymbol{F}_0$, $\boldsymbol{F}_2 = 1.125\ \boldsymbol{F}_0$, $\boldsymbol{F}_3 = 1.875\ \boldsymbol{F}_0$, $\boldsymbol{F}_4 = 3.125\ \boldsymbol{F}_0$). The plastic limit load multiplier (m_p) was equal to and the initial load ($\boldsymbol{F}_0$) was 4 kN. Taking into account that the Monte-Carlo sampling method was used and for reliability purposes. Table 3 shows the obtained results of the proposed technique according to the concept of load multiplier with considering various values of $\boldsymbol{\beta}_{target}$. It can be summed up by saying that the increasing in the applied load was resulted in the increasing of the percentage of the yielded elements within the models.

Furthermore, the effect of changing $\boldsymbol{\beta}_{target}$ which it was related to the changing of V_f was clear since the resulted topologies were changed as $\boldsymbol{\beta}_{target}$ changed for each load condition.

5 Recent developments of evolutionary RBTO in the case of thermal applied loads

The third paper to consider in this investigation is the developed BESO method to consider reliability-based thermoelastic problems which was done by Habashneh and Rad [41]. The proposed approach integrated the concept of the reliability-based topology optimization into geometrically nonlinear thermoelastic problems. The proposed work assumed the volume fraction random variable to represent the uncertainties.

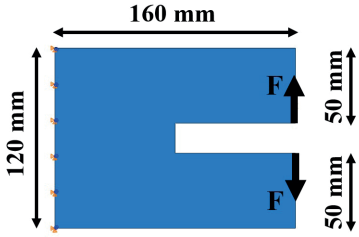


Fig. 3 2D U-shaped problem

5.1 Developed formulations

The problem of thermoelastic reliability-based optimization is constructed as:

$$Minimize: C = u^T Ku\,, \tag{30}$$

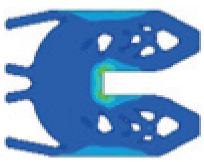
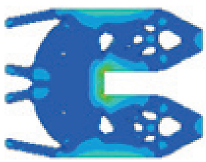
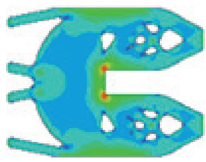
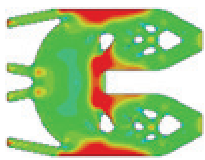
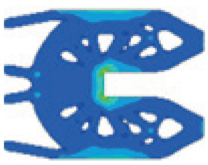
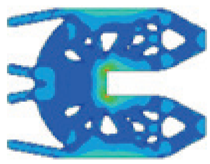
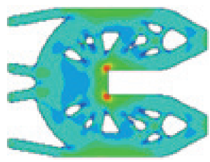
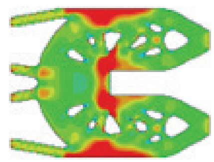
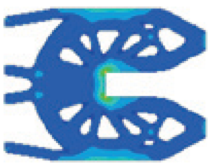
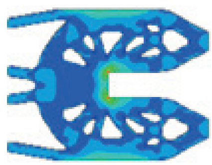
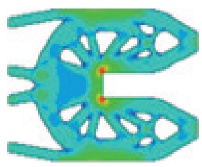
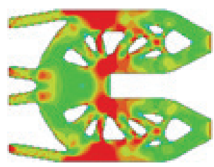
$$Subject\ to: V^* - \sum_{i=1}^{N} V_i x_i = 0\,, \tag{31}$$

$$\boldsymbol{x}_i \in \{0,1\}\,, \tag{32}$$

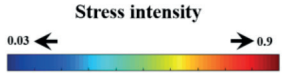
$$Ku = f\,, \tag{33}$$

$$\beta_{target} - \beta_{calc} \le 0\,, \tag{34}$$

Table 3 Mean stress and optimal topologies according to various load multiplier and $\boldsymbol{\beta}_{target}$

$\boldsymbol{\beta}_{target}$		Applied load			
		$\boldsymbol{F}_1 = 0.9\ \boldsymbol{F}_0$	$\boldsymbol{F}_2 = 1.125\ \boldsymbol{F}_0$	$\boldsymbol{F}_3 = 1.875\ \boldsymbol{F}_0$	$\boldsymbol{F}_4 = 3\ \boldsymbol{F}_0$
4.83	Optimal solution				
	Mean stress	179.47	190.24	194.61	219.74
4.41	Optimal solution				
	Mean stress	181.08	190.33	195.60	224.07
3.64	Optimal solution				
	Mean stress	183.78	190.44	196.77	225

Stress intensity



where f is the vector of loads and u is the vector of displacements. It's important to note that the loading vector $\boldsymbol{f} = \boldsymbol{f}_m + \boldsymbol{f}_{th}$ includes not only the mechanical loading $\boldsymbol{f}_m$ but also the thermal loading $\boldsymbol{f}_{th}$. A semi-coupled thermoelasticity theory was considered. First, the heat regulating equations had to be solved to produce the temperature field. The total response of the elastic body was then calculated by adding the forces exerted on it by the temperature field to those exerted by all other forces.

As a result, the given objective function considers the thermal expansion displacements. It is also worth noting that Eqs. (30), (31), (32), and (34) have the same functions as Eqs. (18), (19), (21), and (20), respectively. While Eq. (33) shows the new conditions which is related to thermomechanical loading.

5.2 Thermoelastic optimization results

The considered example to show the results of the recent developed work is a reliability-based topology optimization problem considering a geometrically nonlinear 2D L-shaped beam. Fig. 4 shows the geometry and load conditions of the considered optimization problem.

As mentioned in the previous section, the applied load (F) is the sum of thermal and mechanical load. Taking into consideration that the four different thermal loads were ΔT = 0 °C, 5 °C, 10 °C and 15 °C. The resulted layouts in the case of probabilistic designs were compared with those which was obtained in the case of deterministic designs for both linear and geometrically nonlinear analysis. The displacement measurements were also considered in the proposed work based on W^C. The obtained optimized shapes and the resulted W^C according to two various V_f for deterministic design and according to two different $\boldsymbol{\beta}_{target}$ in the case of linear designs are presented in Table 4. While the obtained optimized shapes and the resulted W^C in the case of geometrically nonlinear designs are presented in Table 5.

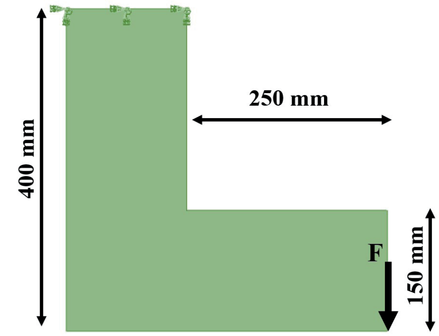


Fig. 4 2D L-shaped beam model

Table 4 Optimized shapes and complementary work –linear design

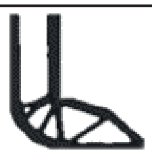
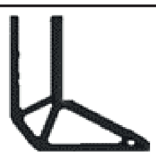
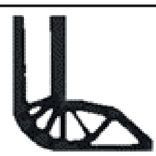
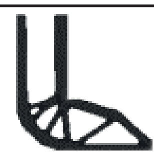
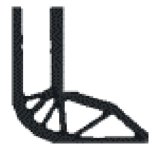
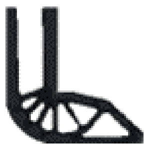
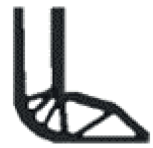
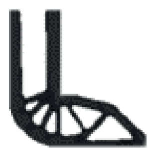
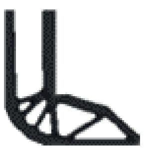
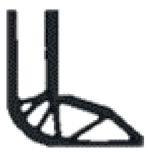
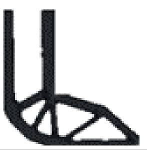
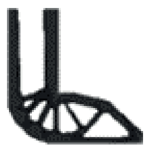
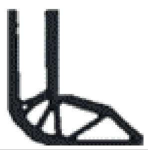
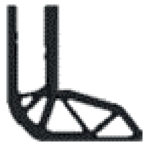
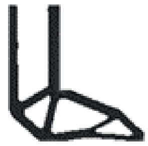
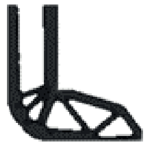
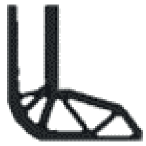
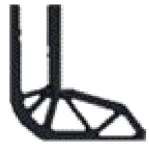
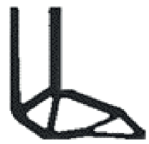
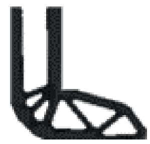
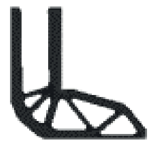
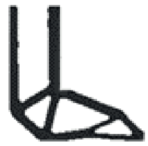
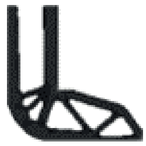
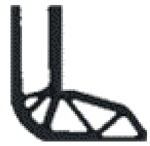
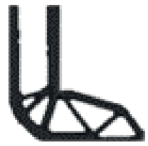
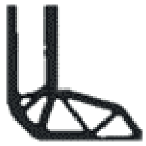
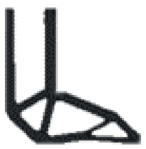
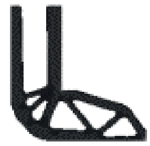
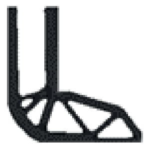
F / ΔT	12 kN							
	Deterministic				Probabilistic			
	V_f				$\boldsymbol{\beta}_{target}$			
	0.45		0.40		5.61		3.23	
	Optimized shapes	W^C (kJ)	Optimized shapes	W^C (kJ)	Optimized shapes	W^C (kJ)	Optimized shapes	W^C (kJ)
0 °C		25.92		28.82		23.11		25.31
5 °C		26.02		28.88		23.31		25.56
10 °C		26.13		29.31		23.31		25.56
15 °C		26.28		29.71		23.49		25.67

Table 5 Optimized shapes and complementary work –geometrically nonlinear design

F	12 kN							
ΔT	Deterministic				Probabilistic			
	V_f				β_{target}			
	0.45		0.40		5.61		3.23	
	Optimized shapes	W^C (kJ)	Optimized shapes	W^C (kJ)	Optimized shapes	W^C (kJ)	Optimized shapes	W^C (kJ)
0 °C		25.41		28.78		23		24.82
5 °C		25.74		28.83		23.02		24.94
10 °C		26.17		29.25		23.19		25.09
15 °C		26.25		29.64		23.43		25.25

According to the obtained results, we can say that introducing reliability constraint into deterministic design effectively changed the final optimized shapes of the model. Furthermore, the optimum shapes that emerged for each scenario of the applied thermal load demonstrated that considering varied β_{target} would alter the optimized shape according to probabilistic design. The less complementary work needed by nonlinear designs shows that they operate better under load situations. This emphasizes the necessity for a nonlinear topology optimization approach for large-displacement cases like those with snap-through effects, where the linear design integrating buckling and snap-through effects does more complementing work than the nonlinear one.

6 Conclusions

The proposed work in this paper is a survey of recent developments in evolutionary algorithms for reliability-based structural topology optimization. The presented work also seeks to highlight the breadth of themes and advancements addressed in reliability-based topology optimization. Different mathematical formulations for the most recent developed evolutionary optimization algorithms in the case of reliability-based designs were examined. Furthermore, the obtained results of these algorithms were considered for discussion. A lot of effort has been put into RBTO already. Considering that uncertainty may come from several places, including the load, the material, and the geometry, thus several methods have been developed to account for these factors. RBTO found its way in the problems of compliant mechanisms considering geometrically and materially nonlinear problems. From the detailed reviews, it can be noted that when introducing a reliability-based design into elastoplastic problems, the yielded state became smaller than in the deterministic case, which can effectively show how the reliability constraint works as a limit to control the optimization of structures. Also, it is noted that the changing of reliability constraints can effectively change the resulted topological shapes and the stress distribution of the models.

Reliability-based BESO is a relatively new field that combines the principles of structural optimization and reliability analysis. The goal of Reliability-based BESO is to improve the reliability of engineering structures by optimizing their design using evolutionary algorithms. As with any emerging field, and based on current trends, it is likely that Reliability-based BESO will continue to grow in importance and become an increasingly popular tool in the field of engineering design. One reason for this is that it has already demonstrated significant potential in a range of applications, including in the aerospace, automotive, and civil engineering industries. By optimizing the design of structures using BESO, engineers can improve their performance while minimizing their weight and cost. Another reason for the potential growth of Reliability-based BESO is the increasing availability of computational resources. As the power and availability of high-performance computing continue to increase, such algorithms will become more efficient and effective, allowing for more complex optimization problems to be tackled.

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